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Delegation and Limited Liability in a Modern Capitalistic Economy

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Delegation and Limited Liability in a Modern Capitalistic Economy*

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Abstract

We examine an effect of *limited liability* on *strategic delegation* in a Cournot duopoly with demand uncertainty. We establish that owners of each firm *always* delegate their tasks, decisions, and responsibility to a manager *under limited liability*, while they do *not always* do so *under unlimited liability.* This result is consistent with the fact that separation of ownership and management as well as limited liability prevail in many modern large companies.

Keywords: limited liability, delegation, managerial incentives, and Cournot duopoly
JEL classification: G32, L13, L12

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1 Introduction

Modern large companies have two typical features: separation of ownership and management, which was observed by Berle and Means (1932), and limited liability. Separation of ownership and management allows firms to commit to behavior other than profit-maximizing behavior. Certain empirical evidence implies that they do not behave as profit-maximizers. For example, Amihud and Kamin (1979) supported "Baumol (1958)'s hypothesis that revenue-maximizing behavior is more prevalent among oligopolistic, management-controlled firms ". From a managerial incentive perspective, Vickers (1985) and Fershtman and Judd (1987) theoretically justified this empirical evidence. Vickers (1985) established that firms may obtain higher profits by delegating to managers who do not behave as profit-maximizers. Fershtman and Judd (1987) examined the incentives that owners of competing firms in an oligopoly give managers as compensation, inducing the managers to maximize linear combinations of the objectives of profits and sales. These authors explored both how competing owners may strategically manipulate these incentive contracts and the resulting impact on the oligopoly equilibrium outcome. Fershtman and Judd (1987) showed that owners of duopolistic firms always give more incentives weighted to sales than those weighted to profit if costs are sufficiently low at the equilibrium and that while this equilibrium output of each firm exceeds the Cournot output, both profit and price are lower than in the Cournot equilibrium. [1]. These works have motived a large amount of research concerning *strategic delegation* [2].

The other feature (limited liability) is adopted by modern large companies. According to Spulber (2009, pp. 264–65), corporations, which can be regarded as limited liability companies, earn almost 85% of the total revenue a year, though the share of cooperations is over 19% in terms of total number of firms in the United States. Brander and Lewis (1986) theoretically considered the relationships between an oligopolistic product market and financial structure, and they showed that limited liability may induce a leveraged firm to a more aggressive output stance [3]. As is the case for strategic delegation, there

[1] These analytical frameworks seem to be supported by the result of McGuire, Chiu, and Elbing (1962). They present the average correlation coefficients for executive compensation and sales and for executive compensation and profits from the data on revenues, profits, and compensation for 45 enterprises from 1953 to 1959; they then show that the former is larger than the latter and that the significance of the t values for the former is consistently higher than for the latter.

[2] See Sengul, Gimeno and Dial (2012) for a recent good survey on strategic delegation in economics and management literature.

[3] Etro (2010) characterized the optimal financial structure as a strategic device to optimize the value

are also many works concerning financial structure. [4]

As previously mentioned, a modern large company can be considered an oligopolistic, management-controlled firm with limited liability. From this perspective, previous works concerning each topic have the following drawback. An oligopolist is assumed to be a limited liability company in financial structure literature, while one is assumed to be a profit-maximizer in the strategic delegation literature. The previous literature is also limited by focusing only on a symmetric equilibrium.

In a Cournot duopoly with demand uncertainty, we examine the effect of limited liability on whether an owner delegates its tasks, decisions, and responsibility to a manager. In this sense, our research can be regarded as an amalgam of the works of Fershtman and Judd (1987) and Brander and Lewis (1986). To derive a clear result and to ensure an asymmetric equilibrium, we modify the models of Fershtman and Judd (1987) and Brander and Lewis (1986) as follows. Although Fershtman and Judd (1987) assumed that a firm's managerial incentive chosen by its owner represents a linear combination of its sales and its profits, we assume that this incentive is either its sales or its profits. We consider the situation in which a firm's managerial incentive is its sales (resp. its profits) as the situation in which its owner delegates (resp. does not delegate) its tasks to its manager. While this assumption is certainly restrictive, it allows us to generate an asymmetric equilibrium, that is, one firm chooses delegation, while another firm selects no delegation at the equilibrium and it is supported by empirical evidence in McGuire, Chiu, and Elbing (1962) and Amihud and Kamin (1979). While Brander and Lewis (1986) assumed that a firm finances a fixed start-up or project cost, we assume that it finances to pay for variable production costs, following Povel and Raith (2004). Cleary et al. (2007) empirically supported this assumption by Povel and Raith (2004), which makes our model tractable.

We consider a two-stage duopoly game with demand uncertainty under *both unlimited liability and limited liability*. In the first stage, shareholders of each firm simultaneously choose the mode of delegation by designing an incentive scheme for the manager of their firm, either *no strategic delegation* (profit maximization) or *strategic delegation* (sales maximization)[5]. In the second stage, the manager of each firm simultaneously chooses

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of a firm competing in a market for which entry is endogenous, and he showed the general optimality of moderate debt financing under both quantity and price competition in an oligopoly with cost uncertainty.

[4] For example, see Neff (2003) for the related literature.

[5] In this paper, we do not consider the choice of organizational form by owners of oligopolistic firms, that is the choice between "limited liability" and "unlimited liability." But Ohkawa, Shinkai, Okamura,

her output quantity after she observes the objective of her rival firm. That is, in the second stage game, following the Brander-Lewis framework, we first consider à la Cournot three types of duopoly with demand uncertainty, each of which is composed of firms with no strategic delegation or with a strategic delegation under *unlimited liability* and *mixed delegation-type duopoly*, in which a no delegation firm and a delegation firm coexist. We derive a subgame perfect equilibrium of the game. By comparing the equilibria of subgames, we characterize the equilibrium outputs, prices, and total outputs in these equilibria. Then, we proceed to the similar analysis under *limited liability.* We explore how different modes of delegation under unlimited liability or limited liability affect the strategic behavior of firms and the outcomes in these equilibria.

Consequently, we show that *delegation equilibrium* always occurs as a whole game equilibrium under *limited liability*, although delegation equilibrium occurs when potential demand is not too small; however, *mixed delegation-type equilibria exist* if *potential demand is small* under *unlimited liability.*

The result we derive in this paper illustrates how heterogeneous firms compete in an oligopoly under risk, for example *the U.S. S&L crisis context in* the seminal book by Milgrom and Roberts (1992)[6]. That is, by the derived result, we present an explanation of how *the effect of competition caused by the intrinsic characteristics of* the Federal Savings and Loan Insurance Corporation(FSLIC)'s insurance for bank deposit (which plays a similar role to *a limited liability system* in our model) works, though we do not explicitly address the moral hazard problem.

Furthermore, we define the expected social welfare at the equilibria and compare the expected social welfare at the three subgame equilibria previously derived. Using the sum of potential demand and demand risk, we characterize which of the three subgame equilibria would be desirable from a social welfare perspective. We find that *a Delegation duopoly equilibrium* in a whole game under *limited liability is most desirable* from a social welfare perspective for *any* value of the demand parameter.

In the next section, we describe the structure of our model. In section 3, under unlimited liability, we derive a two-stage duopoly game in which each firm chooses the

and Harimaya (2011) endogenize the choice of organizational form of the oligopolistic firms in the first stage, and these firms then compete in a Cournot fashion in the second stage.

[6]In subsection 'The Perverse Effect of Competition' in Chapter 6, Milgrom and Roberts (1992) described, "Normally we think of competition, which tends to drive out those executives who are unwilling to take the profit-maximizing actions, as promoting efficiency. In the context of S&L industry in 1980s, however, competition had a perverse effect. Many conservative S&L executives had no choice but to gamble on risky investments if they were to survive in the circumstances we have described."

mode of delegation, that is, either no strategic delegation or strategic delegation at the first stage, then competes in à la Cournot fashion at the second stage under *unlimited liability*. In subsection 4.1, we consider a Cournot duopoly subgame that is composed of two firms with no strategic delegation under limited liability and derive an equilibrium of this subgame. In subsection 4.2, we derive a Cournot equilibrium in the duopoly subgame that is composed of two firms with strategic delegation; in subsection 4.3, we derive a Cournot equilibrium in the mixed type duopoly subgame that is composed of one firm with no strategic delegation *and* another firm with strategic delegation under limited liability. In subsection 4.4, we consider the first stage, in which each firm chooses the mode of delegation (its objective), either strategic delegation or no strategic delegation, and derive a subgame perfect equilibrium of the whole game by combining it with the three two-stage Cournot duopoly subgames considered in the preceding subsections. In section 5, we evaluate the equilibrium of the whole game from a social welfare perspective. The final section contains our discussion and concluding remarks.

2 The Model

We consider a duopoly in which two firms produce a homogeneous good with an identical constant marginal cost. There also exists additive demand uncertainty. We assume that the shareholders of each firm are protected by limited liability effects in this duopoly market. Their objective is to maximize the expected profit of the shareholders of the firm they own. They delegate their decision of the firm's output to a manager; however, they control the manager by designing incentives scheme to attain their objective, that is, net profit maximization (after deducting rewards for their manager). Fershtman and Judd (1987) assumed that the manager of firm i is given an incentive to maximize $\alpha_i \pi_i + (1 - \alpha_i) R_i$, where π_i and R_i are the profit and revenue of firm i, respectively, and α_i is the weight assigned to the profit of the manager's incentive. They showed that the shareholders of each firm always give an incentive weighted more to sales than to profit (i.e., small $\alpha_i < 1$) to their manager at the equilibrium if the marginal cost of production of their firm is sufficiently low. Vickers (1985) also presented an example in which shareholders strategically adjust their manager's incentive not to behave as a profit-maximizer[7].

[7]Vickers (1985) assumed that the manager of oligopolistic firm $i (= 1, 2, \cdots, n)$ has the objective to maximize $M_i = \pi_i + \theta_i q_i$, where π_i, q_i, and θ_i are profit, quantity of output, and some strategic parameter

In this paper, we thus restrict our attention to two polar cases, $\alpha_i = 0$ (sales maximization) and $\alpha_i = 1$ (profit maximization) because the profit and sales of each firm is observable (known) to one another[8]. The shareholders can ask for debt D_i $(i = 1, 2)$ from outside investors if the equity capital is not sufficient to finance production. According to Brander and Lewis (1986), the debt holders are residual claimants in case of bankruptcy. Hence, the shareholders of the firm do not care about the returns in the bad state; they are only concerned with the returns in the good state. When the firm takes debt, it is more inclined to follow strategies that provide more returns in the good state and fewer returns in the bad state. That is, we say that *the firm is protected by the limited liability effect of debt financing.* The limited liability effect induces the firm to assume more risk. As Brander and Lewis (1986) established, a leveraged firm behaves more aggressively than does the unleveraged firm. In this paper, we consider *the strategic delegation effect in addition to this limited liability effect.*

Suppose that each firm can choose its mode of strategic delegation from *strategic delegation* (sales maximization) and *no strategic delegation* (profit maximization) in the first stage. Then given the mode of delegations, two firms compete in à la Cournot in the second stage.

The inverse demand function is assumed to be linear with an additive uncertainty,

$$p = a + \widetilde{z} - Q = a + \widetilde{z} - (q_1 + q_2), \tag{1}$$

where a denotes the magnitude of the market and $\widetilde{z}$ is a uniformly distributed random variable with support $[-\overline{z}, \overline{z}]$, $a - \overline{z} > 0$ and with the probability density function

$$\begin{aligned} \phi(z) &= \frac{1}{2\overline{z}}, \text{ for } z \in [-\overline{z}, \overline{z}] \\ &= 0, \quad \text{otherwise.} \end{aligned} \tag{2}$$

¿From (2), we observe that $\widetilde{z}$ has mean 0 and variance $\frac{1}{3}\overline{z}^2$. We also assume that firm

variable on the incentive for manager of firm i.

[8]If we set $\theta_i = c$ or $\theta_i = 0$ in the objective of firm i's manager function with constant returns to scale technology

$M_i = \pi_i + \theta_i q_i = (p(Q) - (c - \theta_i))\, q_i$

then Vickers(1985)'s example reduces to the cases $\alpha_i = 0$ (sales maximization) and $\alpha_i = 1$ (profit maximization), respectively.

i $(=1,2)$ has a linear cost function

$$C_i(q_i) = cq_i, a > c > 0.$$

We normalize $c = 1$. Here we make a key assumption in our analysis of leveraged firms under limited liability. That is, we assume that firms are financially constrained and must finance all or part of their variable costs by borrowing from their investors or banks, following Povel and Raith (2004). Most of the debt contract literature assumes that a firm or an entrepreneur must finance a fixed start-up or project cost, as Brander and Lewis (1986) assumed in their paper. In these papers, the equilibrium output and the equilibrium debt level of each firm are not derived explicitly on account of the nonlinearity of the reaction function of each firm as described in the analysis of the Brander-Lewis framework[9]. Povel and Raith (2004), however, have considered a Cournot duopoly in which one of the firms is financially constrained and must finance all or part of its variable cost by borrowing from an investor and in which another firm is not financially constrained[10]. Under their assumption, the choice of output of each firm uniquely determines its level of debt, thus making our analysis more tractable. We thus assume that debt level of each firm is a linear cost function of the firm's output under limited liability. We take the debt assumed by the firm as endogenous. The firm takes on debt only to finance its production. That is,

$$D_i = cq_i = q_i.$$

The profit of firm $i(=1,2)$ is defined as

$$\pi_i(q_i, q_j, \widetilde{z}) = R_i(q_i, q_j, \widetilde{z}) - C_i(q_i) = (a + \widetilde{z} - q_i - q_j - 1)q_i. \tag{3}$$

[9]In the Brander-Lweis framework, R^i (the gross profit function) is assumed to depend on the outputs q_i , q_j and the random shock $\widetilde{z}_i$ with support $[-\overline{z}, \overline{z}]$. A threshold value of realization z of $\widetilde{z}_i$, $\widehat{z}_i$ is also assumed such that the firm is bankrupt for $z_i < \widehat{z}_i$ and that equity holders are residual claimants only in good state of nature $(z_i \geq \widehat{z}_i)$. Then, the value of $\widehat{z}_i$ depends not only on the debt level of $B_{i,}$ but also on q_i and q_j. Therefore, the reaction function of a firm with respect to q_i becomes a nonlinear function of q_i. For example, see Franck and Pape (2008).

[10]As Povel and Raith (2004) stated in their paper, "internal funds" denotes the firm's own funds that it can use to pay for variable production costs, $w_0 \equiv r_0 - F$, where r_0 and F denote the firms retained earnings and fixed cost, respectively. Cleary et al. (2007) show that $w_0 < 0$, that is, *"negative internal funds"* are empirically relevant using 20 years of annual Compustat data, so we can expect that a firm must finance variable costs in such cases. Hence, we think that the role of a risky debt contract on product rivalry in a duopoly under the assumption in Povel and Raith (2004) must merit investigation.

Because the revenue of firm i ($R_i(q_i, q_j, z) = (a + z - q_i - q_j - c)q_i$) is increasing in z, we can define the repayment function under limited liability as $r \equiv \min\{R_i(q_i, q_j, z), D_i\}$ for any given realized value z of $\widetilde{z}$:

$$
\begin{aligned}
r &= R_i(q_i, q_j, z), \text{ if } -\overline{z} \leq z < \widehat{z} \\
&= D_i = q_i, \quad \text{if } \overline{z} \geq z \geq \widehat{z},
\end{aligned}
$$

where $\widehat{z}$ is defined as

$$D_i = q_i = (a + \widehat{z} - q_i - q_j)q_i = R_i(q_i, q_j, \widehat{z}).$$

$$\widehat{z} = -(a - q_i - q_j - 1). \tag{4}$$

Following the assumption of Brander and Lewis (1986) for $\widehat{z}$, we assume that [11]

$$-\overline{z} < \widehat{z} < \overline{z}. \tag{5}$$

The expected profit function of firm i under limited liability is given by

$$
\begin{aligned}
E_{\widetilde{z}}\left[\pi_i(q_i, q_j, \widetilde{z})\right] &= \int_{-\overline{z}}^{\overline{z}} \left(R_i(q_i, q_j, z) - C_i(q_i) + D_i - r\right)\phi(z)dz \\
&= \int_{\widehat{z}}^{\overline{z}} (a + z - q_i - q_j - 1)q_i \frac{1}{2\overline{z}}dz + \int_{-\overline{z}}^{\widehat{z}} 0\phi(z)dz \\
&= \frac{1}{4\overline{z}} q_i(\overline{z} - \widehat{z})^2.
\end{aligned} \tag{6}
$$

We assume the following to guarantee a positive output in equilibrium.

[Assumption1]
$\overline{z} > \frac{1}{3}(a - 1)$.

[11] This assumption guarantees that $\widehat{z}$, the break-even realized value of $\widetilde{z}$, at which the expected net profit (sales) of the firms after full repayment D_i exists between the closed interval $[-\overline{z}, \overline{z}]$.

3 A Two-stage Game under Unlimited Liability

We first derive a two-stage duopoly game in which each firm chooses the mode of delegation, either *No delegation* (profit maximization) or *Delegation* (sales maximization), at the first stage; they then compete à la Cournot at the second stage under *unlimited liability.*

We consider second-stage games. Given that each firm chooses *No delegation* as its mode of delegation, we have an equilibrium in the Cournot duopoly game.

From (3), the first order condition is given by

$$a - 2q_i^{UN} - q_j^{UN} - 1 = 0, i, j = 1, 2, \tag{7}$$

where the superscript "UN" of q_i denotes that the mode of delegation for each firm is *No delegation* under *Unlimited liability.*

From (7) and (1), we can easily obtain each firm's output, total output, and expected price at the equilibrium:

$$\begin{aligned} q_i^{UN} &= \frac{1}{3}(a-1), i = 1, 2, \\ Q^{UN} &= \frac{2}{3}(a-1), \\ Ep^{UN} &= \frac{1}{3}(a+2). \end{aligned} \tag{8}$$

By (1) and (3), we have

$$\begin{aligned} E\pi_i^{UN} &= E[(p^{UN}-1)q_i^{UN}] = (q_i^{UN})^2 = \frac{1}{9}(a-1)^2, i = 1, 2, \\ EPS^{UN} &= 2E\pi_i^{UN} = \frac{2}{9}(a-1)^2, \\ ECS^{UN} &= \frac{1}{2}E[(a+\widetilde{z}-p^{UN})Q^{UN}] = \frac{1}{2}(Q^{UN})^2 = \frac{2}{9}(a-1)^2, \\ ESS^{UN} &= EPS^{UN} + ECS^{UN} = \frac{4}{9}(a-1)^2, \end{aligned} \tag{9}$$

where PS, CS, and SS denote producers' surplus, consumers' surplus, and social surplus, respectively.

Next, given that each firm chooses Delegation as its mode of delegation, a simple calculation provides us with the Cournot equilibrium. Each firm i maximizes its expected sales (revenue),

$$ER_i^{UD} = \max_{q_i} E[(a + \widetilde{z} - q_i^{UD} - q_j^{UD})q_i^{UD}],$$

where the superscript "UD" of q_i denotes that the mode of delegation for each firm is *Delegation* under *unlimited liability.*

The first order condition is

$$a - 2q_i^{UD} - q_j^{UD} = 0, i, j = 1, 2. \tag{10}$$

From (10) and (1), we can easily obtain each firm's output, the total output, and the expected price at the equilibrium:

$$\begin{aligned} q_i^{UD} &= \frac{1}{3}a, i = 1, 2, \\ Q^{UD} &= \frac{2}{3}a, \\ Ep^{UD} &= \frac{1}{3}a. \end{aligned} \tag{11}$$

By (1) and (3), we have[12]

$$\begin{aligned} E\pi_i^{UD} &= E[(p-1)q_i^{UD}] = \frac{1}{3}(a-3) \cdot \frac{1}{3}a = \frac{1}{9}a(a-3), i = 1, 2, \\ EPS^{UD} &= 2E\pi_i^{UD} = \frac{2}{9}a^2, \\ ECS^{UD} &= \frac{1}{2}E[(a + \widetilde{z} - p)Q^{UD}] = \frac{1}{2}(Q^{UD})^2 = \frac{2}{9}a^2, \\ ESS^{UD} &= EPS^{UD} + ECS^{UD} = \frac{2}{9}a(2a-3). \end{aligned} \tag{12}$$

We examine *the mixed delegation type Cournot duopoly*, in which one firm (say, firm 1) adopts *No delegation* (profit maximization) while another (say, firm 2) adopts *Delegation* (sales maximization) under *Unlimited liability.* We denote by *superscript "UN"* of the associated variables the *mixed-delegation type Cournot duopoly.*

[12] To guarantee positive expected profit , we assume that $a > 3$.

Setting $i = 1, j = 2$ in (7) and $i = 2, j = 1$ in (10) yields

$$a - 2q_1^{UND} - q_2^{UND} - 1 = 0,$$

$$a - 2q_2^{UND} - q_1^{UND} = 0,$$

where the superscript "UND" of $q_1(q_2)$ shows that the mode of delegation of firm 1 (firm 2) is *No delegation* (*Delegation*) under *Unlimited liability*.

Thus, we obtain

$$q_1^{UND} = \frac{1}{3}(a-2), q_2^{UND} = \frac{1}{3}(a+1). \tag{13}$$

From (1), we have

$$\begin{aligned} EQ^{UND} &= \frac{1}{3}(2a-1), \\ Ep^{UND} &= \frac{1}{3}(a+1). \end{aligned} \tag{14}$$

By (1) and (3), we have

$$\begin{aligned} E\pi_1^{UND} &= E[(p^{UND}-1)q_1^{UND}] = \frac{1}{9}(a-2)^2, \\ E\pi_2^{UND} &= E[(p^{UND}-1)q_2^{UND}] = \frac{1}{9}(a-2)(a+1), \\ EPS^{UND} &= E\pi_1^{UND} + E\pi_2^{UND} = \frac{1}{9}(a-2)(2a-1), \\ ECS^{UND} &= \frac{1}{2}E[(a+\widetilde{z}-p^{UND})Q^{UND}] = \frac{1}{2}(Q^{UND})^2 = \frac{1}{18}(2a-1)^2, \\ ESS^{UND} &= EPS^{UND} + ECS^{UND} = \frac{1}{18}(2a-1)(4a-5). \end{aligned} \tag{15}$$

From above equalities, we derive the following lemma.

Lemma 1 Suppose that $a > 3$. Then, we have
$q_2^{UND} > q_i^{UD} > q_i^{UN} > q_1^{UND}$, $Q^{UD} > Q^{UND} > Q^{UN}$, and $Ep^{UD} < Ep^{UND} < Ep^{UN}$. If

$a \geq 4$, then $E\pi_2^{UND} > E\pi_i^{UN} \geq E\pi_i^{UD} \geq E\pi_1^{UND}$. If $3 < a < 4$, then $E\pi_2^{UND} > E\pi_i^{UN} > E\pi_1^{UND} > E\pi_i^{UD}$.

The intuition for the lemma is clear. The *Delegation* (sales maximizer) firm in the *UD equilibrium* produces more aggressively than does the *No delegation* (profit-maximizer) firm in the *UN equilibrium* because the former acts without considering its cost. This result intrinsically corresponds to the result presented in Fershtman and Judd (1987). In the *UND* mixed-delegation type duopoly equilibrium, in addition to no consideration on cost, the strategic substitute property in Cournot competition makes the *Delegation* (sales maximization) firm 2 act more aggressively, so the *No delegation* (profit maximization) firm 1 reacts by shrinking its output as compared to the *UN* equilibrium.

To illustrate the result on the expected firm profits in the lemma, from (9) and (15), we have

$$\begin{aligned} E\pi_1^{UN} - E\pi_1^{UND} &= E[p^{UN} - 1]q_1^{UN} - E[p^{UND} - 1]q_1^{UND} \\ &= E[p^{UN} - 1](q_1^{UN} - q_1^{UND}) + E[p^{UN} - p^{UND}]q_1^{UND}. \end{aligned} \tag{16}$$

We consider the first stage game summarized in Table 1.

Firm 1/Firm 2	N	D
N	$E\pi_1^{UN}, E\pi_2^{UN}$	$E\pi_1^{UND}, E\pi_2^{UND}$
D	$E\pi_1^{UDN}, E\pi_2^{UDN}$	$E\pi_1^{UD}, E\pi_2^{UD}$

Let "N" and "D" denote the the modes of delegation "No delegation" and "Delegation," respectively.

Table 1 First stage game (under *unlimited liability*)

Note that $E\pi_2^{UDN} = E\pi_1^{UND}$ and $E\pi_1^{UDN} = E\pi_2^{UND}$. From Lemma 1 and Table 1, we derive;

Proposition 1 Suppose that $a \geq 4$. Then, the equilibrium mode of delegation is (D, D). Suppose that $3 < a < 4$. Then, the equilibrium mode is mixed, either (D, N) or (N, D).

We also present the following result.

Proposition 2 $ESS^{UD} > ESS^{UN} > ESS^{UN}$.

The Delegation duopoly is most desirable and it is attained as an equilibrium in the two-stage game for large demand ($a \geq 4$) under *unlimited liability.*

Note that there exists the mixed-delegation type equilibrium ((D, N) or (N, D)) when the potential demand is small because both no consideration on cost and the strategic substitute property in a Cournot competition makes the *Delegation* (sales maximization) firm 2 act more aggressively, so the *No delegation* (profit maximization) firm 1 reacts by greatly shrinking its output because of the small residual demand after deducting the expanding output of the *Delegation* firm 2 if the *No delegation firm 1 still may stay (may not exit from the market) in duopoly market.* However, owners of firm 1 choose *Delegation* instead of *No delegation* if the potential demand is large enough and she expects that residual demand after deduction of the expanding output of the *Delegation* firm 2 is sufficiently abandoned, and the equilibrium that may be attained is Delegation one, (D, D).

4 A Two-stage Game under Limited Liability

In this section, we derive a two-stage duopoly game in which each firm chooses its mode of delegation from *No delegation* and *Delegation* in the first stage; they then compete in à la Cournot fashion in the second stage under *limited liability.*

4.1 Cournot Subgame Composed of No delegation under Limited Liability

In this subsection, we consider Cournot duopoly comprising two firms with *No delegation* under limited liability. We denote the duopoly in which the mode of delegation of each firm is No delegation (profit-maximization) under *Limited liability* by *superscript "LN."* We name the Cournot equilibrium in this duopoly "*the LN equilibrium*," hereafter. Because the firm under limited liability repays $r \equiv \min\{R_i(q_i, q_j, z), D_i\}$ for some realized value z of $\widetilde{z}$, firm i ($i = 1, 2$) maximizes its *expected profit after deducting of repayment* to its investors.

The first order condition is given by

$$
\begin{aligned}
\frac{\partial \pi_i^{LN}}{\partial q_i} &= \int_{\widehat{z}^{LN}}^{\overline{z}} \frac{\partial}{\partial q_i}\left[R_i(q_i, q_j, z) - q_i\right]\phi(z)dz + (a + \widehat{z}^{LN} - q_i - q_j - 1)q_i \cdot \frac{\partial \widehat{z}^{LN}}{\partial q_i}\phi(\widehat{z}^{LN}) \\
&= \int_{\widehat{z}^{LN}}^{\overline{z}} (a + z - 2q_i - q_j - 1)\frac{1}{2\overline{z}}dz(\because (4)) \\
&= \frac{1}{2\overline{z}}\frac{(\overline{z} - \widehat{z}^{LN})}{2}\left[-q_i + \frac{\overline{z} - \widehat{z}^{LN}}{2}\right] = 0.
\end{aligned}
$$

Because $\overline{z} - \widehat{z} > 0$ holds from (5), we see that $-q_i + \frac{\overline{z}-\widehat{z}}{2} = 0$ holds.
Substituting (4) into this equality, we obtain

$$\frac{1}{2}\left(a - 3q_i^{LN} - q_j^{LN} - 1 + \overline{z}\right) = 0, i = 1, 2. \tag{17}$$

From the symmetry of q_i^{LN} and q_j^{LN}, setting $q_i^{LN} = q_j^{LN} = q^{LN}$ in (17), we obtain

$$q^{LN} = \frac{1}{4}(t - 1) > 0, \tag{18}$$

where $t \equiv a + \overline{z}$.[13]
By (18) and (1), we see that

$$E\left[p^{LN}\right] \equiv E\left[a + \widetilde{z} - 2q^{LN}\right] = \frac{1}{2}(2a + 1 - t).$$

$$\widehat{z}^{LN} = -\frac{1}{2}(a - \overline{z} - 1). \tag{19}$$

We can show that $\overline{z} < \widehat{z}^{LN} < \overline{z}$.[14]
Hence, we obtain the equilibrium net expected profit of firm i from (6), (19), and (20)

[13] The inequality holds from the assumption 1. From $t = a + \overline{z} > 1 + \frac{1}{3}(a-1) > 1$ holds because $a > 1 > 0$.

[14]

$$\overline{z} - \widehat{z}^{LND} = \frac{1}{2}(t-1) = \frac{1}{2}(a + \overline{z} - 1) > \frac{1}{2}\left(a + \frac{1}{3}(a-1)\right) > \frac{1}{6}(a-1) > 0 \tag{20}$$

and

$$(\widehat{z}^{LND}) - (-\overline{z}) = \overline{z} + \widehat{z}^{LND} = \frac{1}{2}(3\overline{z} - a + 1) > 0,$$

where the inequality holds from assumption 1.

$$
\begin{aligned}
\pi_i^{LN} \equiv \pi^{LN} &= \int_{\widehat{z}^{LN}}^{\overline{z}} (a + z - 2q^{LN} - 1)q^{LN} \cdot \frac{1}{2\overline{z}} dz \\
&= \frac{1}{2\overline{z}} q^{LN} \cdot \frac{1}{2} \left(\overline{z} - \widehat{z}^{LN}\right)^2 (\because (17)) \\
&= \frac{1}{\overline{z}} \left(q^{LN}\right)^3 = \frac{1}{64\overline{z}} (t-1)^3, i = 1, 2.
\end{aligned} \tag{21}
$$

4.2 Cournot Subgame Composed of Delegation Firms under Limited Liability

In this subsection, we derive a Cournot duopoly composed of two firms with *Delegation* under *Limited liability.* We denote the duopoly in which the choice of the mode of delegation of each firm is *Delegation* (sales maximization) by *superscript "LD"* Furthermore, we call the Cournot equilibrium in this duopoly *"the LD equilibrium."* Although a firm with *Delegation mode has to repay all of her sales to investors when its sales less are than* D_j, it does not care about any repayment when its sales are more than D_j under limited liability. That is, firm j maximizes its net sales $\overline{R}_j(q_j, q_i, z)$, defined by

$$
\begin{aligned}
\overline{R}_j(q_j, q_i, z) &= R_j(q_j, q_i, z) - r = 0, \text{ if } \text{-}\overline{z} \leq z < \widehat{z}^{LD} \\
&= R_j(q_j, q_i, z), \text{ otherwise.}
\end{aligned}
$$

Hence, the expected sales maximization problem for firm j $(j = 1, 2)$ under limited liability is given by

$$
\begin{aligned}
R_j^{LD} &\equiv \max_{q_j} \int_{-\overline{z}^{LD}}^{\overline{z}} \overline{R}_j(q_j, q_i, z)\phi(z)dz \\
&= \max_{q_j} \int_{\widehat{z}^{LD}}^{\overline{z}} R_j(q_j, q_i, z)\phi(z)dz \\
&= \max_{q_j} \frac{1}{2\overline{z}} (\overline{z} - \widehat{z}^{LD})^2 \frac{1}{2} q_j \\
&= \max_{q_j} \frac{1}{4\overline{z}} q_j \left\{ (a + \overline{z} - q_j - q_j)^2 - 1 \right\}.
\end{aligned} \tag{22}
$$

The first order condition is

$$\frac{\partial R_j^{LD}}{\partial q_j} = \frac{1}{4\overline{z}} \left[\left(a + \overline{z} - q_j^{LD} - q_i^{LD}\right)\left(a + \overline{z} - 3q_j^{LD} - q_i^{LD}\right) - 1\right] = 0,$$

or

$$\left(a + \overline{z} - q_j^{LD} - q_i^{LD}\right)\left(a + \overline{z} - 3q_j^{LD} - q_i^{LD}\right) - 1 = 0. \tag{23}$$

From the symmetry of q_i^{LD} and q_j^{LD}, and by setting $q^{LD} = q_i^{LD} = q_j^{LD}$ in (23), we then obtain the quadratic equation of q^{LD},

$$t^2 - 1 - 6tq^{LD} + 8(q^{LD})^2 = 0.$$

This quadratic equation has two distinct real solutions,

$$q^{LD} = \frac{1}{8}\left(3t + \sqrt{t^2+8}\right),\ \frac{1}{8}\left(3t - \sqrt{t^2+8}\right).$$

Because the former solution violates the condition (5), that is, $\overline{z} - \widehat{z}^{LD} = a + \overline{z} - 2q^{LD} - 1 = t - Q^{LD} - 1 > 0$.

Consequently, the equilibrium output of each firm and $\widehat{z}^{LD}$ are

$$q^{LD} = \frac{1}{8}\left(3t - \sqrt{t^2+8}\right), \tag{24}$$

$$\widehat{z}^{LD} = -\frac{1}{4}\left(4(a-1) - 3t + \sqrt{t^2+8}\right). \tag{25}$$

Lemma 2 If $t \equiv a + \overline{z} > 1$and $t > \frac{1}{12}(7(2a-1) + \sqrt{4a^2 - 4a + 25})$, then $\widehat{z}^{LD}$ satisfies assumption (5).

Proof: See the appendix.

Hence, we obtain the *ex ante* equilibrium expected net profit of firm j from (6),(24), and (25),

$$\pi_j^{LD} = \pi^{LD} = \int_{\widehat{z}^{LD}}^{\overline{z}} (a + z - 2q^{LD} - 1)q^{LD} \cdot \frac{1}{2\overline{z}} dz$$
$$= \frac{1}{512\overline{z}} \left(3t - \sqrt{t^2+8}\right) \left(t - 4 + \sqrt{t^2+8}\right)^2. \tag{26}$$

Comparing each firm's output, total output, expected price, and expected net profit derived in the preceding section with those derived in this section, we obtain the following proposition.

Proposition 3 Each firm produces more and earns higher profits in the *LD equilibrium* than in the *LN equilibrium*. Consequently, the total output in the former equilibrium is greater and the expected price in the former is lower than in the latter one. Formally, $q^{LD} > q^{LN}, Q^{LD} > Q^{LN}, E[p^{LD}] < E[p^{LN}]$ and $\pi_j^{LD} > \pi_i^{LN}$.

Proof: See the appendix.

We provide the following intuitive explanation of the results. From (21) and (22), each firm maximizes its expected profit *after deducting the repayment* to its investor for *any* realized value of z in the *LN equilibrium*. In the *LD equilibrium*, each firm maximizes its sales *after repayment all of its sales* to investors when the state of nature is *bad* ($-\overline{z} \leq z \leq \widehat{z}$), so her sales are less than the debt. She maximizes its sales *without considering any repayment* to its investors when the state of nature is *good* ($\widehat{z} < z \leq \overline{z}$) so its sales are more than the debt. Therefore, each firm in the *LD equilibrium* produces its output more aggressively than it does in the *LN equilibrium*. Consequently, the latter earns more than the former.

4.3 Cournot Subgame Composed of Mixed-Delegation under Limited Liability

In this subsection, we consider a new type of duopoly that has yet to be considered in the related literature, including Brander and Lewis (1986) and Fershtman and Judd (1987). That is, we consider *the mixed-delegation* Cournot duopoly, in which the mode of delegation of firm 1 is *No delegation* (profit maximization), while that of another is *Delegation* (sales maximization) under *limited liability*. We denote this mixed-delegation type duopoly by superscript *"LND(LDN)."* Furthermore, we call the Cournot equilibrium in this duopoly *"the Mixed Delegation equilibrium."*

We assume that the mode of delegation of firm 1 (2) is *No delegation* (*Delegation*).

From (4) and (17), the first order condition for firm 1 is given by

$$\frac{1}{2}(t-1-3q_1^{LND}-q_2^{LND})=0.$$

From (4) and (23), the first order condition for firm 2 with Delegation in the mixed delegation type duopoly is given by

$$\left(t-q_2^{LND}-q_1^{LND}\right)\left(t-3q_2^{LN}-q_1^{LND}\right)-1=0 \tag{27}$$

From the above equalities, we have

$$q_1^{LND}=\frac{1}{16}(2t-7+\sqrt{4t^2+4t+17}), \tag{28}$$

where $t>1$. For a detailed derivation of q_1^{LND}, see the appendix. Thus, we also have

$$q_2^{LND}=\frac{1}{16}(5(2t+1)-3\sqrt{4t^2+4t+17}). \tag{29}$$

That is, when the sum of latent demand a and the degree of the demand risk $\overline{z}$ is sufficiently larger than the marginal cost $c=1$, the sales maximizing firm 2 expands its output aggressively under limited liability, so that profit maximizing firm 1 shrinks its output on account of stronger strategic substitute effect in *the Mixed Delegation equilibrium* than that in the *LN* and *LD equilibria*.

By (28), (29) and (1), we see that

$$E\left[p^{LND}\right]\equiv E\left[a+\widetilde{z}-q_2^{LND}-q_1^{LND}\right]=a-\frac{1}{8}(6t-1-\sqrt{4t^2+4t+17})>0, \tag{30}$$

for $t>1$. For $\widehat{z}^{LND}$ to satisfy (5), we derive the following lemma.

Lemma 3 If a satisfies the condition that

$$\frac{1}{16}(7(2t+1)-\sqrt{4t^2+4t+17})>a>\frac{1}{8}(6t+7-\sqrt{4t^2+4t+17}), \tag{31}$$

then $\widehat{z}^{LND}$ satisfies assumption (5)[15].

Proof: See the appendix.

Hence, for the Mixed Delegation equilibrium to exist, both (5) and (31) must hold. That is, these conditions imply that there must exist sufficient latent demand or some extent of demand risk (high value of $\overline{z}$ and a) for Delegation firm 2 to expand its output aggressively, while No delegation firm 1 can survive in the Mixed Delegation duopoly. We derive the expected profit of the No delegation (profit maximization) firm 1, π_1^{LND}, and the expected profit of the Delegation (sales maximization) firm 2, π_2^{LND}.

$$\begin{aligned}\pi_1^{LND} &= \int_{\widehat{z}^{LND}}^{\overline{z}} (a+z-q_1^{LND}-q_2^{LND}-1)q_1^{LND}\cdot\frac{1}{2\overline{z}}dz \\ &= \frac{1}{\overline{z}}\left(q_1^{LND}\right)^3 = \frac{1}{16^3\overline{z}}\left(2t-7+\sqrt{4t^2+4t+17}\right)^3, \end{aligned} \tag{32}$$

$$\begin{aligned}\pi_2^{LND} &= \int_{\widehat{z}^{LND}}^{\overline{z}} (a+z-q_1^{LND}-q_2^{LND}-1)q_2^{LND}\cdot\frac{1}{2\overline{z}}dz \\ &= \frac{1}{2\overline{z}}q_2^{LND}\cdot\frac{1}{2}\left(\overline{z}-\widehat{z}^{LND}\right)^2 \\ &= \frac{1}{\overline{z}}\left(q_2^{LND}\right)(q_1^{LND})^2 = \frac{1}{16^3\overline{z}}(5(2t+1)-3\sqrt{4t^2+4t+17})\left(2t-7+\sqrt{4t^2+4t+17}\right)^2. \end{aligned} \tag{33}$$

From (32) and (33), we have

$$\pi_1^{LND} - \pi L_2^{LND} = \frac{1}{\overline{z}}\left(q_1^{LND}\right)^2(q_1^{LND}-q_2^{LND}) < 0.$$

From (28) and (29), we can easily show that $q_1^{LND} - q_2^{LND} < 0 \Leftrightarrow t > 1$. However, both ((A.5)) and (31) must hold for the existence of the Mixed Delegation equilibrium. We next present the following proposition.

Proposition 4 If the sum of latent demand and the degree of the demand risk is sufficiently large compared to its marginal cost, then Delegation firm 2 produces more

[15] We can easily show that $\frac{1}{16}(7(2t+1)-\sqrt{4t^2+4t+17}) > \frac{1}{8}(6t+7-\sqrt{4t^2+4t+17})$,for $t>1$.

output and earns more than No delegation firm 1 does in the Mixed Delegation *LND* equilibrium. Formally, if $t > 1$, then $q_1^{LND} < q_2^{LND}$ and $\pi_1^{LND} < \pi_2^{LND}$ hold.

Comparing each firm's equilibrium output in the *LN*, *LD*, and the Mixed Delegation (*LND, LDN*) equilibria from the arguments in the preceding two sections, we can present the following lemma and proposition. We restrict our attention to the case in which there exists the Mixed Delegation (*LND,LDN*) equilibrium, that is, we assume that the condition on a and t, (31) hold.

Lemma 4 If t, the sum of latent demand and the degree of the demand risk, is larger than its marginal cost 1, then Delegation firm 2 in the *LND* equilibrium produces more output than each No delegation firm 1 does in the *LN* equilibrium and each Delegation firm in the *LD* equilibrium. If t is larger than 1, then No delegation 1 in the *LND* equilibrium produces less than each Delegation firm does in the *LD* equilibrium and each No delegation firm in the *LN* equilibrium. Formally, if $t > 1$, then $q_2^{LND} > q^{LN}$, $q_2^{LND} > q^{LD}$, $q_1^{LND} < q^{LD}$, and $q_1^{LND} < q^{LN}$.

Proof: See the appendix.

From Proposition 4 and Lemma 4, we can easily show the following proposition.

Proposition 5 *If* $t > 1$, *then* $q_1^{LND}(= q_2^{LDN}) < q^{LN} < q^{LD} < q_2^{LND}(= q_1^{LDN})$.

From (18), (24), (29), (28) and the fact that

$$Q^{LN} = 2q^{LN}, Q^{LM} \equiv q_1^{LND} + q_2^{LND} = q_1^{LDN} + q_2^{LDN}, Q^{LD} = 2q^{LD} \tag{34}$$

and

$$Ep^k = E\left[a + \widetilde{z} - Q^k\right], \, k = LN, LD, LM. \tag{35}$$

Using the fact above, we can show the corollary.

Corollary 1 If t is larger than 1, then *the total output in LD equilibrium* is *the most*, *the one in the LN equilibrium the least*, and the one in the *Mixed Delegation (LND or LDN) equilibrium* lies between them. In consequent, *the expected price in LD equilibrium* is *the highest*, *the one in the LN equilibrium the lowest*, and the one in the *Mixed Delegation(LND or LDN) equilibrium* lies between them. Formally, *if* $t > 1$, *then* $Q^{LN} < Q^{LM} < Q^{LD}$ *and* $Ep^{LN} > Ep^{LM} > Ep^{LD}$,where $LM = LND$ or LDN.

Proof: See the appendix.

4.4 First Stage Equilibrium in the Whole Game under Limited Liability

We examine the first stage under limited liability, and derive a subgame perfect equilibrium of the whole game. The first stage is summarized as Table 2.

Firm 1/Firm 2	N	D
N	π_1^{LN}, π_2^{LN}	π_1^{LND}, π_2^{LND}
D	π_1^{LDN}, π_2^{LDN}	π_1^{LD}, π_2^{LD}

Table 2 First stage game (under *Limited liability*)

Let "N" and "D" denote the modes of delegation"No delegation" and "Delegation," respectively.

We compare the expected profits derived in previous sections. Note that $t = a+\overline{z} > 1$ must hold to guarantee the existence of the Mixed Delegation equilibrium. We see that $\pi_1^{LDN}(= \pi_2^{LND})$ and π_2^{LDN} $(= \pi_1^{LND})$are positive from (33) and (32) if $t > 1$. From (6), we have

$$\begin{aligned}\pi_1^{LDN} - \pi_1^{LN} &= \frac{1}{4\overline{z}}[q_1^{LDN}(a+\overline{z}-1-Q^{LM})^2 - q_1^{LN}(a+\overline{z}-1-Q^{LN})^2] \\ &> \frac{1}{4\overline{z}}(q_1^{LDN} - q_1^{LN})(a+\overline{z}-1-Q^{LN})^2 > 0. \end{aligned} \tag{36}$$

Note that $\pi_2^{LND} - \pi_2^{LN} = \pi_1^{LDN} - \pi_1^{LN}$. From (32) and (26), we find

$$\begin{aligned}\pi_1^{LND} - \pi_1^{LD} &= \frac{1}{4\overline{z}}\{q_1^{LND}(a+\overline{z}-1-Q^{LM})^2 - q_1^{LD}(a+\overline{z}-1-Q^{LD})^2\} \\ &< \frac{1}{4\overline{z}}(q_1^{LND} - q_1^{LD})(a+\overline{z}-1-Q^{LM})^2 < 0 \end{aligned} \tag{37}$$

Note that $\pi_1^{LND} - \pi_1^{LD} = \pi_2^{LDN} - \pi_2^{LD}$. Then, we can straightforwardly show that the next lemma from (36) and (37).

Lemma 5 *If* $t > 1$, then the *Delegation firm* in the *LDN or LND* (Mixed Delegation) equilibrium earns more than the *No delegation* firm in the *LN* equilibrium, and the *No delegation firm* in the *LDN or LND* (Mixed Delegation) equilibrium earns less than the *Delegation firm* in the *LD* equilibrium. Formally, if $t > 1$, then $\pi_1^{LDN}(= \pi_2^{LND}) > \pi_1^{LN}(= \pi_2^{LN})$ *and* $\pi_1^{LND}(= \pi_2^{LDN}) < \pi_1^{LD}(= \pi_2^{LD})$.

By Table 2 and Lemma 5, we can immediately derive the equilibrium of the two-stage game.

Proposition 6 If $t > 1$, the equilibrium mode of delegation of the whole game is the *LD* equilibrium, that is, (D, D).

As we have already shown in section 3, the Mixed Delegation equilibrium may be attained at least when the potential demand is small under *unlimited* liability. However, the shareholders may *be too eager to undertake risky investment* by debt finance under *limited* liability [16]. Therefore firm owners choose *Delegation* instead of *No delegation* when facing the Delegation rival firm for any potential demand and demand risk over unit cost under *limited* liability. As suggested in Corollary 1, the equilibrium total output is largest and the equilibrium expected price is lowest in the *LD* equilibrium. Hence, the equilibrium is expected to be socially efficient, and in the next section we show that this expectation holds.

4.5 Application case: The Perverse Effect of Competition Mechanism in the U.S. S&L Crisis

The result we previously derived illustrates how the competition among heterogeneous firms works in an oligopoly *in the U.S. S&L crisis context* in Milgrom and Roberts (1992). In our LDN equilibrium given in Lemma 5, *Delegation firm 1* and No delegation firm 2 in our duopoly correspond to *the S&L that directly saw the chance to exploit the FSLIC deposit insurance system by moving into more risky investments* and the S&L who held out and made only safe investments in the S&L industry, respectively. As we have shown

[16] Milgrom and Roberts (1992) state in subsection 'Confliction Interests: Current Lenders versus Other Capital Suppliers' of Chapter 15 that "This is because the owners of shares enjoy virtually all the benefits if returns on the risky investments turn out to be high, but the lenders suffer a major portion of losses if the returns turnout to be low."

in Lemma 5, however, the profit maximizing shareholders of No delegation firm 2 are better off if they change the mode of delegation from No delegation to Delegation. This result implies that in a S&L crisis context, the S&L sound-manager is better off if she changes her management to gamble on risky investments. In this sense, we present an explanation of how the perverse effect of competition works caused by intrinsic characteristics of *the FSLIC deposit insurance system* which plays a similar role to *a limited liability system* in our model by a simple but formal model analysis, although we do not explicitly address the moral hazard problem.

In our model, investors (banks) to each firm, the manager of firm, and the limited liability system corresponds to depositors of each S&L association, the S&L manager, and the Federal Savings and Loan Insurance Corporation (FSLIC)'s insurance for bank deposit, respectively, if we apply our setting to the U.S. S&L crisis [17]. In the result given in the paper, *Delegation* (sales maximizing) *firm 1* and *No delegation* (profit maximizing) firm 2 in our duopoly correspond to *the S&L that directly saw the chance to exploit the FSLIC deposit insurance system by moving into more risky investments* and the S&L who held out and made only safe investments in the S&L industry, respectively.

5 Welfare Comparison

In this section, we compare the expected social welfare at the three equilibria derived in the preceding section, and we evaluate the equilibrium of the whole game from a social welfare perspective.

The expected social surplus is the sum of the net expected producer surplus, the expected surplus of the bank (investor), and the expected consumer surplus.

The net expected producer surplus at the k $(= LN, LD$, and LM $(LND$ or $LDN))$

[17] See footnote 5 in section 1 for introduction, in detail.

equilibrium is expressed as

$$
\begin{aligned}
EPS^k &= \int_{-\overline{z}}^{\overline{z}} PS^k(Q^k, z)\phi(z)dz \\
&= \int_{-\overline{z}}^{\overline{z}} (\pi_1(q_i, q_j, z) + \pi_2(q_i, q_j, z) - r_\pi)\phi(z)dz \\
&= \frac{1}{2\overline{z}} \int_{-\overline{z}}^{\widehat{z}^k} \{(a + z - Q^k)Q^k - (a + z - Q^k)Q^k\}dz + \frac{1}{2\overline{z}} \int_{\widehat{z}^k}^{\overline{z}} (a + z - Q^k - 1)Q^k dz \\
&= \frac{1}{2\overline{z}} \int_{\widehat{z}^k}^{\overline{z}} (a + z - Q^k - 1)Q^k dz. \qquad (38)
\end{aligned}
$$

The expected profit (losses) of bank at the k $(= LN, LD$, and $LM)$ equilibrium is given by

$$
\begin{aligned}
EBP^k &= \frac{1}{2\overline{z}} \int_{-\overline{z}}^{\widehat{z}^k} \{(a + z - Q^k)Q^k - D_i\}dz + \frac{1}{2\overline{z}} \int_{\widehat{z}^k}^{\overline{z}} (D_i - Q^k)dz \\
&= \frac{1}{2\overline{z}} \int_{-\overline{z}}^{\widehat{z}^k} (a + z - Q^k - 1)Q^k dz + \frac{1}{2\overline{z}} \int_{\widehat{z}^k}^{\overline{z}} (Q^k - Q^k)dz \\
&= \frac{1}{2\overline{z}} \int_{-\overline{z}}^{\widehat{z}^k} (a + z - Q^k - 1)Q^k dz \qquad (39)
\end{aligned}
$$

The expected consumers' surplus at the k $(= LN, LD$, and LM $(LND$ or $LDN))$ equilibrium is

$$
\begin{aligned}
ECS^k &= \int_{-\overline{z}}^{\overline{z}} CS^k(Q^k, z)\phi(z)dz \\
&= \int_{-\overline{z}}^{\overline{z}} \frac{1}{2}\{a + z - (a + z - Q^k)\}Q^k \phi(z)dz \\
&= \frac{1}{2\overline{z}} \int_{-\overline{z}}^{\overline{z}} \frac{1}{2}(Q^k)^2 dz = \frac{1}{2}(Q^k)^2. \qquad (40)
\end{aligned}
$$

We define the expected social surplus at the k $(= LN, LD$, and LM $(LND$ or $LDN))$ equilibrium as

$$
ESS^k = EPS^k + EBP^k + ECS^k. \qquad (41)
$$

Substituting (38), (39), and (40) into (41) and rearranging yield

$$\begin{aligned}
ESS^k &= \frac{1}{2\overline{z}}\int_{\widehat{z}^k}^{\overline{z}}(a+z-Q^k-1)Q^k dz + \frac{1}{2\overline{z}}\int_{-\overline{z}}^{\widehat{z}^k}(a+z-Q^k-1)Q^k dz + \frac{1}{2}(Q^k)^2 \\
&= \frac{1}{2\overline{z}}\int_{-\overline{z}}^{\overline{z}}(a+z-Q^k-1)Q^k dz + \frac{1}{2}(Q^k)^2 \\
&= \frac{1}{2\overline{z}}Q^k\left[(a-Q^k-1)z+\frac{z^2}{2}\right]_{-\overline{z}}^{\overline{z}} + \frac{1}{2}(Q^k)^2 \\
&= (a-Q^k-1)Q^k + \frac{1}{2}(Q^k)^2 \\
&= (a-\frac{1}{2}Q^k-1)Q^k \equiv F(Q^k). \qquad (42)
\end{aligned}$$

That is, the expected social surplus at the k equilibrium is expressed by a concave quadratic function of the total output of each equilibrium.

The function F has a maximum at $Q^* = a-1$. Then, we have

$$F'(Q^k) \gtreqless 0, \text{ if } Q^k \lesseqgtr Q^* = a-1. \qquad (43)$$

Because $E[p^k - 1] = E[a+\overline{z}-1-Q^k] = a-1-Q^k = Q^* - Q^k > 0$, $Q^* > Q^k$ holds.

Note that $t = a+\overline{z} > 1$ guarantees the existence of the Mixed Delegation equilibrium. We can easily show the following proposition from Corollary 1. Hence, the proof of the proposition is omitted.

Proposition 7 If t is larger than 1, then *the expected social surplus in the LD equilibrium* is *the largest*, *that in the LN equilibrium the least*, and that in the Mixed Delegation equilibrium lies between them under *limited liability*. Formally if $t > 1$, then $ESS^{LN} < ESS^{LM} < ESS^{LD}$ ($LM = LND$ or LDN) holds.

From the results derived in the preceding and this sections, we see that *Delegation duopoly equilibrium* is always desirable most a social welfare perspective, and it can be attained as an equilibrium in the two-stage game under *limited liability*. That is, we can interpret this result as managerial incentives with less-weighed to profit prevail and it always attains efficient *Delegation duopoly equilibrium* in duopoly under a *limited liability system* irrespective of potential demand and demand risk under *unlimited liability*, although investors (banks) do not play any positive role as economic agents to achieve their own objectives, but only lend an amount equal to the requested variable cost.

From the result derived in section 3, note that the *Mixed delegation* (one firm chooses No delegation and another chooses Delegation) duopoly prevails as an equilibrium when potential demand is small, however the Delegation duopoly prevails when this demand becomes large.

This result of course holds only for the scenario in which the moral hazard problem never exists, as is the case for this work. In the real business world, lenders (banks) take precautions to ensure that their money is not squandered, or put at unnecessary risk by those who have borrowed it, because there exists the moral hazard problem of borrowers. That is, they monitor what they lend by examining the firms' financial condition and credit history and by placing restrictions on how their funds may be used. Our analysis ignores this moral hazard problem and so also ignores such monitoring activities of investors. The results of this welfare analysis may differ if the analysis considers the moral hazard problem.

6 Discussion and Concluding Remarks

In this article, we consider a duopoly with additive demand uncertainty in which there are two firms producing and supplying a homogeneous good with an identical constant return to scale technology. We consider *No delegation firm* and *Delegation firm* the two polar types of incentive contracts, one of which attaches great importance to *profit* and another of which places less emphasis on profit but more on *sales*. First, we derive a two-stage duopoly game in which shareholders of each firm choose the mode of delegation between No delegation (profit maximization) and Delegation (sales maximization) at the first stage, then compete in à la Cournot fashion at the second stage under *unlimited liability*. By deriving a subgame perfect equilibrium in this two-stage game, we show that a Mixed delegation-type duopoly occurs as an SPE for small potential demand; however, a Delegation duopoly occurs as an equilibrium for large potential demand under *unlimited liability*. Furthermore, we show that *a Delegation duopoly* is always the most efficient from a social welfare perspective and that it is attained as an equilibrium of the two-stage game for large potential demand under *unlimited liability*.

Next, by deriving three equilibria of Cournot duopoly subgames under *limited liability*, we explore how differences in the mode of delegation and limited liability affect the strategic behavior of firms and the outcomes. Then, we derive a subgame perfect equilibrium this two-stage entire game in which each firm's mode of delegation choices are endogenized under *limited liability*. Consequently, we show that *a Delegation duopoly* is always attained as an equilibrium under *limited liability*. Though in our model investors (banks) do not play any positive role as economic agents to achieve their own objectives but only lend an amount equal to the requested variable cost, we present an explanation of how the competition among heterogeneous firms in an oligopoly works *caused by the intrinsic characteristics of a limited liability system* by a simple but formal model analysis by the derived result. We present the perverse effect of competition mechanism in the U.S. S&L crisis context in Milgrom and Roberts (1992) as an application to the real case, in which our result illustrates how the competition among heterogeneous firms in an oligopoly works caused by intrinsic characteristics of *the FSLIC deposit insurance system* which plays a similar role to *a limited liability system* in our model.

Furthermore, we define an expected social welfare at the equilibria and compare the expected social welfare at the three equilibria previously derived. We find that *a Delegation duopoly* under *limited liability is the most desirable* from a social welfare perspective for *any* value of the sum of potential demand and demand risk that is larger than unit cost.

There remains much work to consider. In the paper, we consider the relationship between strategic delegation and the limited liability system only in a Cournot duopoly with strategic substitute model structure. The first issue to address is the extension of our analysis to an oligopoly setting. Furthermore, as Etro (2012) emphasizes in his paper, it is important to consider the relationship between strategic delegation and the limited liability system in a price competition setting with strategic complementarity. This issue also remains to be addressed by our future research.

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Appendix

Proof of Lemma 2

Proof: For $\widehat{z}^{LD}$ to satisfy the assumption (5), we also have from assumption 1 and (20)

$$\begin{aligned}\overline{z} - \widehat{z}^{LD} &= a + \overline{z} - Q^{LD} - 1 \\ &= \frac{1}{4}\left(\sqrt{t^2+8} - (t-4\right) \\ &= \frac{1}{4}\left(\sqrt{t^2+8} - (t-4)\right) \quad > 0 \Leftrightarrow 8(t-1) > 0 \Leftrightarrow t > 1,\end{aligned} \tag{A.1}$$

where $t \equiv a + \overline{z} > 1$.

We can also show that

$$\begin{aligned}\overline{z} - (-\widehat{z}L^{LD}) &= \overline{z} + z^{LD} \\ &= \frac{1}{4}\left(-a - \overline{z} + 8\overline{z} + 4 - \sqrt{(a+\overline{z})^2+8}\right) \\ &= \frac{1}{4}\left(7t - 8a + 4 - \sqrt{t^2+8}\right) > 0 \\ &\Leftrightarrow 6t^2 - 7(2a-1) + 8a^2 - 8a + 1 > 0\end{aligned}$$

equivalently

$$t < \frac{1}{12}(7(2a-1) - \sqrt{4a^2-4a+25}), \frac{1}{12}(7(2a-1) + \sqrt{4a^2-4a+25}) < t \tag{A.2}$$

In order for both (A.1) and (A.2) to hold , we see that

$$1 < t < \frac{1}{12}(7(2a-1) - \sqrt{4a^2 - 4a + 25}) \Leftrightarrow a > \frac{7}{4},$$
$$t > \frac{1}{12}(7(2a-1) + \sqrt{4a^2 - 4a + 25}) \Leftrightarrow a > 1 \text{ hold.}$$

However, the first inequality of the above is invalid since $t \equiv a + \overline{z} > 1$. Hence, the lemma holds.■

Proof of Proposition 3

Proof: From(18)and (24), $q^{LD} - q^{LN} = \frac{1}{8}\left(3(a+\overline{z}) - \sqrt{(a+\overline{z})^2+8}\right) - \frac{1}{4}(a+\overline{z}-1) = \frac{1}{8}(t+2-\sqrt{t^2+8})$. $(t+2)^2 - (\sqrt{t^2+8})^2 = 4(t-1) > 0$ for $t > 1$,where $t = a + \overline{z}$. From (**??**) and (26), we see that the expected net profit at the LN andLD equilibrium can be expressed as $\pi^k(q^k) = \frac{1}{4\overline{z}} q^k(\overline{z} - \widehat{z}^k)^2$, $k = LN, LD$. We also see that $\frac{d\widehat{z}^k}{dq^k} = 1 > 0$ from (4) and the symmetry of each firm output at the LND and LDN equilibria.$\frac{\partial \pi^t(q^k)}{\partial q^k} = \frac{1}{4\overline{z}}\left((\overline{z} - \widehat{z}^k)^2 + 2q^k(\overline{z} - \widehat{z}^k)\frac{\partial \widehat{z}^k}{\partial q^k}\right) = \frac{1}{4\overline{z}}\left((\overline{z} - \widehat{z}^k)^2 + 2q^k(\overline{z} - \widehat{z}^k)1\right) > 0$, $k = LN, LD$ form (6) and (4). Hence the result follows from $q^{LN} > q^{LD}$.■

Derivation of q_1^{LNDPS}

Solving (27) with respect to q_1^{LND}, we obtain

$$q_1^{LND} = \frac{1}{3}\left(t - 1 - q_2^{LND}\right) \tag{A.3}$$

Substituting ((A.3)) into (27) and rearranging it yields the quadratic equation of q_2^{LND},

$$\frac{1}{9}\{((2t+1)^2 - 10(2t+1))q_2^{LND} + 16(q_2^{LND})^2 - 9\} = 0.$$

Solving this equation, we get

$$q_2^{LND} = \frac{1}{16}(5(2t+1) \pm 3\sqrt{4t^2+4t+17}).$$

Substituting this into ((A.3)) and rearranging it, we have

$$q_1^{LND} = \frac{1}{16}(2t - 7 \mp \sqrt{4t^2 + 4t + 17}).$$

Since $q_1^{LND} > 0$ for $t > 1$, however, we have

$q_1^{LND} = \frac{1}{16}(2t - 7 + \sqrt{4t^2 + 4t + 17})$,where $t > 1$.■

Proof of Proposition 3

Proof: From(18)and (24), $q^{LD} - q^{LN} = \frac{1}{8}\left(3(a + \overline{z}) - \sqrt{(a + \overline{z})^2 + 8}\right) - \frac{1}{4}(a + \overline{z} - 1) = \frac{1}{8}(t + 2 - \sqrt{t^2 + 8})$. $(t + 2)^2 - (\sqrt{t^2 + 8})^2 = 4(t - 1) > 0$ for $t > 1$,where $t = a + \overline{z}$. From (**??**) and (26), we see that the expected net profit at the LN andLD equilibrium can be expressed as $\pi^k(q^k) = \frac{1}{4\overline{z}}q^k(\overline{z} - \widehat{z}^k)^2$, $k = LN, LD$. We also see that $\frac{d\widehat{z}^k}{dq^k} = 1 > 0$ from (4) and the symmetry of each firm output at the LND and LDN equilibria.$\frac{\partial \pi^t(q^k)}{\partial q^k} = \frac{1}{4\overline{z}}\left((\overline{z} - \widehat{z}^k)^2 + 2q^k(\overline{z} - \widehat{z}^k)\frac{\partial \widehat{z}^k}{\partial q^k}\right) = \frac{1}{4\overline{z}}\left((\overline{z} - \widehat{z}^k)^2 + 2q^k(\overline{z} - \widehat{z}^k)1\right) > 0$, $k = LN, LD$ form (6) and (4). Hence the result follows from $q^{LN} > q^{LD}$.■

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Solving (27) with respect to q_1^{LND}, we obtain

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Substituting (A.3) into (27) and rearranging it yields the quadratic equation of q_2^{LND},

$$\frac{1}{9}\{((2t + 1)^2 - 10(2t + 1))q_2^{LND} + 16(q_2^{LND})^2 - 9\} = 0.$$

Solving this equation, we get

$$q_2^{LND} = \frac{1}{16}(5(2t + 1) \pm 3\sqrt{4t^2 + 4t + 17}).$$

Substituting this into ((A.3)) and rearranging it, we have

$$q_1^{LND} = \frac{1}{16}(2t - 7 \mp \sqrt{4t^2 + 4t + 17}).$$

Since $q_1^{LND} > 0$ for $t > 1$, however, we have

$q_1^{LND} = \frac{1}{16}(2t - 7 + \sqrt{4t^2 + 4t + 17})$,where $t > 1$.■

Proof of Lemma 3

Proof: From the above equation and (4), (29) , (28) and (30), we have

$$\begin{aligned}\widehat{z}^{LND} &= -(a-1-Q^{LND}) = -(E[p^{LND}]-1) = -(a-\frac{1}{8}(6t+7-\sqrt{4t^2+4t+17})) < 0,\\ &\Leftrightarrow \ a > \frac{1}{8}(6t+7-\sqrt{4t^2+4t+17}). \qquad \text{(A.4)}\end{aligned}$$

Therefore, in order for $\widehat{z}^{LND}$ to satisfy the assumption (5), we have $\widehat{z}^{LND}$ to satisfy (5)

$$\begin{aligned}\overline{z}-\widehat{z}^{LND} &= a+\overline{z}-\frac{1}{8}(6t+7-\sqrt{4t^2+4t+17})\\ &= \frac{1}{8}(2t-7+\sqrt{4t^2+4t+17}) > 0 \text{ for } t>1 \qquad \text{(A.4)}\end{aligned}$$

and

$$\begin{aligned}\widehat{z}^{LND}-(-\overline{z}) = \overline{z}+\widehat{z}^{LND} &= \overline{z}-a+\frac{1}{8}(6t+7-\sqrt{4t^2+4t+17})\\ &= \frac{1}{8}(7(2t+1)-\sqrt{4t^2+4t+17})-2a > 0,\\ &\Leftrightarrow \frac{1}{16}(7(2t+1)-\sqrt{4t^2+4t+17}) > a \qquad \text{(A.5)}\end{aligned}$$

In order to hold both of (A.4) and (A.5), a have to satisfy
$\frac{1}{16}(7(2t+1)-\sqrt{4t^2+4t+17}) > a > \frac{1}{8}(6t+7-\sqrt{4t^2+4t+17})$.[18] ■

Proof of Lemma 4

Proof: $q_2^{LND}-q^{LD} = \frac{5}{8}t-\frac{3}{16}\sqrt{4t^2+4t+17}+\frac{5}{16}-\frac{1}{8}(3t-\sqrt{t^2+8}) = \frac{1}{8}(3t-\sqrt{t^2+8}) > 0$ for $t>1$, since $(3t)^2-(\sqrt{t^2+8})^2 = 8(t-1)(t+1) > 0$ for $t>1$. From proposition 3, we have $q_2^{LND} > q_1^{LND}$ for $t>1$. We can show that $q_1^{LND}-q^{LD} = \frac{1}{8}t+\frac{1}{16}\sqrt{4t^2+4t+17}-\frac{7}{16}-\frac{1}{8}(3t-\sqrt{t^2+8}) = \frac{1}{16}\sqrt{4t^2+4t+17}-\frac{1}{4}t+\frac{1}{8}\sqrt{t^2+8}-\frac{7}{16}$. Let $F(t) = \frac{1}{16}\sqrt{4t^2+4t+17}-\frac{1}{4}t+\frac{1}{8}\sqrt{t^2+8}-\frac{7}{16}$. We see that $F(1)=0$ and $F'(t) = \frac{1}{32}\frac{8t+4}{\sqrt{4t^2+4t+17}}+\frac{1}{8}\frac{t}{\sqrt{t^2+8}}-\frac{1}{4} = \frac{1}{8}\frac{t\sqrt{4t+4t^2+17}+2t\sqrt{t^2+8}-2\sqrt{t^2+8}\sqrt{4t+4t^2+17}+\sqrt{t^2+8}}{\sqrt{t^2+8}\sqrt{4t+4t^2+17}}$. However, we have $F'(1) = -\frac{2}{15} < 0$,

[18] We can easily show that $\frac{1}{16}(7(2t+1)-\sqrt{4t^2+4t+17}) > \frac{1}{8}(6t+7-\sqrt{4t^2+4t+17})$ for $\forall t>1$.

$F''(t) = \frac{1}{4\sqrt{4t^2+4t+17}} - \frac{1}{64}\frac{(8t+4)^2}{(4t^2+4t+17)^{\frac{3}{2}}} - \frac{1}{8}\frac{t^2}{(t^2+8)^{\frac{3}{2}}} + \frac{1}{8\sqrt{t^2+8}} = \frac{\left(4t^2+4t+17\right)^{\frac{3}{2}}+4\left(t^2+8\right)^{\frac{3}{2}}}{(t^2+8)^{\frac{3}{2}}(4t^2+4t+17)^{\frac{3}{2}}} > 0$, $\lim_{t\to\infty} F'(t) = \lim_{t\to\infty} \frac{1}{8}\frac{t\sqrt{4t+4t^2+17}+2t\sqrt{t^2+8}-2\sqrt{t^2+8}\sqrt{4t+4t^2+17}+\sqrt{t^2+8}}{\sqrt{t^2+8}\sqrt{4t+4t^2+17}}$

$= \lim_{t\to\infty} \frac{1}{8}\frac{\sqrt{4/t+4+17/t^2}+2\sqrt{1+8/t^2}-2\sqrt{1+8/t^2}\sqrt{4/t+4+17/t^2}+\sqrt{1+8/t^2}/t}{\sqrt{1+8/t^2}\sqrt{4/t+4+17/t^2}} = \frac{2+2-2\cdot 2+0}{1\cdot 2} = 0$. So we see that $F(t) < 0$, for $t > 1$.$\therefore$ $q_1^{LND} < q^{LD}$ for $t > 1$. We can show that $q_2^{LND} - q^{LN} = \frac{5}{8}t - \frac{3}{16}\sqrt{4t^2+4t+17} + \frac{5}{16} - \frac{1}{4}(t-1) = \frac{3}{8}t - \frac{3}{16}\sqrt{4t^2+4t+17} + \frac{9}{16} = \frac{1}{16}(6t+9-3\sqrt{4t^2+4t+17}) > 0$ for $t > 1$ since $(6t+9)^2 - (3\sqrt{4t^2+4t+17})^2 = 72t - 72 = 72\,(t-1) > 0$, for $t > 1$. Finally, we can see that $q_1^{LND} - q^{LN} = \frac{1}{8}t + \frac{1}{16}\sqrt{4t^2+4t+17} - \frac{7}{16} - \frac{1}{4}(t-1) = \frac{1}{16}\sqrt{4t^2+4t+17} - \frac{1}{8}t - \frac{3}{16} = \frac{1}{16}(\sqrt{4t^2+4t+17} - (2t+3)) < 0$, for $t > 1$,

since $(\sqrt{4t^2+4t+17})^2 - (2t+3)^2 = 8 - 8t = -8\,(t-1) < 0$. In consequent, we get the result. ■

Proof of Corollary 1

Proof: From (34), (28) and (29), we have $Q^{LND} = Q^{LDN} = Q^{LM} = \frac{3}{4}t - \frac{1}{8}\sqrt{4t^2+4t+17} - \frac{1}{8} = \frac{1}{8}(6t-1-\sqrt{4t^2+4t+17})$. So

from (34), (18) and the above, we can show that $Q^{LM} - Q^{LN} = \frac{1}{8}(6t-1-\sqrt{4t^2+4t+17}) - \frac{1}{2}(t-1) = \frac{1}{4}t - \frac{1}{8}\sqrt{4t^2+4t+17} + \frac{3}{8} = \frac{1}{4}t - \frac{1}{8}\sqrt{4t^2+4t+17} + \frac{3}{8} = \frac{1}{8}(2t+3-\sqrt{4t^2+4t+17}) > 0$ for $t > 1$, since $(2t+3)^2 - (4t^2+4t+17) = (2t+3)^2 - 4t - 4t^2 - 17 = 8t - 8 = 8\,(t-1) > 0$ for $t > 1$.$\therefore$ $Q^{LM} > Q^{LN}$ for $t > 1$. From (34), (24) the above, we have $Q^{LM} - Q^{LD} = \frac{1}{8}(6t-1-\sqrt{4t^2+4t+17}) - \frac{1}{4}(3t-\sqrt{t^2+8}) = \frac{1}{4}\sqrt{t^2+8} - \frac{1}{8}\sqrt{4t^2+4t+17} - \frac{1}{8} = \frac{1}{4}\sqrt{t^2+8} - \frac{1}{8}\sqrt{4t^2+4t+17} - \frac{1}{8}$,and let $H(t) = \frac{1}{4}\sqrt{t^2+8} - \frac{1}{8}\sqrt{4t^2+4t+17} - \frac{1}{8}$. We see that $H(1) = 0$, $H'(t) = \frac{1}{4}\frac{t}{\sqrt{t^2+8}} - \frac{1}{16}\frac{8t+4}{\sqrt{4t^2+4t+17}} = -\frac{1}{4\sqrt{t^2+8}\sqrt{4t^2+4t+17}}\left(2t\sqrt{t^2+8} - t\sqrt{4t^2+4t+17} + \sqrt{t^2+8}\right) < 0$ since $((2t+1)\sqrt{t^2+8})^2 - (t\sqrt{4t^2+4t+17})^2 = (2t+1)^2\,(t^2+8) - t^2\,(4t^2+4t+17) = 16t^2 + 32t + 8 > 0$.Hence, $H(t) < 0$ for $t > 1$,$\therefore$ $Q^{LM} < Q^{LD}$. That is, we have shown that $Q^{LN} < Q^{LM} < Q^{LD}$ for $t > 1$.■