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Strategic product R&D investment policy under international rivalry in the presence of demand spillover effects

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Abstract

This paper first presents the optimal conditions for strategic R&D investment policy in the cases of noncooperative and cooperative R&D investment policies with international rivalry. Then we deal with a model of strategic product (i.e., quality-improving) R&D investment competition. In particular, we analyze an optimal R&D investment policy with regard to the two cases in the presence of demand spillover effects associated with improving the quality of a product. We show how optimality depends on the strength of demand spillover effects. We also consider the same problems assuming heterogeneous consumers and alternative utility functions.

Keywords: strategic R&D investment policy; quality choice; international rivalry; demand spillover effects

JEL classification: F12, F13, L13
1. Introduction

The purpose of this paper is to analyze strategic product (i.e., quality-improving) R&D investment policy in a standard framework of a third-country market. In particular, we use a horizontal product differentiation model that includes endogenous quality choice. Furthermore, assuming the presence of demand spillover effects associated with improving the quality of a product, we consider how demand spillovers affect optimality of R&D investment policy in the cases of a noncooperative and a cooperative policy decision. Thus, we show that the optimality depends on the strength of demand spillover effects.

Seminal papers that deal with Cournot duopoly in a homogeneous product market and consider strategic process (i.e., cost-reducing) R&D investment competition include Brander and Spencer (1983), Spencer and Brander (1983), and others. These authors analyze how R&D investments affect the environment for market competition. In addition, they show that an R&D investment subsidy is an optimal policy in the case of international rivalry.

With regard to the literature that analyzes product R&D investment competition, i.e., endogenous quality choice, based on a model of vertical product differentiation, some researchers have analyzed strategic product R&D investment policy with international rivalry, for example, Park (2001), Zhou et al. (2002), and Jinji (2003). These researchers implicitly assume the case of partial market coverage, in which there are some consumers who do not purchase any products. However, in the case of full market coverage, in which all consumers purchase either any product, a firm producing a low-quality product chooses the lower limit quality level, i.e., a corner solution. Thus, the government’s R&D investment policy does not affect the low-quality firm’s activity. However, assuming a Hotelling model with quality choice (e.g., Sanjo, 2007; Ishibashi and Kaneko, 2008) and a model of a horizontally and vertically differentiated products market (e.g., André, et al., 2009), we examine the case of
complete market coverage.

As mentioned above, introducing demand spillover effects into a model of horizontally differentiated products with quality choice (e.g., Häckner, 2000; Symeonidis, 2003), we address strategic product R&D investment policy. Formally, our model is similar to d’Aspremont and Jacquemin (1988) and De Bondt and Henriques (1995), in which they treat the spillover effect of cost-reducing R&D investment on the marginal cost of production, i.e., the technology-side spillover effect. Closely related to our model dealing with the spillover effect on the demand side associated with improving the quality of horizontally differentiated products, we have Foros et al. (2002) and Stühmeier (2012), in which they discuss the role of roaming endogenously determined by competitive firms in a mobile phone industry. Roaming in their models corresponds to the parameter indicating demand spillover effects in our model.

The remainder of this paper is structured as follows. In Section 2, as preliminary, we present the optimal conditions with respect to a noncooperative and a cooperative R&D investment policy. In Section 3, we consider strategic product R&D investment policy in the presence of demand spillover effects and then examine the optimal policy. Furthermore, in Section 4, assuming heterogeneous consumers and alternative utility functions associated with a horizontal product differentiation model with quality choice, we address the same problems as in the previous section. Section 5 summarizes our results and raises remaining issues.

2. Preliminary: The conditions of an optimal R&D investment policy

2.1 R&D investment competition with international rivalry

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1 Symeonidis (2003) deals with the spillover effect of quality-improving R&D investment on the investment costs.
To consider a strategic product R&D investment policy, we employ a standard framework of the third-country market. That is, there are two countries, home and foreign, in which one firm exists respectively. The firm located in each country sells the product to the third-country market, but it does not sell it to the markets in the home and foreign countries. Hereafter, the firm located in the home (foreign) country is denoted as firm $I$ ($2$).

The firms compete in a two-stage game. In stage 1, each firm simultaneously chooses its investment level, which affects the market competition in the subsequent stage. The investment can be interpreted as R&D. In stage 2, each firm simultaneously chooses its market action, i.e., either price or output. As analyzed below, the home and foreign governments either noncooperatively or cooperatively commit to an R&D investment policy in stage 0, prior to the game played by the firms. Throughout, we restrict our attention to pure-strategy equilibria and focus on the interior solutions. We confine our attention to subgame-perfect equilibria in two stages by solving the model backward.

The Nash equilibrium outcome for firm $i$ in stage 2 (i.e., product market competition) is expressed by the equilibrium revenue, $R_i[z_i, z_j], i, j = 1, 2, i \neq j$, where $z_i$ is firm $i$’s activity (e.g., investment and quality levels) and $z_j$ is the rival firm’s activity. Furthermore, the investment cost function of firm $i$ is given by $F_i[z_i]$, where $F_i[0] = F_i'[0] = 0$, $F_i'[z_i] > 0$, $F_i''[z_i] > 0$, for $z_i > 0$, $\lim_{z_i \to \infty} F_i'[z_i] = \infty$, and $F_i'''[z_i] \geq 0$. Thus, the profit function of firm $i$ in stage 1 is expressed as

$$\Pi_i[z_i, z_j] = R_i[z_i, z_j] - (1 - s_i)F_i[z_i], i, j = 1, 2, i \neq j,$$

where $s_i(< 1)$ is an investment subsidy from the government of country $i$. In addition, negative $s_i$ implies an investment tax.

The first-order condition (FOC) to maximize the profit of firm $i$ is given by
\[ \frac{\partial \Pi_i}{\partial z_i} = \frac{\partial R_i}{\partial z_i} - (1 - s_i)F_i' = 0, \quad i, j = 1, 2, i \neq j. \] (1)

For the following analysis, taking eq. (1) into account, we derive the effects of a subsidy on the activities of the two firms as

\[ \frac{\partial z_i}{\partial s_i} = -\frac{F_i'}{\Delta} \frac{\partial^2 \Pi_j}{\partial z_i^2} > 0, \] (2)

\[ \frac{\partial z_j}{\partial s_i} = \frac{F_i'}{\Delta} \frac{\partial^2 \Pi_j}{\partial z_j \partial z_i} \geq (\leq)0 \Leftrightarrow \frac{\partial^2 \Pi_j}{\partial z_j \partial z_i} \geq (\leq)0, \] (3)

where \( \Delta = \frac{\partial^2 \Pi_i}{\partial z_i^2} \frac{\partial^2 \Pi_j}{\partial z_j^2} - \frac{\partial^2 \Pi_i}{\partial z_j \partial z_i} \frac{\partial^2 \Pi_j}{\partial z_j \partial z_i} > 0, \quad i, j = 1, 2, \ i \neq j. \)

It is clear in eq. (2) and eq. (3) that a subsidy of government \( i \) always increases firm \( i \)'s activity, whereas the effect on firm \( j \)'s activity depends on the sign of the cross effect of its profit function.

2.2 The optimal R&D investment policy in the noncooperative case

We consider the unilaterally set R&D investment policy at stage 0, i.e., noncooperatively, government \( i \) chooses an investment subsidy/tax on its domestic firm, \( s_i \), to maximize domestic social welfare, \( W_i \), which is given by

\[ W_i[s_i] = \Pi_i[z_i, z_j] - s_i F_i[z_i] = R_i[z_i, z_j] - F_i[z_i], \]

where \( z_i = z_i[s_i, s_j] \), \( i, j = 1, 2, \ i \neq j. \)

Based on the FOC to maximize the social welfare of country \( i \), i.e., \( \frac{\partial W_i}{\partial s_i} = 0 \), given \( z_j \), we derive the optimal R&D investment policy in the noncooperative case as
Based on eq. (2) and eq. (3), eq. (4) can be rewritten as

\[
\begin{aligned}
    s_i^* &= \frac{1}{F_i} \left( \frac{\partial R_i}{\partial z_j} \left( \frac{\partial z_i}{\partial s_i} \right) \right) \\
    &= \frac{1}{F_i} \left( \frac{\partial R_i}{\partial z_j} \left( \frac{\partial^2 \Pi_j}{\partial z_j^2} \right) - \frac{\partial^2 \Pi_j}{\partial z_j \partial z_i} \right) \\
    &= \frac{1}{F_i} \left( \frac{\partial R_i}{\partial z_j} \right) \left( \frac{dz_j}{dz_i} \right).
\end{aligned}
\]

In this case, we obtain

\[
\begin{aligned}
    \text{sign}(s_i^*) &= \text{sign} \left( \frac{\partial R_i}{\partial z_j} \left( \frac{dz_j}{dz_i} \right) \right). \tag{5}
\end{aligned}
\]

Because it holds that \( \text{sign} \left( \frac{dz_j}{dz_i} \right) = \text{sign} \left( \frac{\partial^2 \Pi_j}{\partial z_j \partial z_i} \right) \), eq. (5) can be rewritten as

\[
\begin{aligned}
    \text{sign}(s_i^*) &= \text{sign} \left( \frac{\partial R_i}{\partial z_j} \left( \frac{\partial^2 \Pi_j}{\partial z_j \partial z_i} \right) \right). \tag{6}
\end{aligned}
\]

Therefore, as already shown in Spencer and Brander (1983), Park (2001), Toshimitsu and Jinji (2008), and others, in view of eq. (6), we summarize as follows.

**Lemma 1**

A noncooperative optimal R&D investment policy by the home government depends on

(i) the sign of the externality of the foreign firm’s R&D activities toward the home firm,

i.e., \( \frac{\partial R_i}{\partial z_j} \), \( i, j = 1, 2, i \neq j \), and
(ii) the sign of the slope of the foreign firm’s R&D investment reaction curves, i.e., \( \frac{dz_j}{dz_i} \).

In other words, the sign of the cross effect of the foreign firm’s profit function, i.e.,

\[
\frac{\partial^2 \Pi_j}{\partial z_j \partial z_i} \left( = \frac{\partial^2 R_j}{\partial z_j \partial z_i} \right), \quad i, j = 1, 2, i \neq j,
\]

where firm \( i \) (\( j \)) is the home (foreign) firm.

Lemma 1 (ii) implies that the strategic relationship between firms is either substitute or complement, i.e., \( \frac{dz_j}{dz_i} < 0 \) or \( \frac{dz_j}{dz_i} > 0 \), \( i, j = 1, 2, i \neq j \).

2.3 The optimal R&D investment policy in the cooperative case

In the case of a cooperative R&D investment policy by both governments, we assume that each government decides an R&D investment policy to maximize joint social welfare of both countries, given by

\[
W[s_1, s_2] \equiv W_1[s_1, s_2] + W_2[s_1, s_2] = \Pi_1[s_1, s_2] + \Pi_2[s_1, s_2]
\]

That is, as shown in the equation above, the joint welfare is the same as the joint profits of both firms in the framework of the third-country market.

Thus, the FOC to maximize the joint social welfare with respect to the cooperative R&D investment policy of the home government is represented as

\[
\frac{\partial W}{\partial s_1} = \left( \frac{\partial R_2}{\partial z_1} - s_1 F_1' \right) \left( \frac{\partial z_1}{\partial s_1} \right) + \left( \frac{\partial R_1}{\partial z_2} - s_2 F_2' \right) \left( \frac{\partial z_2}{\partial s_1} \right) = 0. \tag{7}
\]

Similarly, for the cooperative R&D investment policy of the foreign government, we have
\[
\frac{\partial W}{\partial s_2} = \left( \frac{\partial R_2}{\partial z_1} - s_1 F_1' \right) \frac{\partial z_1}{\partial s_2} + \left( \frac{\partial R_1}{\partial z_2} - s_2 F_2' \right) \frac{\partial z_2}{\partial s_2} = 0. \tag{8}
\]

Taking eq. (2), eq. (3), eq. (7), and eq. (8) into account, an optimal cooperative R&D investment policy is given by

\[
s_i^C = \frac{1}{F_i} \left( \frac{\partial R_j}{\partial z_i} \right), \quad i, j = 1, 2, i \neq j, \tag{9}
\]

where superscript \(C\) denotes the cooperative case. Therefore, we sum up the result as follows.

**Lemma 2**

*An optimal cooperative R&D investment policy by the home government depends on the sign of the externality of the home firm’s R&D activities toward the foreign firm, i.e., \(\frac{\partial R_j}{\partial z_i}\), \(i, j = 1, 2, i \neq j\), where firm \(i\) (\(j\)) is the home (foreign) firm.*

3. **Optimal R&D investment policy and quality competition**

3.1 **Horizontal product differentiation model with quality choice and demand spillover effects**

Here we employ the same setting as in Section 2: that is, there are two countries, home and foreign, in which one firm exists, respectively. The firms compete in a two-stage game. In stage 1, a firm simultaneously chooses the quality level, hereafter, \(z_i = q_i\), \(i = 1, 2\). In stage 2, a firm simultaneously chooses the price, \(p_i\), \(i = 1, 2\).

We assume that the utility of a representative consumer in the third-market is given by a

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2 This section is based on Toshimitsu (2012).
quality-augmented version of a standard quasilinear function, which is attached with the term of demand spillover effects by improving quality as

\[ V = U + y, \]

\[ U = \left\{ \alpha(x_1 + x_2) - \frac{1}{2}(x_1^2 + x_2^2) - \theta x_1 x_2 \right\} + \{(q_1 + \epsilon_2 q_2)x_1 + (q_2 + \epsilon_1 q_1)x_2 \}, \quad (10) \]

where \( y \) is the numeral good, and \( p_y = 1 \). In addition, \( \alpha > 0 \) and \( 1 > \theta \geq 0 \). \( \theta \) is a parameter indicating substitutability between the products. In particular, for the second term composed of quality arguments in eq. (10), we assume that \( \epsilon_i, i = 1,2 \ (1 > \epsilon_i \geq 0) \), implies demand spillover effects of an increase in the quality level of product \( i^3 \).

The budget constraint is given by \( I \geq p_1 x_1 + p_2 x_2 + x_0 \), so that we obtain the optimal behavior of a representative consumer as

\[ \frac{\partial U}{\partial x_i} = \alpha + q_i + \epsilon_j q_j - x_i - \theta x_j = p_i, \quad i, j = 1,2, i \neq j. \]

Thus, the demand function of product \( i \) is given by

\[ x_i = \frac{\alpha(1 - \theta) + (1 - \theta \epsilon_i)q_i + (\epsilon_j - \theta)q_j - p_i + \theta p_j}{(1 - \theta)(1 + \theta)}, \quad i, j = 1,2, i \neq j. \quad (11) \]

Given eq. (11), we have

\[ \frac{\partial x_i}{\partial q_j} \geq (\leq)0 \iff \epsilon_j \geq (\leq)\theta, \quad i, j = 1,2, i \neq j. \quad (12) \]

As shown in eq. (12), if the strength of demand spillover effects of product \( j \)'s quality is larger (smaller) than substitutability, an increase in the quality level increases (decreases) the demand of the rival firm \( i \).

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3 If \( \epsilon_i = 0 \), the utility function is formally similar to that of Häckner (2000). See also the appendix in Symeonidis (2003).
3.2 The analysis

In Stage 2, firm $i$ decides the price to maximize the profit, i.e., \[ \Pi_i = p_i x_i - (1 - s_i)F_i[q_i]. \]

The FOC to maximize the profit of firm $i$ is expressed by \[ \frac{\partial \Pi_i}{\partial p_i} = x_i - \Lambda = 0, \]

where \[ \Lambda = \frac{1}{(1 - \theta)(1 + \theta)} > 0. \] Thus, taking eq. (11) into account, we have

\[ \alpha(1 - \theta) + (1 - \theta \epsilon_i)q_i + (\epsilon_j - \theta)q_j - 2 p_i + \theta p_j = 0, \quad i, j = 1, 2, i \neq j. \quad (13) \]

Therefore, the price of product $i$ in the Nash equilibrium is given by

\[ p_i = \frac{\Lambda + \Phi_j q_i + \Gamma_j q_j}{(2 - \theta)(2 + \theta)}, \quad i, j = 1, 2, i \neq j, \quad (14) \]

where \[ \Lambda = \alpha(1 - \theta)(2 + \theta) > 0, \quad \Phi_i = 2 - \theta \epsilon_i - \theta^2 > 0, \quad \text{and} \quad \Gamma_j = (2 - \theta^2) \epsilon_j - \theta. \]

Hereafter, we omit the indexes of the firms in each equation, i.e., \[ i, j = 1, 2, i \neq j, \] unless we refer to them specifically.

In this case, it holds that

\[ \Gamma_j \geq (\leq) 0 \iff \Theta[\epsilon_j] \geq (\leq) \theta, \quad \text{where} \quad \Theta[\epsilon_j] = \frac{\sqrt{-2} \epsilon_j^2 + 8 - \epsilon_j^{-1}}{2} < 1, \quad \text{and} \quad \Theta'[\epsilon_j] > 0. \quad (15) \]

Eq. (15) shows that if the parameter composed of demand spillovers with product $j$'s quality, i.e., \[ \Theta[\epsilon_j] \], is larger (smaller) than a certain value of the substitutability, \( \theta \), then quality improvement of product $j$ raises the price of product $i$ as well as product $j$. That is, firm $i$ raises (reduces) the price, because the positive effect by demand spillovers is larger (smaller) than the negative effect by substitutability between the products.

In Stage 1, firm $i$ decides the quality level that maximizes the profit, which is given by
\[ \Pi_i = R_i[q_i, q_j] - (1 - s_i)F_i[q_i], \quad \text{where} \quad R_i[q_i, q_j] = \Lambda p_i^2. \]

Thus, the FOC is given by
\[
\frac{\partial \Pi_i}{\partial q_i} = 2\Lambda p_i \frac{\Phi_i}{(2 - \theta)(2 + \theta)} - (1 - s_i)F_i' = 0.
\]

In addition, the second-order condition is given by
\[
\frac{\partial^2 \Pi_i}{\partial q_i^2} = 2\Lambda \left\{ \frac{\Phi_i}{(2 - \theta)(2 + \theta)} \right\}^2 - (1 - s_i)F_i'' < 0.
\]

Furthermore, with respect to the cross effect, we obtain
\[
\frac{\partial^2 \Pi_i}{\partial q_i \partial q_j} = \frac{\partial^2 R_i}{\partial q_i \partial q_j} = 2\Lambda \left\{ \frac{\Phi_i}{(2 - \theta)(2 + \theta)} \right\} \left\{ \frac{\Gamma_j}{(2 - \theta)(2 + \theta)} \right\}. \quad (16)
\]

Thus, it holds that
\[
\frac{\partial^2 R_i}{\partial q_i \partial q_j} \geq (<)0 \iff \Gamma_j \geq (<)0. \quad (17)
\]

Eq. (17) implies that the slope of reaction functions, in other words, the strategic relationships between the firms, depends on the strength of a demand spillover effect of the rival firm. Furthermore, the external effect on the profit (revenue) is given by
\[
\frac{\partial \Pi_i}{\partial q_j} = \frac{\partial R_i}{\partial q_j} = 2\Lambda p_i \frac{\Gamma_j}{(2 - \theta)(2 + \theta)}.
\]

Thus, we obtain
\[
\frac{\partial R_i}{\partial q_j} \geq (<)0 \iff \Gamma_j \geq (<)0. \quad (18)
\]

Eq. (19) illustrates the external effect on the firm’s revenue. The sign depends on the strength of demand spillover effects of the rival firm only, irrespective of the mode of market competition.

For the following analysis, we assume as follows.
Assumption 1: Asymmetric demand spillover effects

If the demand spillover effect of product 1 is larger than that of product 2, i.e., $\varepsilon_1 > \varepsilon_2$, then it holds that $\Theta_1 = \Theta[\varepsilon_1] > \Theta_2 = \Theta[\varepsilon_2]$.

Taking into account eq. (15), eq. (17), eq. (19), and Assumption 1, I derive the following lemma.

Lemma 3

1. If $\Theta_1 > \Theta_2 > \theta$, each firm’s reaction curve is sloping upward. Hence, an increase in the quality of product 2 (1) increases the revenue of firm 1 (2).

2. If $\theta > \Theta_1 > \Theta_2$, each firm’s reaction curve is sloping downward. Hence, an increase in the quality of product 2 (1) decreases the revenue of firm 1 (2).

3. If $\Theta_1 > \Theta > \Theta_2$, firm 1’s reaction curve is sloping downward, whereas firm 2’s is sloping upward. Hence, an increase in the quality of product 2 (1) decreases (increases) the revenue of firm 1 (2).

For Lemma 3 (1) (Lemma 3 (2)), if the strength of demand spillovers of both firms’ products is larger (smaller) than a certain value of substitutability, an increase in the quality level of one firm expands (reduces) the rival firm’s demand, and this, in turn, increases (decreases) the rival firm’s price and quality. Thus, the rival firm’s profit increases (decreases). Similarly, the firm’s profit increases (decreases) with an increase in the rival firm’s quality. Therefore, the strategic complementary (substitutionary) relationship between the firms holds. See Figure 1 (Figure 2).
Furthermore, for Lemma 3 (3), when the strength of demand spillovers is asymmetric between the firms, i.e., the strength of a demand spillover effect of product 1 is larger and that of product 2 is smaller than a certain value of substitutability, an increase in product 1 expands firm 2’s demand, while an increase in product 2 reduces firm 1’s demand. In this case, firm 2 increases its price and quality, while firm 1 decreases its price and quality. As a result, firm 2’s profit increases, while firm 1’s falls. Thus, the strategic substitutionary relationship sustains for firm 1, while the strategic complementary relationship sustains for firm 2. See Figure 3.

3.3 The optimal R&D investment policy in the noncooperative case

An optimal R&D investment policy in the noncooperative case is one in which the government makes the domestic firm a leader in a Stackelberg game. This aspect has already been addressed in the context of strategic trade and industrial policies. Therefore, based on Lemmas 1 and 3, we easily derive the following results.

Proposition 1

(1) If either $\Theta_1 > \Theta_2 > \theta$ or $\theta > \Theta_1 > \Theta_2$, then a noncooperative R&D investment subsidy is optimal, i.e., $s_i^* > 0$, $i = 1,2$.

(2) If $\Theta_1 > \theta > \Theta_2$, then a noncooperative R&D investment tax is optimal, i.e., $s_i^* < 0$, $i = 1,2$. 

Place Figures 1 and 2 approximately here

Place Figure 3 approximately here
First, as to Proposition 1 (1), if the strength of demand spillovers of both firms is either larger or smaller than a certain value of substitutability of the products, each government gives its domestic firm a subsidy to upgrade its quality level. Incidentally, in the case of a large enough demand spillover effect, subsidizing the domestic firm increases the foreign firm’s quality level as well as that of the domestic firm. This, in turn, leads to an increase in the profits of both firms and thus the welfare of both countries (see point $S_1$ in Figure 1).

However, in the case of a smaller demand spillover effect, subsidizing the domestic firm reduces the foreign firm’s quality level, but increases that of the domestic firm. This, in turn, leads to a decrease in the foreign firm’s profit, and an increase in that of the domestic firm (see point $S_1$ in Figure 2). This case is similar to that of Spencer and Brander (1983), in which they consider the cost-reducing (i.e., process) R&D investment policy in a Cournot competition.

In the case of a large demand spillover effect, because the market size increases through quality improvement of the products, both governments will subsidize the domestic firm. On the other hand, in the case of a small demand spillover effect, because market size does not increase through quality improvement of the products, but the firm’s share reduces by an increase in the quality level of the rival firm’s product, both governments will subsidize the domestic firm to expand its market share.

Second, as to Proposition 1 (2), in the case of asymmetric demand spillovers, each government taxes the domestic firm to reduce its quality level. That is, a decrease in the quality level of firm 1 (2) reduces (increases) the quality level of firm 2 (1). In this case, the profit of firm 1 (2) increases by the negative (positive) externality effect on the revenue of
firm 1 (2). However, a country’s R&D investment tax policy, in which the firm produces the product associated with a larger (smaller) demand spillover effect, reduces (increases) the profit of the other firm, and thus the welfare of the other country. See point $S_1$ ($S_2$) in Figure 3.

Taxing firm 1 producing a large demand spillover product reduces the quality level of the rival firm 2 as well as its quality. This mitigates the degree of a fall of the price of firm 1 and expands its share. Accordingly, R&D tax policy increases the profit of firm 1 and thus the welfare of the government, although it reduces the profit of the rival firm 2.

On the contrary, taxing firm 2 producing a small demand spillover product reduces its quality level, but increases the quality level of the rival firm 1. For firm 2, this mitigates the degree of a fall of the price of firm 2 and expands its demand. In addition, the profit of firm 1 increases. Accordingly, the R&D tax policy increases the profits of both firms and thus the welfare of both countries. This implies that the R&D tax on the firm producing a small demand spillover product is Pareto improving.

3.4 The optimal R&D investment policy in the cooperative case

An optimal cooperative R&D investment policy of the governments is determined by joint welfare maximization. This implies that both firms collusively determine their quality levels to maximize their joint profits; however, they noncooperatively compete in prices in the product market\(^4\). Therefore, both firms choose the quality level existing in Pareto superior sets, which are the shaded areas surrounded by the iso-profit curves of both firms, i.e., $\Pi_i^N, i = 1,2$. For example, in the case of large demand spillover effects, i.e.,

\(^4\) This situation illustrates semicollusion similar to the cooperative R&D investment considered by d’Aspremont and Jacquemin (1988) and others.
If $\Theta_1 > \Theta_2 > \theta$, both firms choose higher quality levels than the levels in the Nash equilibrium. In other words, each government subsidizes its domestic firm (see Figure 1). With respect to the other cases of demand spillover effects, we can similarly explain the optimal cooperative R&D investment policy.

Therefore, based on Lemmas 2 and 3, we easily derive the following proposition.

**Proposition 2**

1. If $\Theta_1 > \Theta_2 > \theta \quad (\theta > \Theta_1 > \Theta_2)$, then an optimal cooperative R&D investment policy is a subsidy (tax), i.e., $s_i^C > 0 \quad (s_i^C < 0) \quad i = 1, 2$.

2. If $\Theta_1 > \theta > \Theta_2$, then a cooperative R&D investment subsidy on firm 1 and tax on firm 2 are optimal, i.e., $s_1^C > 0$ and $s_2^C < 0$.

Note that superscript $C$ denotes the cooperative case.

We have an interesting case, in which there are asymmetric demand spillovers, presented by Proposition 2 (2). In the model of vertical product differentiation, Proposition 3 in Zhou, et al. (2002) is that jointly optimal policies involve an investment subsidy in the developed country and an investment tax in the less-developed country. In their model, the firm producing the high- (low-) quality product locates in the developed (less-developed) country. That is, the jointly optimal policies expand the difference in quality levels and thus mitigate price competition. This, in turn, increases the joint profits of the firms and thus the joint welfare of their countries.

For our model, if we can assume that a firm producing the product associated with large (small) demand spillovers locates in the developed (less-developed) country, by the same reasons derived in the results of Zhou, et al. (2002), we state that cooperative R&D policy is a mix of an investment subsidy in the developed country and an investment tax in the
less-developed country.

4. Discussion: Alternative utility functions and heterogeneous consumers

4.1 A Hotelling model with quality choice

In the previous section, we assumed a homogeneous consumer with a standard quasilinear utility function involved with quality arguments. Here, we first assume heterogeneous consumers and use a standard Hotelling spatial model of duopoly associated with quality choice (e.g., Sanjo, 2007; Ishibashi and Kaneko, 2008). There is a continuum of consumers uniformly distributed on the $[0,1]$ interval. The density of consumers is assumed to be one. Firm $i$ ($i = 0,1$) supplies product at a product quality $q_i$ with price $p_i$. Each consumer purchases at most one unit of a product. A consumer located at $x \in [0,1]$ has net utility $U_0 = v + q_0 - p_0 - tx$ when he or she purchases product 0. Similarly, when purchasing product $i$, the net utility is $U_i = v + q_i - p_i - (1-x)t$. Here, $v(>0)$ is the utility obtained from consuming a single unit of the product, irrespective of the quality level. In addition, the parameter $t(>0)$ implies the transportation cost.

Comparing net utilities, i.e., $U_i$, the demand function of product $i$ is given by

$$x_i = \frac{1}{2} + \frac{1}{2t} (q_i - q_j - p_i + p_j), \quad i, j = 0,1, i \neq j. \quad (20)$$

To simplify, we assume no production costs. Thus, we derive the equilibrium price in Stage 2 as

$$p_i = t + \frac{q_i - q_j}{3}, \quad i, j = 0,1, i \neq j. \quad (21)$$
Therefore, the revenue function of firm \( i \)'s in Stage 1 is given by

\[
R_i[q_i, q_j] = \left( t + \frac{q_i - q_j}{3} \right) \left( \frac{1}{2} + \frac{q_i - q_j}{6t} \right), \quad i, j = 0, 1, i \neq j. \tag{22}
\]

Based on (22), we obtain the externality of firm \( j \) toward the revenue of firm \( i \), and the factor deciding the strategic relationship between firms (i.e., the slope of the reaction function).

That is, with regard to these factors, we have

\[
\frac{\partial R_i}{\partial q_j} = -\frac{1}{3} \left( \frac{1}{2} + \frac{q_i - q_j}{6t} \right) - \frac{1}{6} \left( t + \frac{q_i - q_j}{3} \right) < 0, \tag{23}
\]

\[
\frac{\partial^2 R_i}{\partial q_i \partial q_j} = -\frac{1}{9t} < 0, \tag{24}
\]

where \( i, j = 0, 1, i \neq j \).

In view of (23) and (24), and taking Lemmas 1 and 2 into account, we derive the following results.

**Corollary 1**

(1) An optimal noncooperative R&D investment policy is a subsidy, i.e., \( s^*_i > 0, \quad i = 0, 1 \).

(2) An optimal cooperative R&D investment policy is a tax, i.e., \( s^C_i < 0, \quad i = 0, 1 \).

These results are similar to those of Spencer and Brander (1983, Proposition 2) and the case of a small demand spillover effect, as in Propositions 1 (1) and 2 (1) of our model. Although we do not assume demand spillover effects, even with the presence of demand spillover effects, the results do not change (see Appendix A).
4.2 A horizontal and vertical product differentiation model

We exploit the utility function composed of horizontally and vertically differentiated products presented by André, et al. (2009, Section 6). That is, there is a continuum of consumers uniformly distributed on the \([0,1]\) interval. The density of consumers is assumed to be one.

Firm \(i\) \((i = 0,1)\) supplies product at quality \(q_i\) with price \(p_i\). Each consumer purchases at most one unit of a product. A consumer located at \(x \in [0,1]\) has net utility

\[
U_0 = (1-x)q_0 - p_0
\]

when he or she purchases product 0. Similarly, when purchasing product 1, the net utility is given by

\[
U_1 = xq_1 - p_1.
\]

Therefore, we derive the demand function of firm \(i\)

\[
x_i = \frac{q_i - p_i + p_j}{q_i + q_j}, \quad x_j = 1 - x_i, \quad i, j = 0,1, i \neq j.
\] (25)

To simplify, we assume no production costs. In this case, we derive the equilibrium price in Stage 2 as follows.

\[
p_i = \frac{2q_i + q_j}{3}, \quad i, j = 0,1, i \neq j.
\] (26)

Thus, based on eq. (25) and eq. (26), firm \(i\)’s revenue function of qualities in Stage 1 is given by

\[
R_i[q_i,q_j] = \left(\frac{2q_i + q_j}{3}\right)\left(\frac{2q_i + q_j}{3(q_i + q_j)}\right), \quad i, j = 0,1, i \neq j.
\] (27)

Based on eq. (27), we obtain the externality of firm \(j\) toward the revenue of firm \(i\), and the factor deciding the strategic relationship between firms. That is, with regard to these factors, we have as

\[
\frac{\partial R_i}{\partial q_j} = \frac{(2q_i + q_j)q_j}{9(q_i + q_j)^2} > 0,
\] (28)
\[
\frac{\partial^2 R_i}{\partial q_i \partial q_j} = -\frac{2q_i q_j}{9(q_i + q_j)^3} < 0,
\tag{29}
\]

where \( i, j = 0,1, i \neq j \).

In view of eq. (28) and eq. (29), and taking Lemmas 1 and 2 into account, we derive the following results.

**Corollary 2**

1. An optimal noncooperative R&D investment policy is a tax, i.e., \( s_i^* < 0, \ i = 0,1 \).

2. An optimal cooperative R&D investment policy is a subsidy, i.e., \( s_i^C > 0, \ i = 0,1 \).

These results are the reverse of **Corollary 1** derived in the Hotelling model with the quality choice model analyzed in Section 4.1. This is because the sign of the externality are different between eq. (23) and eq. (28), although the sign indicating the strategic relationship between the firms is the same (i.e., strategic substitutes). That is, in view of eq. (20) and eq. (25), an increase in the quality level of the rival firm’s product reduces the demand of the other firm, irrespective of the type of the utility functions. However, as in eq. (21), the increase reduces the price of the other firm, whereas, as in eq. (26), it conversely increases the price of the other firm. Hence, the sign of the externality is negative, as in eq. (23). On the other hand, because the effect of an increase in the price is larger than that of a decrease in the demand, the sign of the externality is positive, as in eq. (28).

Although we do not allow for the demand spillover effects of the models in this section, even with the presence of demand spillover effects, the results do not change (see Appendix B).
5. Concluding remarks

There is no doubt that international price and quality competition among firms prevails in many industries, such as those manufacturing electronic appliances, computers, HDTV, and others. There is international rivalry not only among firms in advanced countries, but also between firms in advanced countries and those in newly industrialized countries including China and India. Furthermore, governments support their domestic firms’ R&D investment activities in many ways, for example, with subsidies, tax credits, and other regulations, although they cannot publicly promote these firms’ export activities under the WTO system.

Using a vertical product differentiation model, Zhou et al. (2002) and others have considered an optimal product R&D policy in the cases of noncooperation and cooperation. In a sense, they illustrate the differences in the competition and strategic policies of an advanced and a newly industrialized country.

In contrast, in this paper, we have analyzed the same issues as in previous studies, based on a horizontal differentiated model with quality choice. Furthermore, we have assumed demand spillover effects in our model. That is, an increase in the quality level of the firm’s product may increase the other rival firm’s demand and profit. In other words, by focusing on both horizontal product differentiation (e.g., brand name and locations) and vertical product differentiation (i.e., quality), in the presence of similar demand spillover effects between the firms’ products, we show the example of strategic policy and competition between advanced countries such as the U.S., the EU, and Japan. In addition, in the presence of asymmetric demand spillover effects, we illustrate the example of strategic policy and competition between an advanced country (e.g., the U.S.) and a newly industrialized country (e.g., China).

Under the above assumptions, we have found the following. In the cases of large and
small demand spillovers with respect to the firms’ products, i.e., the situation of similar demand spillover effects between firms in advanced countries, an R&D investment subsidy is an optimal policy. However, in the case of asymmetric demand spillover effects, an R&D investment tax is an optimal policy for the advanced country and the newly industrialized country. In particular, a government in a newly industrialized country rather reduces the domestic firm’s R&D activity to exploit large demand spillovers of the firm in the advanced country.

Furthermore, we have considered optimality of cooperative R&D investment policy. In particular, in the case of asymmetric demand spillovers, we have shown the same results as in Zhou et al. (2002). That is, if a firm producing the product associated with large (small) demand spillovers locates in the advanced (newly industrialized) country, then an optimal cooperative R&D investment policy combines a subsidy by the advanced country with a tax by the newly industrialized country.

We underline a specificity of our model. For example, we have exogenously introduced demand spillover effects associated with qualities into the quasilinear standard utility function. Thus, in future work, we need to research a general utility function that includes demand-side spillover effects.
Appendix A

Let us introduce a demand spillover effect by quality upgrade into a Hotelling model with quality. In this case, a consumer located at $x \in [0,1]$ has net utility $U_0 = v + q_0 + \epsilon_1 q_1 - p_0 - tx$ when he or she purchases product 0. Similarly, when purchasing product 1, the net utility is $U_1 = v + q_1 + \epsilon_0 q_0 - p_1 - (1-x)t$. The parameters $\epsilon_i (< 1), i = 0,1,$ denote the marginal coefficient of a demand spillover effect.

Thus, the demand function of product $i$ is given by

$$x_i = \frac{1}{2} + \frac{q_i(1-\epsilon_i) - q_j(1-\epsilon_j) - p_i + p_j}{2t}, \quad i, j = 0,1, i \neq j.$$ 

We derive the equilibrium price in Stage 2 as

$$p_i = t + \frac{q_i(1-\epsilon_i) - q_j(1-\epsilon_j)}{3}, \quad i, j = 0,1, i \neq j.$$ 

Therefore, the revenue function of firm $i$ in Stage 1 is given by

$$R_i[q_i, q_j] = \left\{ t + \frac{q_i(1-\epsilon_i) - q_j(1-\epsilon_j)}{3} \right\} \left( \frac{1}{2} + \frac{q_i(1-\epsilon_i) - q_j(1-\epsilon_j)}{6t} \right),$$

where $i, j = 0,1, i \neq j$.

Because $0 \leq \epsilon_i < 1, i = 0,1,$ we derive the same results of the externality on the firm’s revenue and cross effects as those in the case of nondemand spillover effects.

Appendix B

In the case of a horizontal and vertical product differentiation model, a consumer located at $x \in [0,1]$ has net utility $U_0 = (1-x)(q_0 + \epsilon_1 q_1) - p_0$ when he or she purchases product 0. Similarly, when purchasing product 1, the net utility is $U_1 = x(q_1 + \epsilon_0 q_0) - p_1$. The
parameter $\varepsilon_i (<1)$, $i = 0, 1$, denotes the marginal coefficient of a demand spillover effect.

In this case, we derive the demand function of firm $i$ as

$$x_i = \frac{q_i + \varepsilon_j q_j - p_i + p_j}{(1 + \varepsilon_i)q_i + (1 + \varepsilon_j)q_j}, \quad i, j = 0, 1, i \neq j.$$ 

The equilibrium price in Stage 2 is given by

$$p_i = \frac{(2 + \varepsilon_i)q_i + (1 + 2\varepsilon_j)q_j}{3}, \quad i, j = 0, 1, i \neq j.$$ 

Therefore, the revenue function of firm $i$ in Stage 1 is given by

$$R_i[q_i, q_j] = \frac{\{(2 + \varepsilon_i)q_i + (1 + 2\varepsilon_j)q_j\}^2}{9(1 + \varepsilon_i)q_i + (1 + \varepsilon_j)q_j}, \quad i, j = 0, 1, i \neq j.$$ 

Although we omit the tedious calculations, we obtain the same results of the externality on the firm’s revenue and the cross effect as those in the case of nondemand spillover effects.
References


Toshimitsu, T. 2012, Quality competition and a demand spillover effect: A case of product differentiated duopoly*, School of Economics, Kwansei Gakuin University, Discussion Paper Series, No. 89


Figure 1

$\Theta_1 > \Theta_2 > \theta$

$N$ : Nash Equilibrium

$S_1$ : Stackelberg Equilibrium, in which firm $1$ is the leader

$\Pi^k_i, i=1,2, k=L,F$ : The profit of firm $i$ being $k$  ($L$ : leader, $F$ : follower)

Shaded area illustrates Pareto superior sets
Figure 2

$\theta > \Theta_1 > \Theta_2$

$N$: Nash Equilibrium

$S_1$: Stackelberg Equilibrium, in which firm $i$ is the leader

$\Pi^k_i, i = 1, 2, \ k = L, F$: The profit of firm $i$ being $k$ (L: leader, F: follower)

Shaded area illustrates Pareto superior sets
Figure 3

\[ \Theta_1 > \theta > \Theta_2 \]

\( N \): Nash Equilibrium

\( S_i, i = 1, 2, \): Stackelberg Equilibrium, in which firm \( i \) is the leader

\( \Pi_i^k, i = 1, 2, \ k = L, F \): The profit of firm \( i \) being \( k \) (\( L \): leader, \( F \): follower)

Shaded area illustrates Pareto superior sets