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Henry George Theorem in a Dynamic Framework without Accumulation of Public Goods

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Abstract

The Henry George Theorem, which is originally established in a static model, asserts that the cost of public good provision should be equal to the total revenue of the land rent to achieve the optimal size of population of each region. This paper examines this theorem in a dynamic framework of overlapping generations model, assuming that the government maximizes the sum of the utilities of the generations of finite periods. We show that the optimal path converges to the stationary state, however, it does not stay on it. We derive that the theorem is valid only in the stationary state, and no longer valid along the optimal path.

JEL Classification: R51,F11
Keywords: Henry George theorem, local public good, overlapping generations model

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1. Introduction

The purpose of this paper is to examine the Henry George theorem in a dynamic framework.

The Henry George Theorem is about the optimal distribution of the population among regions in a country, or in other words, about the optimal number of regions in a country assuming the total population is fixed. This theorem asserts that the optimal level of the expenditure on the public good equals the aggregate land rent. That is, the revenue of the land rent should be all taxed and the tax-revenue should be all spent only for the provision of the public good.[1] Then the optimal number of the population in each region is realized.

The Henry George theorem is related with the efficiency of the production side of the economy, and it guarantees the maximum per-capita consumption level for any given level of the public good. It does not relate to the optimal supply of the public goods. Together with the condition of the optimal supply of the public good shown by Samuelson (1954), the total optimality is attained. The Henry George theorem was first introduced into regional economics by Flatters, Henderson and Mieszkowski (1974) as the "Golden Rule". The name "the Henry George theorem" was first used by Arnott and Stiglitz (1979), based on the Single Tax theory of Henry George (1942). [1]

Henry George theorem was discussed in the framework of statics. Later a number of variations of this theorem had been published, however, most of them were in the static framework. Hartwick (1980) examined the theorem when the regions are heterogeneous, and derived that the theorem should be modified. Kanemoto, *et al.,* (1996) discussed whether Tokyo exceeds the optimal size by using the idea of the Henry George theorem.

We examine the Henry George theorem in a dynamic framework, because it is important for policy makers to know whether taking the Henry George policy is beneficial or not in a growing economy, or non-stationary economy in general. We can imagine that, because the Henry George theorem holds in a static model, it also holds at the stationary state in dynamics. Hence our main problem is whether it holds or not at transient states.

Fu (2005) discussed the theorem in a dynamic setting. He used a simple continuous dynamic model and derived that the present value of land rents over all time periods equals the present value of public goods expenditure over all time periods.

Our model uses a two-period overlapping generations model. The individuals

[1] The book was originally published in 1879.

determine the consumption of the two periods, the young age and the old age, given the public goods and the population of each region at each period. The government maximizes the sum of the utilities of each generation of the finite planning periods, and the control variables are the public goods and the size of the population of each period of each region. The government plays the role of the Stackelberg leader, while the residents the followers. We derive the result that the Henry George theorem holds at the stationary state. In fact, the optimal path converges to the stationary state, however, the path does not stay on it. Thus, the Henry George theorem does not hold in any period. However, it nearly holds when the path converges and stays in the neighborhood of it.

We assumed that the durable good in this economy is only the land, hence, in order to save the income and convey the asset from the young age to the old one, people have to buy the land. At the final period T, any people do not want to buy the land because there is no period $T+1$. So if there is no net land rent income at period $T-1$, people in this period do not buy the land. Thus if the government taxes 100% on the land rent at each period, then the people of all periods do not want to buy the land. We want to check that this really happens in the optimal path or not. This is the reason why we assume the finite horizon planning period.

In our dynamics, even the public goods are not storable. The public good in our model can be interpreted as a consumption good jointly consumable without congestion such as defense service.

In the next section, we will present the basic model. In section III, the government policy to maximize the welfare is discussed, and we derive the optimal path of the economy. In section IV, we examine the property of the optimal path and consider whether the Henry George theorem holds or not. In section V, we conclude our discussions.

II. The Model

The population of a country is distributed among homogeneous regions. We assume that the number of the population of a country is fixed and there is no population growth. Each individual lives for two periods. He works at the first period in the agricultural sector, and earns the wage income w, which equals the marginal productivity of labor. He consumes a part of the wage income and saves the rest. As the means of the saving, he buys the land. In the next period, he does not work, but he receives the land rent and sells the land at the end of the period. The income of the old period is spent on the consumption at the period. The generations overlap, and there are young workers and the same number of old land owners, since we assume that there is no population growth. Population is equally distributed over the homogeneous regions in a country.[2]

Let us assume, for the sake of simplicity of explanation, that the square of the land available for region is limited to unity from geographical reason.[3] All regions are homogeneous. Since the number of the regions is the control variable of the government, there exists a possibility that some lands may not be used for economic activities and remain vacant. For technical reason, we assume that the number of available lands are finite.

Since the size of the land is fixed, the production of each region is given only by the number of workers,

$$Y_t = \sqrt{N_t}. \tag{1}$$

The people who was born at the beginning of period t is called the generation t. The generation t works at period t and earns the wage income w_t, and he faces the interest rate r_{t+1}. Then the budget constraint is given by

$$w_t = c_t^1 + \frac{c_{t+1}^2}{1+r_{t+1}}. \tag{2}$$

He maximizes the utility function,

$$u_t = \sqrt{c_t^1}\sqrt{c_{t+1}^2} + \sqrt{G_t} + \sqrt{G_{t+1}}, \tag{3}$$

subject to the budget constraint (2). G_t denotes the public good in period *t.* We assume that the public good is not storable. Since the individual of the generation t determines the consumption of the periods t and $t+1$ so as to maximize the utility function (3), we have

$$c_t^1 = \frac{w_t}{2}, \tag{4}$$

$$c_{t+1}^2 = \frac{w_t\left(1+r_{t+1}\right)}{2}. \tag{5}$$

Thus the saving of the generation t per capita is given by

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[2] We do not examine the stability of the equilibrium, where population are equally distributed.

[3] Most of the literatures on the Henry George theorem assume that the size of the land in each region is determined endogenously, however, in result, the regions are all homogeneous since the conditions on those regions are assumed homogeneous.

$$s_t = w_t - c_t^1 = \frac{w_t}{2}. \tag{6}$$

Assume that there is only one way of storing the wealth, that is to buy the land. The saving s_t is spent to buy the land. We assume that the older people of any regions can buy the land of any other regions. Since the government can determine the number of the regions at each period, there should be some regions which happen to be unpopulated suddenly. Then the land owners suffer loss. In order to avoid asymmetry among landowners, we assume that all landowners are homogeneous, and people can buy any lands of any other regions. The important assumption is that it is not known to the people that what regions vanish in the government's plan. Thus, the expected profit rate from the land is common to all. Let P_t denote the value of the land of one region, then the total land value of this economy is given by $P_t n_t$, where n_t stands for the number of the regions at period *t*. . Since the total population is fixed at unity, we have $n_t = 1/N_t$. Then the saving of the total population of this country is given by, since it is all invested into the land,

$$\frac{P_t}{N_t} = s_t \ . \tag{7}$$

The total saving of the society is $S_t = \frac{w_t}{2} N_t n_t = \frac{w_t}{2}$. The land rent income of the generation *t* at period *t+1* is given by $Y_{t+1} - \frac{\partial Y_{t+1}}{\partial N_{t+1}} N_{t+1}$ in each region. They also receives the revenue from selling the land to the younger generation, who are willing to pay the saving. Here, we assume that the tax to supply the public good is levied on the land rent. Then the generation 0's net income of period 1 is given by $\sqrt{N_1} - w_1 N_1 + \frac{w_1}{2} N_1 - G_1$, where G_1, which was already assumed to be the amount of the public good, and we assume here that it also denotes the cost to finance this amount of the public goods. Hence per-capita income is given by $i_0 = \frac{1}{\sqrt{N_1}} - \frac{w_1}{2} - \frac{G_1}{N_1}$. Then the consumption of the old age of the generation 0 is given by

$$c_1^2 = \frac{1}{\sqrt{N_1}} - \frac{w_1}{2} - \frac{G_1}{N_1} \tag{8}$$

For the generation t, for, $2 \le t \le T-1$, the per-capita net income of the old age is

$$i_t = \frac{w_{t+1}}{2} + \frac{\frac{\sqrt{N_{t+1}}}{2} - G_{t+1}}{N_{t+1}}. \tag{9}$$

They save $s_t = \frac{w_t}{2}$ at the end of period *t* and receives the net income i_t at the end of the period *t+1*, hence the interest rate for the saving r_{t+1} is given by

$$r_{t+1} = \frac{i_t}{s_t} - 1 . \tag{10}$$

The interest rate can be negative, but it is greater than -1, otherwise, people do not save. Since the wage rate is determined by the marginal productivity of labor, it is given by

$$w_t = \frac{1}{2\sqrt{N_t}} . \tag{11}$$

Then (10) is rewritten as

$$r_{t+1} = \frac{3w_{t+1} - w_t - 8w_{t+1}{}^2 G_{t+1}}{w_t} . \tag{12}$$

At the final period T, the total income of the old age of the generation $T-1$ is

$$i_T = \frac{\frac{\sqrt{N_T}}{2} - G_T}{N_T} , \tag{13}$$

since the generation T does not succeed the land. The generation *T*'s income w_T is all spent for the consumption and we have

$$c^1{}_T = w_T . \tag{14}$$

Then the interest rate of the final period r_T is given by

$$r_T = \frac{2w_T - w_{T-1} - 8w_T{}^2 G_T}{w_{T-1}} , \tag{15}$$

form (6),(10),(11) and (13).

Thus, the time sequence of the interest rate $\{r_t\}$ is determined when that of the wage rate $\{w_t\}$ and the public good $\{G_t\}$ are given. We assumed that the government's control variables are the wage rate and the population size of each region, however, hereafter we treat the wage rate $\{w_t\}$ and the interest rate $\{r_t\}$ are the control variables for the sake of simplicity.

III. Welfare Analysis

We discuss the government behavior to maximize the welfare. The government aims to maximize the sum of the utilities of the generations of finite periods as

$$W = \sum_{\tau=1}^{T} u_\tau . \tag{16}$$

We solve the finite time horizon problem, because we are interested in the optimal solution of the initial and the final periods. The first generation, who is the old generation at period 1, owns the land initially. His consumption at period 1 is given by (8). We have to redefine the utility function for the generation 0 because he lives only for one period and only consumes at the old age. We define that his utility is composed of his consumption c_1^2 and the public good G_1 given by

$$u_o = \sqrt{\frac{3w_1}{2} - 4G_1 w_1^{\ 2}} + \sqrt{G_1} , \tag{17}$$

from (8). And the utility of the generation t, for $2 \le t \le T-1$, is given by

$$u_t = \frac{w_t \sqrt{1+r_{t+1}}}{2} + \sqrt{G_t} + \sqrt{G_{t+1}} , \tag{18}$$

from (3),(4) and (5). For the final period T, the generation T's consumption is given by (14), and his utility is assumed to be

$$u_T = \sqrt{w_T} + \sqrt{G_T} . \tag{19}$$

The government maximizes the sum of the utilities from the generations 1 to T. The constraint of the government is the sequence of the interest rate given by (12) and (15), which is equivalent to the market equilibrium condition of each period[4]. In order to solve the problem of the government, we form the Lagrangian,

[4] The equations (12) and (15), which shows the interest rate of each period, together with the budget constraint of individuals (2) and the optimal condition of the labor employment (11), lead us to the market equilibrium condition

$$Y_t = \left(c^1{}_t + c^2{}_t\right) n_t + G_t \quad \text{for} \quad 1 \le t \le T . \tag{f1}$$

$$L=\left(\sqrt{\frac{3w_1}{2}-4G_1w_1^{\ 2}}+\sqrt{G_1}\right)+\left(\frac{w_1\sqrt{1+r_2}}{2}+\sqrt{G_1}+\sqrt{G_2}\right)$$
$$+\sum_{\tau=2}^{T-1}\left[\frac{w_\tau\sqrt{1+r_{\tau+1}}}{2}+\sqrt{G_\tau}+\sqrt{G_{\tau+1}}+\lambda_\tau\left(r_\tau-\frac{3w_\tau-w_{\tau-1}-8G_\tau w_\tau^{\ 2}}{w_{\tau-1}}\right)\right] \qquad (20)$$
$$+\sqrt{w_T}+\sqrt{G_T}+\lambda_T\left(r_T-\frac{2w_T-w_{T-1}-8G_Tw_T^{\ 2}}{w_{T-1}}\right).$$

Let us solve this maximization problem. Differentiating (20) *w.r.t.* r_t, G_t, λ_t and w_t, we obtain the first order conditions,

$$\frac{\partial L}{\partial r_t}=\lambda_t+\frac{w_{t-1}}{4\sqrt{1+r_t}}=0, \qquad \text{for } 2\le t\le T \qquad (21)$$

$$\frac{\partial L}{\partial G_t}=\begin{cases}-\dfrac{2w_1^{\ 2}}{\sqrt{\dfrac{3}{2}w_1-4G_1w_1^{\ 2}}}+\dfrac{1}{\sqrt{G_1}}=0, & \text{for } \ t=1, \qquad (22-1)\\[2ex] \dfrac{1}{\sqrt{G_t}}+\lambda_t\dfrac{8w_t^{\ 2}}{w_{t-1}}=0, & \text{for } \ 2\le t\le T \qquad (22-2)\end{cases}$$

$$\frac{\partial L}{\partial \lambda_t}=\begin{cases}r_t-\dfrac{3w_t-w_{t-1}-8G_tw_t^{\ 2}}{w_{t-1}}=0, & \text{for } 2\le t\le T-1 \qquad (23-1)\\[2ex] r_T-\dfrac{2w_T-w_{T-1}-8G_Tw_t^{\ 2}}{w_{T-1}}=0, & \text{for } t=T \qquad (23\text{-}2)\end{cases}$$

$$\frac{\partial L}{\partial w_t}=\begin{cases}\dfrac{\frac{3}{4}-4G_1w_1}{\sqrt{\frac{3}{2}w_1-4G_1w_1^{\ 2}}}+\dfrac{\sqrt{R_2}}{2}+\lambda_2\dfrac{3w_2-8G_2w_2^{\ 2}}{w_1^{\ 2}}=0, & \text{for } t=1 \qquad (24-1)\\[2ex] \dfrac{\sqrt{1+r_t}}{2}-\lambda_t\dfrac{3-w_{t-1}-16G_tw_t}{w_{t-1}}+\lambda_{t+1}\dfrac{3w_{t+1}-8G_{t+1}w_{t+1}^{\ 2}}{w_t^{\ 2}}=0, & \text{for } 2\le t\le T-2, \ (24-2)\\[2ex] \dfrac{\sqrt{1+r_T}}{2}-\lambda_{T-1}\dfrac{3-w_{T-2}-16G_{T-1}w_{T-1}}{w_{T-2}}+\lambda_T\dfrac{2w_T-8G_Tw_T^{\ 2}}{w_{T-1}^{\ 2}}=0, & \text{for } t=T-1, \qquad (24-3)\\[2ex] \dfrac{1}{2\sqrt{w_T}}+\lambda_T\dfrac{-2+16G_Tw_T}{w_{T-1}}=0\ , & \text{for } t=T. \qquad (24-4)\end{cases}$$

From (21), we have

$$\lambda_t = -\frac{w_{t-1}}{4\sqrt{1+r_t}} \qquad \text{for} \quad 2 \le t \le T. \tag{25}$$

From (22-1), we derive G_1, and from (22-2) and (25), G_t for $2 \le t \le T$ as

$$G_t = \begin{cases} \dfrac{3}{8w_1\left(1+w_1^{\,2}\right)} & \text{for} \quad t = 1, \qquad (26-1) \\ \dfrac{1+r_t}{4w_t^{\,4}} & \text{for} \quad 2 \le t \le T. \qquad (26-2) \end{cases}$$

Then from (26-2) and (23-1,2), eliminating G_t, we have the difference equations for w_{t-1} for $2 \le t \le T$,

$$w_{t-1} = \begin{cases} \dfrac{3w_t^{\,3} - 2 - 2r_t}{\left(1+r_t\right)w_t^{\,2}} & \text{for} \quad 2 \le t \le T-1, \qquad (27-1) \\ \dfrac{2\left(w_T^{\,3} - 1 - r_T\right)}{\left(1+r_T\right)w_T^{\,2}} & \text{for} \quad t = T. \qquad (27-2) \end{cases} .$$

For $t = 1$, from (24-1),(25),(26-1,2), we obtain the difference equation, which gives w_1 from w_2 and r_2,

$$\frac{3w_2^{\,3} - 2(1+r_2)}{w_2^{\,2}\sqrt{1+r_2}} - 2w_1\sqrt{1+r_2} + \frac{\sqrt{6}\left(1-w_1^{\,2}\right)}{\sqrt{w_1\left(1+w_1^{\,2}\right)}} = 0 . \tag{28}$$

Then (27-1) and (28) both determine w_1 given w_2 and r_2. Those two equations have to give the same value of w_1. This implies that the initial value of the difference equation system, *i.e.*, either w_T or r_T should be properly determined. For $2 \le t \le T-2$, from (24-2), (25), (26-2) and (27-2), we have

$$3 + \sqrt{1+r_{t+1}}\sqrt{1+r_t} = 4\left(1+r_t\right)\left(\frac{\left(1+r_{t+1}\right)w_{t+1}^{\,2}}{3w_{t+1}^{\,3} - 2(1+r_{t+1})}\right)^3 . \tag{29}$$

For $t = T-1$, from (24-3),(26-2) and (27-2), we have a slightly different equation,

$$3 + \sqrt{1+r_T}\sqrt{1+r_{T-1}} = \frac{\left(1+r_{T-1}\right)}{2}\left(\frac{\left(1+r_T\right)w_T^{\,2}}{w_T^{\,3} - 1 - r_T}\right)^3 . \tag{30}$$

For $t = T$, from (24-4), (25) and (26-2), we have

$$1 + r_T - \frac{w_T^{\,5/2}}{2}\sqrt{1+r_T} - \frac{w_T^{\,3}}{2} = 0 . \tag{31}$$

Then r_t is given by

$$r_t = \begin{cases} \left(\dfrac{\sqrt{1+r_{t+1}} + \sqrt{1+r_{t+1}+6A_{t+1}}}{A_{t+1}} \right)^2 - 1, \\ \qquad \text{where } A_{t+1} = \left(\dfrac{(1+r_{t+1}) w_{t+1}{}^2}{\frac{3}{2} w_{t+1}{}^3 - (1+r_{t+1})} \right)^3 \quad \text{for } \quad 2 \le t \le T\text{ - }2, \quad (32\text{-}1) \\ \left(\dfrac{\sqrt{1+r_{t+1}} + \sqrt{1+r_{t+1}+6B_{t+1}}}{B_{t+1}} \right)^2 - 1, \\ \qquad \text{where } B_{t+1} = \left(\dfrac{(1+r_{t+1}) w_{t+1}{}^2}{w_{t+1}{}^3 - (1+r_{t+1})} \right)^3 \quad \text{for } \quad t = T-1, \quad (32-2) \\ \dfrac{w_T{}^5}{16} \left(1 + \sqrt{1+\dfrac{8}{w_T{}^2}} \right)^2 - 1, \qquad \text{for } \quad t = T. \qquad (32-3) \end{cases}$$

(29),(30) and (31) give (32-1), (32-2) and (32-3) , respectively. (32-3) is not a difference equation, and which shows the relation between w_T and r_T. If r_T is determined, then w_T is determined. Then at $t = T-1$, from (32-2) r_{T-1}, and from (27-2), w_{T-1} are determined. Similarly, from (32-1) and (27-1), w_{T-2} and r_{T-2} are determined, and so forth. And as shown in the above, (27-1) gives w_1, and (28) also gives w_1, those two should be the same value.

(32-1,2) are the difference equations of the form of $r_{t-1} = \Phi(w_t, r_t)$ and (27-1,2) are those of $w_{t-1} = \Psi(w_t, r_t)$.

IV The Property of the Optimal Path

Firstly, we have to check the transient path, and show that the optimal path does not stay on the stationary state. From (27-1) and (32-1) or (29), we have the stationary state of the wage rate and the interest rate as follows:

$$w^* = 1, \; r^* = 0. \qquad (33)$$

If we assume that the optimal path is on the stationary state in $2 \le t \le T-2$, then $w_t = 1, r_t = 0$ holds in this interval. Then from (27-1), we have $w_1 = 1$. But from (28) we cannot derive $w_1 = 1$. This leads to a contradictory. Thus we derived the next theorem.

Theorem 1:

The optimal path does not stay on the stationary state given by (33).

Next, we derive the dynamic behavior of this optimal path. First, let us see the stationary state of $2 \le t \le T-2$, which is given from (27-1) and (32-1) or (29). We can easily have the stationary state level of the variables, $r^* = 0$ and $w^* = 1$. We define $\Delta w_t = w_t - w_{t-1}$, or the increment of w_t from w_{t-1}. Similarly, $\Delta r_t = r_t - r_{t-1}$ is defined. The curve $\Delta w = 0$ is given by

$$r = \frac{2w^3 - 2}{w^3 + 2} \tag{34}$$

from (27-1) setting $w_t = w_{t-1}$. And that of $\Delta r = 0$ is given by

$$4 + r = 4(1+r)\left(\frac{(1+r)w^2}{3w^3 - 2(1+r)}\right)^3. \tag{35}$$

from (29). The graphs of $\Delta w = 0$ and $\Delta r = 0$ are shown in Figure 1. We can check the saddle point property of our difference equation system as follows: Totally differentiating (27-1) and (32-1) or (29), we obtain[5]

$$\begin{pmatrix} dw_{t-1} \\ dr_{t-1} \end{pmatrix} = \begin{pmatrix} 7 & -3 \\ 24 & -\frac{71}{7} \end{pmatrix} \begin{pmatrix} dw_t \\ dw_t \end{pmatrix}. \tag{36}$$

The characteristic roots of the matrix of (36) are $\frac{-11 \pm 6\sqrt{2}}{7}$, or −2.783 and −0.359. One of the absolute values of those two characteristic roots is less than one, and the other is greater than one. This shows that the stationary state is a saddle point. In Figure 1, we show the phase diagram. The arrows in the diagram show the direction of the movement of the point. When under a differential equation system, in this case, there exists a pair of the stable arms which monotonically converge to the stationary state. However, our system is not a differential equation system but a difference equation system. Then there exists the possibility of overshooting. The solution

[5] (35) is a backward differential equation system. We can easily derive the forward system as $\begin{pmatrix} dw_t \\ dr_t \end{pmatrix} = \begin{pmatrix} 7 & -24 \\ 3 & -\frac{71}{7} \end{pmatrix} \begin{pmatrix} dw_{t-1} \\ dw_{t-1} \end{pmatrix}$. The characteristic roots of the matrix are the same.

path does not necessarily converge monotonically to the stationary state. We actually numerically solved our difference equation system and show the result.

[Figure 1 around here]

We have derived the optimal dynamic path of the wage rate $\{w_t\}$ and the interest rate $\{r_t\}$. In the background, the optimal path of the public goods $\{G_t\}$ is determined. We have to note that there exist some regions which vanish or emerge in the process, however for those regions, the optimal path of the public good is on the derived path $\{G_t\}$ as long as those regions exist.

We calculated the optimal path of for the cases of $T=5$ and $T=12$.[6] The path of each case is shown in Table 1 and 2, respectively. Both cases show that the path converges to the stationary state ($w^*=1,\ r^*=0$), and spend most of the time near it as the planning period T increases as shown in Table 1 and 2.[7] The optimal path oscillates around the stationary state. This is due to the overshooting because our system is a difference equation system.

[Table 1 and 2 around here]

The optimal path of $\{w_t\}$ is shown in Figure 2. The path of the interest rate is shown in Figure 3. Those two exhibit the same property that the paths converge to the stationary state in the middle of the planning periods.

[Figure 2 and 3 around here]

IV-2. Reconsideration of the Henry George Theorem.

As shown in Theorem 1, the optimal path does not stay on the stationary state. Then what about the Henry George theorem? In the periods $2 \le t \le T-2$, the land rent minus public goods expenditure is given by

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[6] We used *Maple 11* to calculate the optimal path.

[7] This is so called the "turnpike" property.

$$\frac{\sqrt{N_t}}{2} - G_t = \frac{1}{4w_t} - \frac{1+r_t}{4w_t^4} = \frac{w_t^3 - 1 - r_t}{4w_t^4}, \tag{37}$$

from (11) and (26-2). It is evident from (33) that only at the stationary state (37) vanishes and the Henry George theorem holds. However, if the optimal path is off the stationary state, the theorem does not hold.

Theorem 2:

On the stationary state, the Henry George theorem is valid at each period, but off the stationary state, the theorem is not valid.

Fu (2005) examined the Henry George theorem in an ordinary continuous dynamic model where public good accumulates, and derived the result that in terms of the discounted sum the land rent and public goods expenditure are equal. In our model, however, where there is no accumulation of the public goods, those two are not equal.

We are apprehensive that if the government were to continue to take the Henry George policy then $r_T = -1$ holds and possessing the land does not play the role of the store of value. Hence, for the generation t, for $2 \le t \le T-1$, the utilities are all zero, because they cannot consume at all in the old age . This policy is evidently not optimal. In the reality, in our numerical example, the government does not take the Henry George policy every period, and r_T is actually greater than minus one. Hence, there exists an incentive to posses the land for generation $T-1$, thus, for all generations.

V. Concluding Remarks

We re-examined the Henry George theorem in a dynamic framework of the overlapping generations model where public goods cannot be accumulated. We derived that the Henry George theorem does not hold in general. Only at the stationary state, the theorem holds, however, the optimal path does not stay on the stationary state, but converges to it and stays most of the periods near it as the planning period T increases. In our model, there is no state variable, and any variable can jump to any level. Though there exists a possible choice to jump to the stationary state, but we showed that this jump is not optimal. These findings were possible only in a finite horizon planning, where the starting point and final point are determined endogenously.

There exist regions which vanish in the process, or emerge in the process. The land value of the region which vanishes in the process is smaller than the other regions which continue to exist. Then our assumption that all regions are homogeneous is not satisfied. To avoid the complexity, we assumed that all lands are in a basket and people buy a portion of this basket, so the all portions of the basket are homogeneous. To relax this assumption is the next extension of this model.

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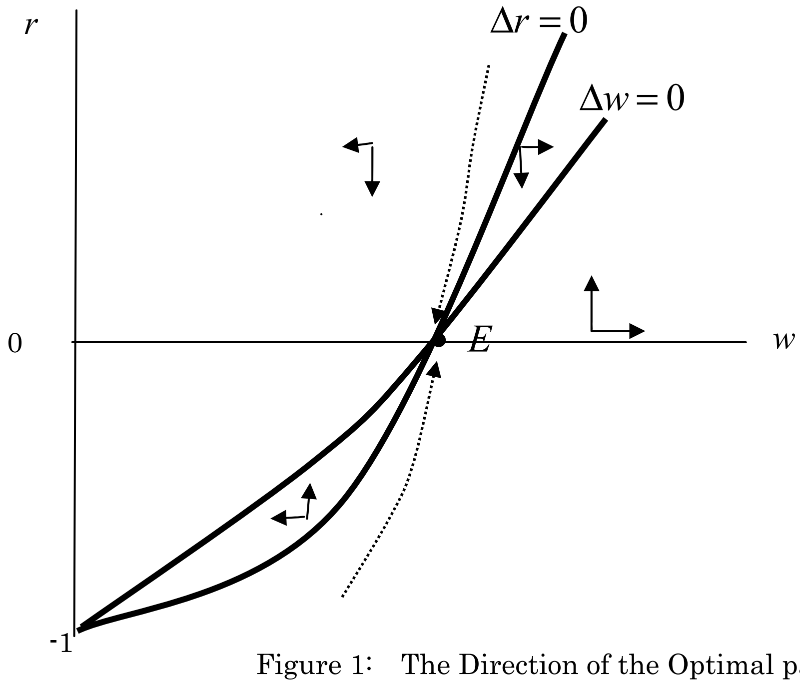

Figure 1:　The Direction of the Optimal path

Table 1: T=5

t	W	r
1	0429	—
2	1.121	0.426
3	0.906	−0236
4	1.212	0.604
5	0.657	−0.976

Table 2: T=12

t	w	r
1	0.740	—
2	1.111	0.413
3	0.962	−0.118
4	1.013	0.045
5	0.995	−0.015
6	1.000	0.003
7	1.002	0.005
8	0.991	−0.020
9	1.023	0.059
10	0.935	−0.152
11	1.197	0.540
12	0.659	−0.773

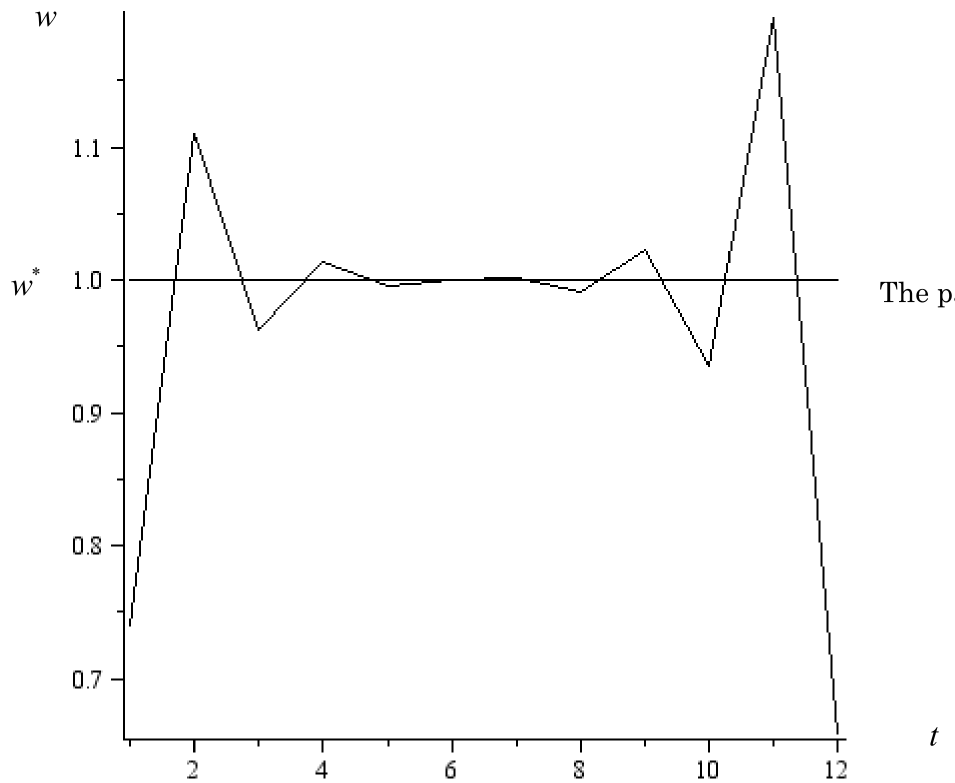

Figure 2:
The path of the wage rate

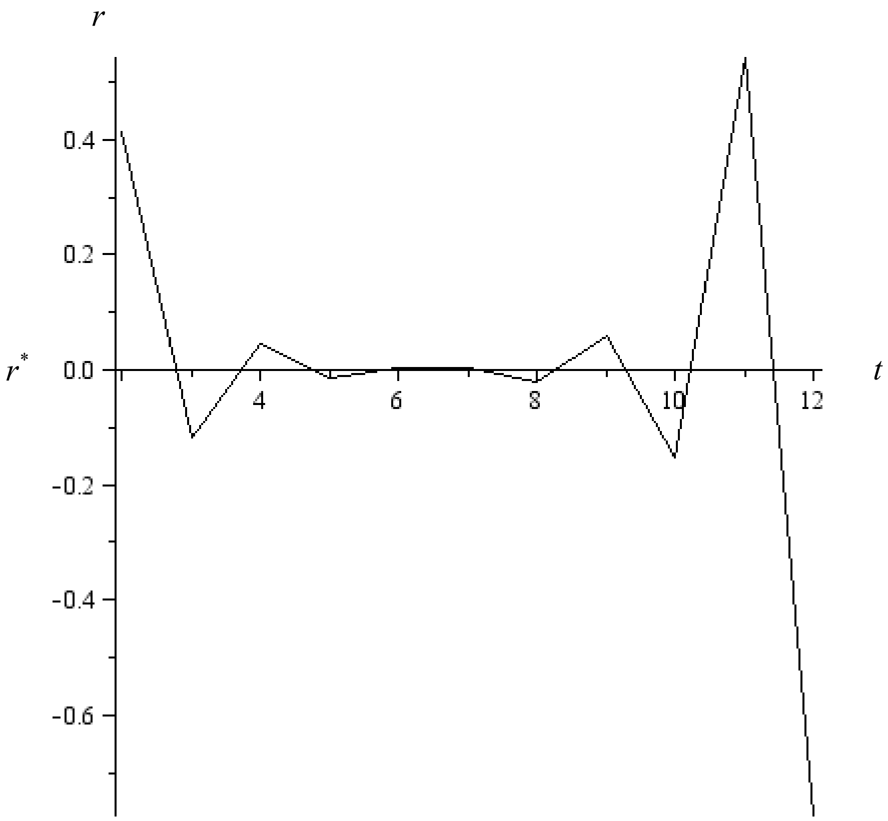

Figure 3:
The path of the interest rate