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Skill-Biased Technological Change, Organizational Change, and

Wage Inequality

Kohei Daido    Ken Tabata
School of Economics, Kwansei Gakuin University

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SCHOOL OF ECONOMICS
KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho
Nishinomiya 662-8501, Japan
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Kohei Daido * Ken Tabata †

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Abstract

We build a general equilibrium model of monopolistic competition with moral hazard contracting to examine the interactions among skill-biased technological change (SBTC), organizational changes, and skill premium and within-group wage inequality. While the existing literature finds that the increase in the skilled labor ratio induces SBTC and raises the skill premium, we show that SBTC leads to organizational change toward decentralization by delegating authority within firms, which influences the reward schedule for delegated skilled managers. This organizational change results in the following: (1) the further increase in the skill premium and (2) the rapid expansion of wage inequality among skilled individuals (between skilled workers and skilled managers). Moreover, we find that there are multiple equilibria where the centralized and decentralized organizational modes simultaneously emerge at the intermediate values of the skilled labor ratio.

JEL classification numbers: D86, J31, L16, L22

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*School of Economics, Kwansei Gakuin University; E-mail address: daido@kwansei.ac.jp
†School of Economics, Kwansei Gakuin University; E-mail address: tabataken@kwansei.ac.jp
1 Introduction

Firms in many developed countries have experienced drastic changes in their organizational forms over the past decades. One of the most prominent features of these changes is a trend toward decentralization by delegating decision-making authority. Along with organizational changes, wage inequality has grown in many developed countries, while the implication of this organizational change for individual wages have however remained largely unexplored thus far. The overall wage inequality has grown in the U.S. since the 1980s, and similar trends have been observed in other developed countries.\textsuperscript{1} A major contributor to rising overall inequality is the skill premium, or differentials between the wages of college degree and high school diploma holders.\textsuperscript{2} Despite a remarkable increase in the supply of college skills, the skill premium has steadily increased in the U.S. since the 1980s. However, observable characteristics, such as education and work experience, have been found to explain no more than half of the variation in wages. Thus, wage dispersion within the same demographic and skill group is also a major component of the increased dispersion in overall wage inequality.\textsuperscript{3}

Few studies have been conducted on understanding the interaction between recent organizational change and growing wage inequality. To provide a unified explanation for this issue, we build a simple general equilibrium model of monopolistic competition with moral hazard contracting by extending Acemoglu (2002b). A large body of literature documents that skill-biased technological change (SBTC) plays a prominent role in explaining this widening wage inequality. Acemoglu’s (2002b) seminal research provides excellent explanations for why SBTC is likely to have accelerated over the past several decades.

He argues that the large increase in the supply of college-educated labor since the 1970s in the U.S. expands the market size of skill-biased technology, which provides greater incentives for firms to develop and adopt such technologies. Such a SBTC responding to the supply of skills results in the acceleration of the demand for skills and the increase in the skill premium.\textsuperscript{4} On the one hand, we follow Acemoglu (2002b) in that the increase in the skilled labor ratio leads to a SBTC that is represented by the increase in the variety of intermediate goods, which improves the productivity of skilled labor, increases the demand for skilled workers, and raises the skill premium. On the other hand, we deviate from Acemoglu

\textsuperscript{1}See, for example, Autor et al. (2008) on the U.S., Haskel and Slaughter (2002) on the U.K. and Dustmann et al. (2009) on Germany.

\textsuperscript{2}Precise definitions of “skill premium” and “within-group inequality” appear in Section 2.

\textsuperscript{3}See, for example, Acemoglu (2002a).

\textsuperscript{4}More precisely, Acemoglu (2002b) shows that the increase in skilled labor supply provides two competing impacts upon the direction of the technical change: the price effect and the market size effect. While the former effect encourages innovations directed at scarce unskilled labor, the latter effect leads to SBTC . Acemoglu derives the conditions for SBTC and the widening of the skill premium.
(2002b) in that we focus on organizational changes within firms and the corresponding changes in the reward schedule for delegated skilled managers, which causes a skill premium and wage inequality within the skilled group.

We show that the increase in the skill premium that is caused by SBTC leads to an organizational change from the centralized mode, in which the firm owner maintains authority, to the decentralized mode, in which the firm owner delegates the authority to a skilled manager. This organizational change and the corresponding change in the reward schedule for skilled managers results in (1) a further increase in the skill premium and (2) a rapid expansion of wage inequality among skilled individuals (between skilled workers and skilled managers). A recent empirical study by Autor et al. (2008) find that the bulk of the widening wage inequality in the U.S. is concentrated in the upper tail of the wage distribution and that there is a similar pattern in terms of residual wage inequality. Further, Caroli and Van Reenen (2001) and Bresnahan et al. (2002) empirically show that SBTC affects the wage structure primarily through organizational changes in the work place. Our result, in which the organizational decentralization complements the appreciation of the skill premium and the wage inequality within the skilled group, could provide an explanation for these empirical findings. Moreover, we show that there are multiple equilibria in which both the centralized and decentralized modes simultaneously emerge at intermediate values of the skilled labor ratio. The presence of multiple equilibria implies that the proportion of firms that adopt the decentralized mode can vary, even among countries that have similar skilled labor ratios or technological conditions. An empirical study by Bloom et al. (2009) find that there are significant differences in the proportion of firms that adopt the decentralized mode, even among developed economies with similar skilled labor ratios or technological conditions. Bloom et al. (2009) argue that cultural factors, such as religion and regional trust, play crucial roles in accounting for cross-regional differences in the organizational mode within firms. The multiple equilibria that we found could explain this empirical observation of organizational diversity.

In related research that studies the relationship between organizational change and wage inequality, Nikolowa (2010) focuses on a skill-biased organizational change rather than a SBTC as a source of the increases in the skill premium. She shows that the increasing supply of skilled labor leads firms to adopt organizational forms that are less hierarchical, and this organizational change increases the demand for skilled labor and results in a surge of the skill premium. Although similar to Nikolowa (2010), this paper focuses on the interaction between organizational change and wage inequality. We also consider the
surge of wage inequality within the skilled group.\footnote{Nikolowa (2010) introduces a moral hazard problem into a knowledge-based hierarchy model that was developed by Garicano (2003). Garicano and Rossi-Hansberg (2006), which extend Garicano (2003), argue that the reduction in the costs of acquiring and communicating information leads to the decline in the firm size and the rise in wage inequality in the U.S.. Following Garicano and Rossi-Hansberg (2006), we also focus on the relationship between organizational change and wage inequality, but we explicitly consider a moral hazard problem and focus on organizational changes in the allocation of decision-making authority inside a firm rather than the formation of knowledge based hierarchies.} Other papers close to ours are Kremer and Maskin (1996) and Acemoglu (1999). These papers find that the increasing supply of skilled labor qualitatively modifies the composition of jobs by affecting a firm’s decision to adopt skill-demanding technology. In our model, the increase in the supply of skilled labor alters the allocation of decision making authority within the firm rather than the decision to adopt. This paper also relates to that of Ishiguro (2010a), who incorporates a firm’s choice of organizational mode for the allocation of internal decision making authority into a search theoretic model. He shows that centralized and decentralized organizations can coexist as multiple equilibria.\footnote{Ishiguro (2010b) incorporates a moral hazard problem into an overlapping generations model to examine the interactions between organizational choice and economic development. The author shows that there are multiple equilibrium paths. Some paths converge to the steady-state with the decentralized organization, while other paths converge to the steady-state with the centralized organization. Marin and Verdier (2008, 2009) combine the Aghion and Tirole (1997) model of firm organizations with the Dixit and Stiglitz (1977) monopolistic competition model to describe the interaction between organizational modes and market competition in general equilibrium.} This paper incorporates a choice of organizational modes à la Ishiguro (2010a) into a simplified skill-biased technological change model à la Acemoglu (2002b) to provide a unified explanation for the recent trend in organizational changes and wage inequalities.

This paper is organized as follows. Section 2 presents our model. Section 3 characterizes equilibrium and illustrates the possibilities of multiple equilibria. Section 4 explains how organizational changes interact with the skill premium and residual wage inequality, and Section 5 concludes the paper.

2 The Model

The economy is populated by two types of individuals: a mass $H$ of skilled and a mass $L$ of unskilled individuals. The total population size of the economy is $N$ and is expressed as $N = H + L = hN + (1-h)N$, where $h$ is the skilled ratio in the population. An unskilled individual can be employed only for producing final goods. A skilled individual works as a skilled worker in the final goods sector or as a manager in the intermediate goods sector.

There is one final good produced by competitive firms with access to two types of production technologies. One of these technologies, which we call the “old technology,” combines skilled labor with unskilled labor, and the other technology, which we call the “new technology,” combines skilled labor with an expanding variety of intermediate goods. Final goods are used for consumption and for manufacturing intermediate goods. The intermediate goods sector consists of firms that produce horizontally
differentiated goods under monopolistic competition with free entry. Each owner of a firm in the intermediate goods sector matches a skilled individual for production. Then, the owner chooses the firm’s organizational modes, which depend on whether the owner delegates decision-making authority to the matched skilled individuals or retains it for herself.\footnote{We use the feminine pronoun for owners and the masculine pronoun for skilled individuals, including managers.} When the matched skilled individual is delegated authority, he can choose whether to accept such an offer and whether to work or shirk if he accepts it. This situation involves a moral hazard problem. If the matched skilled individual accepts the offer, he works as a manager in the firm; if he rejects the offer, he works as a skilled worker in the final goods sector. Our paper defines the “skill premium” as the wage difference between the average wage of the skilled individuals (i.e., managers and skilled workers) and the wage of unskilled workers. We define “within-group wage inequality” as the wage difference among skilled individuals (i.e., between managers and skilled workers).

2.1 Final Goods Sector

Final goods are produced by perfectly competitive firms that can exploit the two types of production technologies. In the new technology, firms combine skilled labor $H_N$ with an expanding variety of intermediate goods $x_j$, $j \in [0, A]$ as follows:

$$ Y_N = H_N^{1-\alpha} \int_0^A x_j^\alpha dj, \quad 0 < \alpha < 1. $$

(1)

In the old technology, firms combine skilled labor $H_O$ with unskilled labor $L_O$ according to the CES production function as follows

$$ Y_O = B[\beta H_O^\rho + (1 - \beta)L_O^{\rho}]^{1/\rho}, \quad \rho \leq \frac{1}{2}, $$

(2)

where $B$ is a technology parameter of the old technology.\footnote{To avoid unnecessary lexicographic explanation, we focus our analysis on the case where $\rho \leq \frac{1}{2}$. Skilled and unskilled workers are gross substitutes (resp. complements) when $\rho > 0$ (resp. $\rho < 0$). Thus, the assumption $\rho < \frac{1}{2}$ holds true for the Leontief case ($\rho \to -\infty$) and the Cobb-Douglas case ($\rho \to 0$) but does not satisfy the case where skilled and unskilled workers are perfect substitutes ($\rho \to 1$).} Because skilled workers are mobile between the old and new technology firms, the wage of skilled workers is equalized across these firms.

Let $p_j$, $w_L$, and $w_H$ denote the price of intermediate goods $j$, the wage of unskilled workers (henceforth, the unskilled wage), and the wage of skilled workers (henceforth, the skilled wage), respectively. Profit maximization in the competitive final goods sector is consistent with the following conditions in factor markets:

$$ p_j = \alpha H_N^{1-\alpha} x_j^{-\rho}, $$

(3)
\[ w_L = (1 - \beta)B[\beta(\frac{H_O}{(1 - h)N})^\rho + (1 - \beta)]^{\frac{1-\rho}{\rho}}, \]  

\[ w_H = (1 - \alpha)H_N^{-\alpha} \int_0^A x_j^\alpha dj \]
\[ = \beta B[\beta + (1 - \beta)(\frac{H_O}{(1 - h)N})^{-\rho}]^{\frac{1-\rho}{\rho}}. \]

Note that (5) and (6) represent the skilled wage in the new technology and the old technology, respectively.

Because unskilled individuals can only be employed to produce the final good, \( L_O = L = (1 - h)N \) holds.

### 2.2 Intermediate Goods Sector

The intermediate goods sector consists of \( A \) horizontally differentiated goods produced under monopolistic competition with free entry à la Dixit and Stiglitz (1977). There are \( A \) firms, each of which has a simple hierarchy consisting of a firm owner and a manager. In each firm, the owner hires one skilled individual as the manager to start up her firm. We assume that the owner cannot exploit the intermediate goods production technology by herself and that she is therefore required to employ one skilled individual as the manager. For clarity, we continue our discussion with the assumption that the owner has already succeeded in employing one manager.\(^9\)

One unit of intermediate good is produced by \( z_j \) units of the final goods. The marginal cost of production in terms of final goods \( z_j \) depends on the organizational mode \( j \), which is chosen by the owner.\(^10\)

Under these specifications, each intermediate goods firm maximizes its gross profit

\[ \pi_j = (p_j - z_j)x_j = \alpha H_N^{1-\alpha}x_j^\alpha - z_jx_j. \]

The optimal choice of \( x_j \) is represented by

\[ x_j = \left[ \frac{\alpha^2}{z_j} \right]^{\frac{1}{1-\alpha}} H_N, \]

which implies an equilibrium price

\[ p_j = \frac{z_j}{\alpha}, \]

and an equilibrium gross profit

\[ \pi_j = \pi H_N z_j^{\frac{1}{1-\alpha}} = R z_j^{\frac{1}{1-\alpha}}, \]

where \( \pi \equiv \frac{1-\alpha}{\alpha} \alpha^{\frac{2}{1-\alpha}} \) and \( R \equiv \pi H_N. \)

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\(^9\)We explain the matching procedure in the next subsection.

\(^{10}\)We illustrate the available organizational modes in section 2.4.
2.3 Matching

At the beginning of a period, each owner of the intermediate goods firm enters the matching market to find one skilled individual who may serve as a manager of the production site. Each skilled individual also enters the matching market to find an opportunity to work as a manager. However, as we will see in Section 3, the number of firms in equilibrium is always smaller than the number of skilled individuals (i.e., $A < H = hN$). Thus, the supply of manager positions is always on the short side of the matching market. For simplicity, we suppose that a player on the short side of the market can find a trading partner. Applying this short-side principle in the current model, we can ensure that each owner meets one skilled individual, while each skilled individual fails to match with an owner with a positive probability. Here, skilled individuals are assumed to be rationed randomly.

On the one hand, a skilled individual who succeeds in matching with an owner can be a manager of an intermediate goods production site. Alternatively, he can freely decline to work as a manager. In that case, he would be a skilled worker in the final goods sector and would earn the competitive labor market skilled wage. Therefore, to exploit the intermediate goods production technologies, the owner must offer an acceptable wage contract to her matched skilled individual. On the other hand, a skilled individual who fails to match with an owner has no alternative other than being a skilled worker in the final goods sector.

2.4 Optimal Contracts and Organizational Modes

This section characterizes the optimal organizational mode of intermediate goods firms. The marginal cost of production in terms of final goods $z_j$ depends on the managerial action $e \in \{0, 1\}$. Here, the owner has two options: one is to choose the action $e \in \{0, 1\}$ by herself, and the other is to delegate the decision-making authority for action $e \in \{0, 1\}$ to the manager. We call the former option the centralized organizational mode (“C-mode”) and the latter option the decentralized mode (“D-mode”). In the C-mode, the owner incurs an action cost $ce$, where $c > 0$ and the marginal cost of production is expressed as $z_j = z(C, e)$. In the D-mode, the manager incurs an action cost $ge$, where $g > 0$ and the marginal cost of production is expressed as $z_j = z(D, e)$.

As in the study by Ishiguro (2010a), we assume that the manager is more efficient than the owner because he can produce intermediate goods more efficiently and at a lower action cost than the owner.

**Assumption 1.** $z(C, 1) > z(D, 1)$ and $c > g$.  

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This assumption is justified by the following two arguments: First, in general, the fact that the manager can complete the task more efficiently than the owner because of his informational advantage is often emphasized as a positive aspect of delegation.\textsuperscript{11} Second, a more specific reason in our model is that the owner generally has more tasks than the manager, such as the choices of organizational modes and the design of wage contracts. Therefore, the productivity of the manager with respect to choosing the efficient method of production would be higher than that of the owner because the manager can concentrate solely on this specific task. For simplicity, we normalize $z(C,1) = \varphi > 1 = z(D,1)$. Moreover, to focus on the case where the owner always wants to implement $e = 1$, irrespective of the organizational mode, we add the following assumption.

**Assumption 2.** Both $z(C,0)$ and $z(D,0)$ are prohibitively high.

This assumption means that when $e = 0$ is implemented, the marginal cost of production will be prohibitively high, regardless of the organizational mode.

We suppose that neither the action taken by the manager nor the associated outcome is verifiable and hence contractible. However, the owner receives a contractible signal $s \in \{G, B\}$ on which contacts can be conditioned.\textsuperscript{12} This signal $s = \{G, B\}$ is correlated with the manager’s action $e \in \{0, 1\}$ as follows:

$$s = \begin{cases} G & \text{with probability } q(e), \\ B & \text{with probability } 1 - q(e). \end{cases}$$

(11)

We suppose that $\Delta q \equiv q(1) - q(0) > 0$. $s = G$ (resp. $s = B$) represents a good (resp. bad) signal for the manager’s action. Thus, $\Delta q > 0$ implies that verifiable signals $s$ are informative; thus, an owner compensates a manager based on the signal she received.

Here, we have the owner’s problem in the D-mode and in the C-mode. First, we consider the D-mode. The owner should minimize the expected wage for the manager, subject to a set of relevant constraints, by specifying a compensation scheme $\{v^G, v^B\}$, where $v^G$ (resp. $v^B$) is the wage when $s = G$ (resp. $s = B$). This problem is represented as follows:

$$\min_{(v^G, v^B)} q(1)v^G + [1 - q(1)]v^B,$$

which is subject to

\textsuperscript{11}See, for example, Aghion and Tirole (1997), Dessein (2002), or Caroli and Van Reenen (2001) on the advantages and disadvantages of delegation for the owner.

\textsuperscript{12}Our setting follows MacLeod (2003).
\[ q(1)v^G + [1 - q(1)]v^B - g \geq w_H, \quad \text{(IR)} \]
\[ q(1)v^G + [1 - q(1)]v^B - g \geq q(0)v^G + [1 - q(0)]v^B + b, \quad \text{(IC)} \]
\[ v^G \geq 0 \text{ and } v^B \geq 0. \quad \text{(LL)} \]

(IR) indicates the individual rationality constraint for the manager. Note that the reservation value for
the manager is supposed to be the skilled wage \( w_H \), which is determined by the labor market. (IC)
is the incentive compatibility constraint, which induces the manager to choose \( e = 1 \) instead of \( e = 0 \)
under the contract \( \{v^G, v^B\} \). As in Tirole (2005), \( b \) indicates the private benefit that the manager can
obtain when he exerts no effort (i.e., \( e = 0 \)); exerting effort (i.e., \( e = 1 \)) yields no private benefit. For
example, a private benefit \( b \) could be interpreted as a manager’s benefit by selling useful information
on the operations of production to other owners. Finally, (LL) is the limited liability constraint that
ensures that the manager receives non-negative rewards.

The optimal solution to the above problem is given as follows:\(^{13}\) (a) the optimal contract is \( (v^G, v^B) = ((g + b)/\Delta q, 0) \) when \( \frac{q(1)}{\Delta q} (g + b) - g > w_H \), where (IC) binds; (b) the optimal contract is \( (v^G, v^B) \), which
satisfies (IR) with equality when \( \frac{q(1)}{\Delta q} (g + b) - g \leq w_H \).

In case (a), (IC) is binding while (IR) becomes slack. The owner must give the manager positive
information rent over his reservation value \( w_H \) to induce \( e = 1 \). Therefore, the expected payoff of the
manager becomes larger than his reservation value. This implies that the owner suffers from the agency
cost. Recalling (10) and \( z(D, 1) = 1 \), the owner’s expected payoff in case (a) is given by \( R - \frac{q(1)}{\Delta q} (g + b), \)
where \( R \) represents the gross profit under the D-mode as \( R \equiv \pi H_N \). In case (b), (IR) is binding while
(IC) becomes slack. In this case, the owner is not required to give the manager positive information rent
to induce \( e = 1 \). Therefore, the expected payoff to the manager equals his reservation value \( w_H \). The
owner’s expected payoff in case (b) is given by \( R - (w_H + g) \). In sum, the expected payoffs for the owner
and for the manager under the D-mode are as follows, respectively:

\[ \hat{\pi}_D = R - \max\{\frac{q(1)}{\Delta q} (g + b), w_H + g\}. \quad \text{(12)} \]
\[ \hat{v}_D = \max\{\frac{q(1)}{\Delta q} (g + b) - g, w_H\}. \quad \text{(13)} \]

\(^{13}\)See, for example, Macho-Stadler and Perez-Castrillo (2001) for finding the optimal contracts of this problem.
Note that the manager always chooses $e = 1$ under Assumption 2.

Next, under the C-mode, the owner who chooses an action offers a fixed wage $w_H$ to the manager to compensate him for his reservation value. Note that the owner needs one unit of skilled labor to start up the project even when she does not delegate the decision-making authority. Hence, recalling (10) and $z(C, 1) = \varphi$, the owner’s expected payoff under the C-mode is

$$\hat{\pi}_C = R\varphi - \frac{\alpha}{\varphi} - (w_H + c).$$

(14)

Note that the owner always chooses $e = 1$ under Assumption 2.

### 2.5 Optimal Organizational Modes

Here, we characterize the optimal organizational modes. Notice that the optimal organizational modes depend on the market size of intermediate goods (henceforth, the market size) and on the skilled wage. From (8), the market size is in direct proportion to the skilled labor size in the new technology $H_N$. Therefore, $H_N$ could be regarded as the proxy of the market size. We find that $\hat{\pi}_D \geq \hat{\pi}_C$ holds if and only if

$$R \equiv \pi H_N \geq \frac{1}{1 - \varphi^{-\frac{\alpha}{\varphi}}} \frac{q(1)}{\Delta q} (g + b) - (w_H + c) \equiv T(w_H).$$

(15)

The optimal organizational mode can be illustrated in Figure 1. The $T(w_H)$ line represents the threshold that determines the optimal organizational mode. In the area above (resp. below) the $T(w_H)$ line, the owner chooses the D-mode (resp. C-mode). Figure 1 implies that the owner prefers the D-mode to the C-mode when the market size is large or the skilled wage is high.

[Figure 1]

The intuition is as follows. The positive effect of the owner adopting the D-mode is that the manager can produce intermediate goods more efficiently than the owner. The marginal benefit of this effect becomes more critical as the market size becomes large. Hence, the owner is more likely to adopt the D-mode when the market size is large. However, when the skilled wage is small enough to satisfy $w_H < \frac{q(1)}{\Delta q} (g + b) - g$, where (IC) is binding under the D-mode, the delegation of the decision-making authority provides the manager with more power to extract a higher reward $\frac{q(1)}{\Delta q} (g + d)$, which reflects his informational advantage over the owner. As in Ishiguro (2010a), this reward is independent of the skilled wage. On the contrary, under the C-mode, the manager’s reward increases in the skilled wage. Hence, as the skilled wage increases, the owner is more likely to prefer the D-mode to the C-mode because...
the expected wage under the C-mode increases, and the expected wage under the D-mode is constant at \( \frac{g(1)}{2\varphi} (g + b) \). However, if the skilled wage is too small to exploit the positive effect of adopting the D-mode, the owner adopts the C-mode because it is costly for her to pay an information rent to the manager under the D-mode.

2.6 Free Entry and Equilibrium Organizational Mode

Firms enter the intermediate goods market until profits are driven down to cover the startup costs (i.e., the cost of employing the manager). From (12), the free-entry condition for firms under the D-mode is given by

\[
R = \max \{ F_D^{IC}, F_D^{IR}(w_H) \} \equiv F_D(w_H),
\]

where \( F_D^{IC} \equiv \frac{g(1)}{2\varphi} (g + b) \) and \( F_D^{IR}(w_H) \equiv w_H + g \). Note that the relation \( F_D^{IC} > F_D^{IR}(w_H) \) holds when \( w_H < \overline{w}_H \equiv \frac{g(1)}{2\varphi} (g + b) - g \). On the contrary, from (14), the free-entry condition for firms under the C-mode is given by

\[
R = \varphi^\frac{g}{2\varphi} (w_H + c) \equiv F_C(w_H).
\]

Figure 2 shows the relationships among \( T(w_H) \) in (15), \( F_D(w_H) \) in (16) and \( F_C(w_H) \) in (17). The \( F_D(w_H) \) line consists of two parts: the horizontal line of \( F_D^{IC} \) when (IC) is binding (i.e., \( \overline{w}_H > w_H \)) and the upward sloping line of \( F_D^{IR}(w_H) \) when (IR) is binding (i.e., \( \overline{w}_H \leq w_H \)), where \( \overline{w}_H \) is the intersection between \( F_D^{IC} \) and \( F_D^{IR}(w_H) \). The \( F_C(w_H) \) line is also upward sloping in \( w_H \), but the slope of the \( F_C(w_H) \) line is larger than that of the \( F_D(w_H) \) line. To avoid unnecessary lexicographic explanations, we add the following parametric assumption:

**Assumption 3.** \( \frac{g(1)}{2\varphi} (g + b)\varphi^{-\frac{g}{2\varphi}} > c \).

Under Assumption 3, the \( T(w_H) \), \( F_C(w_H) \) and \( F_D^{IC} \) lines have a unique intersection at \( E_M \), where \( (w_H, R) = (\overline{w}_H, F_D^{IC}) \) and \( \overline{w}_H \equiv \frac{g(1)}{2\varphi} (g + b)\varphi^{-\frac{g}{2\varphi}} - c \). Hence, recalling that the owner chooses the D-mode (resp. C-mode) in the area above (resp. below) the \( T(w_H) \) line, the free-entry condition in equilibrium is summarized as

\[
R = F(w_H) \equiv \begin{cases} 
\varphi^\frac{g}{2\varphi} (w_H + c) \equiv F_C(w_H), & \text{if } w_H \in [0, \overline{w}_H], \\
\frac{g(1)}{2\varphi} (g + b) \equiv F_D^{IC}, & \text{if } w_H \in [\overline{w}_H, \overline{w}_H], \\
w_H + g \equiv F_D^{IR}(w_H), & \text{if } w_H \in [\overline{w}_H, \infty),
\end{cases}
\]

\(^{14}\)Assumption 3 ensures the existence of parameter regions for which the C-mode can be a possible equilibrium organizational mode.
where $w_H \equiv \frac{q(1)}{\lambda^2} (g + b) - g$. Equation (18) is illustrated in Figure 2 as a bold line. This equation implies that when the skilled wage is sufficiently small enough (resp. large) to satisfy $w_H \leq \bar{w}_H$ (resp. $w_H \geq \bar{w}_H$), only the C-mode (resp. D-mode) is a possible candidate for the equilibrium organizational mode. In particular, under the D-mode, (IC) is binding when $w_H \in [\bar{w}_H, \bar{w}_H]$, while (IR) is binding when $w_H \in [\bar{w}_H, \infty]$.

2.7 Labor Market

The market clearing condition for skilled workers is

$$H_O + H_N + A_C + A_D = H_O + H_N + A = hN,$$

where $A_C$ and $A_D$ represent the number of intermediate goods firms adopting the C-mode and the D-mode, respectively, and $A$ represents the total number of intermediate goods firms (i.e., $A = A_C + A_D$). For clarity of exposition, we denote the share of the intermediate goods firms that adopt the D-mode (henceforth, the share of the D-mode) as $k \in [0, 1]$. Hence, $A_C = (1 - k)A$ and $A_D = kA$, respectively.

Recall that from (5), the skilled wage in the new technology firms is $(1 - \alpha)H_N^{-\alpha} \int_0^A x_j^\alpha dj$. Substituting $x_j$ in (8), we get

$$w_H = \bar{w}[A_D + \varphi^{-\frac{\alpha}{1-\alpha}} A_C] = \bar{w}[k + (1 - k)\varphi^{-\frac{\alpha}{1-\alpha}}] A,$$

where $\bar{w} = (1 - \alpha) \alpha \frac{2}{1-\alpha}$. Note that the skilled wage depends on the share of the D-mode $k \in [0, 1]$ and on the amount of the variety of intermediate goods $A$. Because the marginal cost of producing intermediate goods under the D-mode is lower than it is under the C-mode, the price of intermediate goods under the D-mode is relatively lower. Hence, from (5) and (8), as the share of the D-mode increases given the amount of the variety of intermediate goods $A$, the amount of each intermediate goods input $x_j$ that contributes to the production of the final goods increases. This enhances the marginal productivity of skilled workers in the final goods sector and raises the skilled wage. From (5) and (8), as the amount of the variety of intermediate goods $A$ increases given the amount of each intermediate goods input, the marginal productivity of skilled workers increases, which raises the skilled wage.

In equilibrium, the skilled wage is equalized across the old and new technology firms. Hence, from (5), (6) and (20), we have

$$w_H = \bar{w}[k + (1 - k)\varphi^{-\frac{\alpha}{1-\alpha}}] A = \beta B[\beta + (1 - \beta)(\frac{H_O}{(1 - k)N})^{-\rho}]^{\frac{1 - \rho}{\rho}} \beta B\left[\beta + (1 - \beta)(\frac{H_O}{(1 - k)N})^{-\rho}\right]^{\frac{1 - \rho}{\rho}}.$$

Equation (21) implies that skilled workers are driven out of the old technology firms by the higher skilled wage (i.e., $\frac{\partial H_O}{\partial w_H} < 0$).
3 Equilibrium

This section characterizes the general equilibrium of our model. In the following analysis, we label the equilibrium in which all owners choose the D-mode (i.e., \( k^* = 1 \)) the “D-mode equilibrium,” and we label the equilibrium in which all owners choose the C-mode (i.e., \( k^* = 0 \)) the “C-mode equilibrium.” We also label the equilibrium in which owners are indifferent about whether to choose the C-mode or the D-mode (i.e., \( k^* \in (0, 1) \)) the “mixed equilibrium.”

Recall from (10) that the gross profit under the D-mode \( R \equiv \pi N \) is in direct proportion to the skilled worker size in the new technology firms \( H N \). Using the skilled labor market clearing condition of (19), we have \( R = \pi (hN - HO - A) \). From (6), we obtain \( HO(w_H; h) = (1 - h)N \left[ \frac{w_H}{k + (1 - k)\phi - \beta} \right]^{-\frac{1}{\beta}} \). We also have \( A(w_H, k) = \frac{w_H}{w_H w_{k + (1 - k)\phi - \beta}} \) from (20). Hence, substituting \( HO(w_H; h) \) and \( A(w_H, k) \) into \( R = \pi (hN - HO - A) \), we have

\[
R = \pi [hN - HO(w_H; h) - A(w_H, k)] \equiv G(w_H, k; h). \tag{22}
\]

In Appendix A, we show that there is a unique threshold for the skilled wage \( \tilde{w}_H \) such that

\[
\frac{\partial G(w_H, k; h)}{\partial w_H} =
\begin{cases}
0 & \text{if } w_H = \tilde{w}_H, \\
y > 0 & \text{if } w_H > \tilde{w}_H, \\
y < 0 & \text{if } w_H < \tilde{w}_H.
\end{cases} \tag{23}
\]

As we described in Figures 3, 4, and 5, the \( G(w_H, k; h) \) line is depicted as an inverted-U shaped function of \( w_H \). Moreover, because \( \frac{\partial A(w_H, k)}{\partial k} < 0 \), we have

\[
\frac{\partial G(w_H, k; h)}{\partial k} > 0. \tag{24}
\]

Thus, \( G(w_H, 1; h) > G(w_H, 0; h) \) holds. This implies that the \( G(w_H, k; h) \) line under the D-mode always lies above the \( G(w_H, k; h) \) line under the C-mode. In addition, the \( G(w_H, k; h) \) line for any \( k \in (0, 1) \) is located in the regions between \( G(w_H, 0; h) \) and \( G(w_H, 1; h) \). Furthermore, because \( \frac{\partial H_O(w_H; k)}{\partial w_H} < 0 \), we have

\[
\frac{\partial G(w_H, k; h)}{\partial w} > 0. \tag{25}
\]

Hence, the \( G(w_H, k; h) \) line shifts upward as the skilled ratio in the population \( h \) increases.

Figures 3, 4, and 5 depict both the \( G(w_H, 1; h) \) and \( G(w_H, 0; h) \) lines according to the value of \( h \). Supposing \( h_C < h_M < h_D \), Figures 3, 4, and 5 represent three typical cases where \( h \) is low (i.e., \( h = h_C \)), high (i.e., \( h = h_D \)), and medium (i.e., \( h = h_M \)), respectively. The intersection of \( G(w_H, k; h) \) in (22) and the free-entry condition \( F(w_H) \) in (18) will determine the equilibrium. However, \( G(w_H, k; h) \) and \( F(w_H) \)
generally have two intersections. As discussed in Appendix B, an equilibrium is stable only when the slope of $F(w_H)$ is not smaller than that of $G(w_H, k; h)$ (i.e., $\frac{\partial G(w_H, k; h)}{\partial w_H} \leq \frac{\partial F(w_H)}{\partial w_H}$). For example, with regard to the relationship between $F(w_H)$ and $G(w_H, 0; h)$, the intersection to the left of $E_C$ is unstable even if it exists. Therefore, we focus on $E_C$ (resp. $E_D$) in the case of $k = 0$ (resp. $k = 1$). Because the owner chooses the D-mode (resp. C-mode) in the area above (resp. below) the $T(w_H)$, $E_D$ (resp. $E_C$) can be an equilibrium only when $E_D$ (resp. $E_C$) is in the area above (resp. below) the $T(w_H)$.

3.1 The C-mode and D-mode equilibria

First, let us consider the case of Figure 3, where the skilled ratio $h$ is sufficiently low (i.e., $h = h_C$). $G(w_H, 1; h_C)$ does not have any intersections with $F(w_H)$ in the area above $T(w_H)$. Hence, the D-mode equilibrium is never realized. On the contrary, $G(w_H, 0; h_C)$ intersects with $F(w_H)$ at $E_C$ in the area below $T(w_H)$. Hence, the C-mode equilibrium could be a possible outcome. The mixed equilibrium is realized only when each owner is indifferent about whether to choose the C-mode or the D-mode (i.e., the points on the $T(w_H)$ line) and the free-entry condition $F(w_H)$ are satisfied. Therefore, the mixed equilibrium is realized only when there is an interior value of $k^* \in (0, 1)$ such that $G(w_H, k^*; h_C)$ passes through the point $E_M$ where $T(w_H)$ and $F(w_H)$ have a unique intersection. However, for any interior $k \in (0, 1)$, $G(w_H, k; h_C)$ is located in the region between $G(w_H, 0; h_C)$ and $G(w_H, 1; h_C)$ and cannot pass through the point $E_M$. Hence, the mixed equilibrium (i.e., $k^* \in (0, 1)$) is never realized. Therefore, when the skilled ratio $h$ is sufficiently low, the unique C-mode equilibrium is realized at $E_C$ in Figure 3.

[Figure 3]

Next, let us consider the case in Figure 4 where the skilled ratio $h$ is sufficiently high (i.e., $h = h_D$) such that $G(w_H, 0; h_D)$ cannot intersect with $F(w_H)$ in the area below $T(w_H)$. Although the C-mode equilibrium is never realized, the D-mode equilibrium is a possible outcome, as $G(w_H, 1; h_D)$ intersects with $F(w_H)$ at $E_D$ in the area above $T(w_H)$.

As with the logic in Figure 3, the mixed equilibrium is never realized. As a result, when the skilled ratio $h$ is sufficiently high, the unique D-mode equilibrium is realized at $E_D$ in Figure 4.

[Figure 4]

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15 Appendix C briefly discusses the parameter conditions under which the C-mode equilibrium exists.
16 Appendix C briefly discusses the parameter conditions under which the D-mode equilibrium exists.
3.2 Multiple Equilibria and Their Selection

When the skilled ratio \( h \) has an intermediate value (i.e., \( h = h_M \)), the mixed equilibrium and the D-mode and C-mode equilibria will arise. As described in Figure 5, \( G(w_H, 0; h_M) \) intersects with \( F(w_H) \) at \( E_C \) in the area below \( T(w_H) \). Hence, the C-mode equilibrium is a possible outcome. On the contrary, \( G(w_H, 1; h_M) \) intersects with \( F(w_H) \) at \( E_D \) in the area above \( T(w_H) \). Hence, the D-mode equilibrium is also a possible outcome. Moreover, because \( G(w_H, k; h_M) \), for any \( k \in (0, 1) \) that is located in the regions between \( G(w_H, 0; h_M) \) and \( G(w_H, 1; h_M) \), there is always a unique \( k^* \in (0, 1) \) such that \( G(w_H, k^*; h_M) \) passes through the point \( E_M \). Hence, a mixed equilibrium is also a possible outcome. As a result, when the skilled ratio \( h \) is intermediate, there are the following three rational expectation equilibria: (1) the C-mode equilibrium at \( E_C \), (2) the D-mode equilibrium at \( E_D \), and (3) the mixed equilibrium at \( E_M \).

The mechanism behind multiple equilibria can be explained as follows: As discussed in Section 2.4, each owner prefers the D-mode to the C-mode when the market size is large or the skilled wage is high. The market size is determined by the skilled labor size in the new technology firms (i.e., \( H_N = hN - H_O(w_H) - A \)), and the skilled wage is determined by (21) (i.e., \( w_H = \bar{w}[k + (1 - k)\varphi^{-1}]A \)).

Given the amount of the variety of intermediate goods \( A \), (21) shows that the skilled wage increases when the share of the D-mode increases. The increase of \( w_H \) induces the reallocation of skilled workers from the old to the new technology firms because \( \frac{\partial H_N}{\partial w_H} < 0 \) and \( \frac{\partial H_N}{\partial w_H} > 0 \). This leads to an increase in the skilled labor size in new technology firms, which represents the market size. This process produces the following feedback effect. The increase in the share of the D-mode creates an environment in which each owner is more likely to choose the D-mode. This feedback effect leads to multiple equilibria.

Which equilibrium is realized among \( E_C \), \( E_D \), and \( E_M \) depends on each owner’s expectation about the equilibrium. First, suppose that each firm owner expects that the D-mode equilibrium will occur (i.e., each owner expects that other owners will choose the D-mode). In this case, the skilled wage will become sufficiently high and the market size will become sufficiently large, which in turn will make \( \frac{\partial H_N}{\partial w_H} < 0 \) and \( \frac{\partial H_N}{\partial w_H} > 0 \). This leads to an increase in the skilled labor size in new technology firms, which represents the market size. This process produces the following feedback effect. The increase in the share of the D-mode creates an environment in which each owner is more likely to choose the D-mode. This feedback effect leads to multiple equilibria.

17 Note that the point \( E_M \) satisfies \( (w_H, R) = (w_H, F_{IC}^D) \). Hence, by substituting \( (w_H, R) = (w_H, F_{IC}^D) \) into (22), we obtain the unique interior \( k^* \in (0, 1) \) that satisfies \( G(w_H, k^*; h_M) = F_{IC}^D \).

18 Although the mechanism behind multiple equilibria is analogous to that explained by Ishiguro (2010b), this paper differs from it with respect to how the higher share of the D-mode leads to the larger market size of intermediate goods. In the paper by Ishiguro (2010b), the increase in the share of the D-mode increases the wage of young workers and enhances their capital accumulation, which expands the market size of intermediate goods. On the contrary, in this paper, the increase in the share of the D-mode increases the skilled wage and induces the reallocation of skilled workers from the old to the new technology firms, which expands the market size of intermediate goods.
choosing the D-mode more attractive for each owner. The above feedback effect will occur, and the D-mode equilibrium will be realized as a self-fulfilling equilibrium. Second, suppose that each owner expects that the C-mode equilibrium will occur. In this case, the C-mode equilibrium will become a self-fulfilling equilibrium in a similar way but through an inverse feedback effect. Finally, suppose that each owner expects that a mixed equilibrium will occur such that each owner will expect that a fraction \( k \in (0,1) \) of firms will choose the D-mode, where \( k \) satisfies \( G(w_H, k; h_M) = F^D_D \). In this case, the skilled wage and the market size will be adjusted to ensure that the relation \( \hat{\pi}^D = \hat{\pi}^C \) holds. Thus, each owner will be indifferent about whether to choose the D-mode or the C-mode, and the mixed equilibrium will be realized as a self-fulfilling equilibrium. In sum, the equilibrium organizational mode in Figure 5 depends on how each owner forms her expectation about the intentions of other owners.

We can provide an informal argument of the stability properties of the equilibria we derived above. Suppose that the economy is in the mixed equilibrium at \( E_M \) in Figure 5, and assume that the skilled wage \( w_H \) has decreased (resp. increased) slightly from \( w_H \) for exogenous reasons. In this case, as is easily confirmed from (15) and Figure 5, the relation \( R < T(w_H) \) (resp. \( R > T(w_H) \)) holds, and all firm owners have strict incentives to choose the C-mode (resp. D-mode). Hence, the economy instantly deviates from the mixed equilibrium at \( E_M \) to reach the C-mode equilibrium at \( E_C \) (resp. D-mode equilibrium at \( E_D \)). Therefore, the mixed equilibrium we obtained in Figure 5 is unstable in the sense that the economy cannot be returned to the original equilibrium once it deviates from it. On the contrary, both the C-mode equilibrium at \( E_C \) and the D-mode equilibrium at \( E_D \) are stable. In the following analysis, we focus our analysis on these two stable equilibria.

### 3.3 Results and Evidence

Our results imply that a firm’s organizational form changes from centralized to decentralized as the skilled ratio in the population increases and the skilled wage correspondingly increases. Some empirical studies provide evidence that organizational changes are complementary to skilled labor and that a larger supply of skilled labor is associated with more decentralized decision-making in firms (Caroli and Reenen, 2001; Bresnahan et al., 2002; Bauer and Bender, 2004). In general, the skilled ratio in developed countries is higher than the ratio in developing countries. Additionally, decentralization seems to be more ubiquitous in developed countries than it is in developing countries. Bloom et al. (2009) find that the delegation of decision-making from a firm owner to her manager is positively associated with how developed a country is. Our theoretical result is partly consistent with this finding.
In the intermediate range of the skilled labor ratio, note that there are multiple equilibria from which firms’ degree of decentralization could differ, even among countries with similar skilled labor ratios and technological conditions. Bloom et al. (2009) also find that there are significant differences in the decentralization of firms, even in developed economies with similar skilled ratios and technological conditions. Bloom et al. (2009) emphasize that cultural factors, such as religion and regional trust, play crucial roles in accounting for cross-regional differences in firms’ organizational modes. These cultural factors play substantial roles in coordinating expectations. These results suggest that our finding of multiple equilibria could provide a possible explanation for the empirical findings.

4 Skill Premium and Within-group Inequality

This section focuses on labor market outcomes. In particular, we are interested in how “within-group inequality” and the “skill premium” respond to organizational changes that are caused by changes in the skilled ratio in the population. Our paper defines the skill premium as the wage difference between the average wage of skilled individuals (i.e., the average wage of managers and skilled workers) and the wage of unskilled workers. Additionally, we define within-group inequality as the wage difference between managers and skilled workers (i.e., within skilled group inequality).

4.1 Skill Premium

In this subsection, we first define the skill premium in our model. We describe the relationship between the skilled ratio $h$ and the skilled wage $w_H$ on the right side of Figure 6, and we define the relationship between the skilled wage $w_H$ and the unskilled wage $w_L$ on the left side of Figure 6.

[Figure 6]

On the right side of Figure 6, the $k^* = 0$ line (resp. $k^* = 1$ line) shows the relationship between $h$ and $w_H$ in the C-mode equilibrium (resp. the D-mode equilibrium), while the $k^* \in (0, 1)$ line shows the relationship in the mixed equilibrium. At sufficiently low (resp. high) values of $h$, the unique C-mode equilibrium (resp. D-mode equilibrium) is realized as shown in Figure 3 (resp. Figure 4). However, at an intermediate range of values of $h \in [h_c, h_d]$, there are three possible outcomes: (1) the C-mode equilibrium, (2) the D-mode equilibrium, and (3) the mixed equilibrium. Three vertical lines show the cases where the skilled ratio is sufficiently low (i.e., $h = h_c$), sufficiently high (i.e., $h = h_d$), and medium (i.e., $h = h_M$), respectively. In what follows, we assume that the mixed equilibrium never occurs for the
stability reasons that were discussed above.\footnote{This assumption simplifies the following explanation. Explicit consideration of the mixed equilibrium does not alter our main arguments, but it requires unnecessary lexicographic explanations.}

Note that the skilled wage increases as the skilled ratio in the population increases from $h_C$ to $h_D$. With $h = h_C$ (resp. $h = h_D$), the skilled wage is uniquely given by $w_H^1$ (resp. $w_H^4$) at $E_1$ (resp. $E_4$). However, with $h = h_M$, there are two possible equilibria for the skilled wage: $w_H^2$ at $E_2$ and $w_H^3$ at $E_3$. Which skilled wage is realized depends on each owner’s expectation about the equilibrium. We can easily confirm that the relation $w_H^1 < w_H^2 < w_H^3 < w_H^4$ holds from Figure 6. In particular, when the organizational mode shifts from the C-mode to the D-mode in the range of $h \in [h_C, h_M]$, there is a jump of skilled wages. For example, suppose that an organizational mode change occurs at $h = h_M$, the skilled wage increases from $w_H^2$ to $w_H^3$. Here, we implicitly assume that an organizational mode decision is irreversible, and thus, an organizational change occurs only once.

The left side of Figure 6 shows the negative relationship between the skilled and unskilled wages, which is derived by eliminating $H_O/(1-h)N$ from (4) and (6). With $h = h_C$ (resp. $h = h_D$), the unskilled wage $w_L$ is uniquely given by $w_L^1$ (resp. $w_L^4$) at $E_1$ (resp. $E_4$). However, with $h = h_M$, there are two possible equilibria for the unskilled wages: $w_L^2$ at $E_2$ and $w_L^3$ at $E_3$. It is straightforward that the relation $w_L^1 > w_L^2 > w_L^3 > w_L^4$ holds from Figure 6. Therefore, the increase in the skilled wage $w_H$ that is caused by the increase in the skilled ratio draws the skilled labor out of the old technology firms (i.e., $\frac{\partial H_O}{\partial w_H} < 0$) and decreases the unskilled wage given by (4).

The skill premium is calculated as the ratio of the average of the manager’s wage and the skilled wage to the unskilled wage. Thus, under the D-mode at $E_3$ or $E_4$, the skill premium is provided as

$$\frac{A_h N}{N} \max \left[ w_H + g, \frac{g(1)}{2g} (g + b) \right] \frac{(1 - \frac{A_h N}{hN}) w_H}{w_L},$$

where $\frac{A_h N}{hN} \in (0, 1)$ represents the share of the manager’s wage among skilled individuals (henceforth, the share of the manager). Note that under the C-mode equilibrium at $E_1$ or $E_2$, the manager’s wage equals the skilled wage. Hence, the skill premium is $\frac{w_H}{w_L}$.

4.2 Organizational Changes and the Skill Premium

In this subsection, we examine how the skill premium responds to organizational changes that are caused by changes in the skilled ratio in the population. We first focus on the case where the skilled ratio increases from $h_C$ to $h_M$. As $h$ increases, the equilibrium also changes from $E_1$ to $E_2$ or $E_3$. From Figure 6, we confirm that the relation $\frac{w_H^1}{w_L^1} < \frac{w_H^2}{w_L^2}$ holds. Hence, the skill premium at $E_2$ is larger than
the skill premium at $E_1$. Note that both $E_1$ and $E_2$ exhibit the C-mode equilibrium (i.e., $k^* = 0$). In addition, the skill premium at $E_3$ (i.e., the D-mode equilibrium) is given by

$$\frac{A_k}{h_M N} \max[w_H^3 + g, \frac{q(1)}{\triangle q} (g + b)] + (1 - \frac{A_k}{h_M N}) w_H^3 > w_L^3,$$

(27)

where $\frac{A_k}{h_M N} \in (0, 1)$ represents the share of the manager at $E_3$. Because $\max[w_H^3 + g, \frac{q(1)}{\triangle q} g] > w_H^3 > w_H^2$ and $w_L^3 < w_L^2$,

$$\frac{A_k}{h_M N} \max[w_H^3 + g, \frac{q(1)}{\triangle q} (g + b)] + (1 - \frac{A_k}{h_M N}) w_H^3 > w_L^3$$

holds. Hence, the skill premium at $E_3$ is larger than the skill premium at $E_2$. These results indicate that when the skilled ratio increases from $h_L$ to $h_M$, the skill premium increases regardless of organizational changes. However, the increase in the skill premium becomes larger with organizational changes.

Next, we focus on increasing from $h_M$ to $h_H$, and we compare the skill premiums at $E_2$, $E_3$, and $E_4$. As discussed above, the skill premium at $E_3$ is larger than the skill premium at $E_2$. In addition, the skill premium at $E_4$ (i.e., the D-mode equilibrium) is provided by

$$\frac{A_k}{h_M N} \max[w_H^3 + g, \frac{q(1)}{\triangle q} (g + b)] + (1 - \frac{A_k}{h_M N}) w_H^3 > w_L^3,$$

(28)

where $\frac{A_k}{h_M N} \in (0, 1)$ represents the share of the manager at $E_4$. Analogous to the comparison of $E_2$ and $E_3$, because $\max[w_H^3 + g, \frac{q(1)}{\triangle q} g] > w_H^3 > w_H^2$ and $w_L^3 < w_L^2$, the skill premium at $E_4$ is larger than the skill premium at $E_2$. With regard to the comparison between $E_3$ and $E_4$, there are three possible cases: (1) both $E_3$ and $E_4$ lie in the region where (IC) is binding (i.e., $w_H^3 < w_H^1 \leq \pi_H$), (2) $E_3$ lies in the region where (IC) is binding, while $E_4$ lies in the region where (IR) is binding (i.e., $w_H^3 < \pi_H < w_H^1$), and (3) both $E_3$ and $E_4$ lie in the region where (IR) is binding (i.e., $\pi_H \leq w_H^3 < w_H^1$).\(^{20}\) Because we have $w_H^3 > w_H^1$ and $w_L^3 < w_L^1$, $\frac{A_k}{h_M N} \geq \frac{A_k}{h_M N}$ is the sufficient condition for the skill premium at $E_4$ to be larger than the skill premium at $E_3$. This inequality means that the share of the manager at $E_4$ is not smaller than his share at $E_3$.

We can show that the sufficient condition $\frac{A_k}{h_M N} \geq \frac{A_k}{h_M N}$ holds when both $E_3$ and $E_4$ lie in the regions where (IC) is binding (i.e., $w_H^3 < w_H^1 \leq \pi_H$). From Figure 5, when (IC) is binding, $R \equiv \pi H_N = F_D^{IC}$ holds in the D-mode equilibrium. This implies that the skilled labor size in new technology firms remains constant at $H_N = \frac{1}{\pi} F_D^{IC}$. Therefore, supposing that both $E_3$ and $E_4$ lie in the regions where (IC) is binding, even when the skilled ratio $h$ increases from $h_M$ to $h_H$, the skilled labor size of the new technology firms $H_N$ will remain constant at $H_N = \frac{1}{\pi} F_D^{IC}$. On the contrary, the skilled labor size in the

\(^{20}\)Figure 6 displays the case of (2), where the relation $w_H^3 < \pi_H < w_H^1$ holds.
old technology firms $H_O$ decreases because $H_O(w^4_H) < H_O(w^3_H)$ holds. Therefore, recalling the labor market clearing condition (i.e., $A = hN - H_O - H_N$), $\frac{A_4}{hM} \geq \frac{A_3}{hM}$ is satisfied. Supposing that both $E_3$ and $E_4$ lie in the regions where (IC) is binding (i.e., $w^3_H < w^4_H \leq w_H$), the skill premium at $E_4$ becomes larger than it was at $E_3$.

In the other two possible cases (i.e., $w^3_H < \overline{w}_H < w^4_H$ and $\overline{w}_H \leq w^4_H < w^3_H$), we cannot show that $\frac{A_4}{hM} \geq \frac{A_3}{hM}$ holds analytically true while $A_4 > A_3$ holds.21 The skill premium at $E_4$ may become smaller than the skill premium at $E_3$ when the share of the manager at $E_4$ (i.e., $\frac{A_4}{hM}$) becomes too small relative to $E_3$ (i.e., $\frac{A_3}{hM}$), which offsets the impacts of the increase in the skill premium in the final goods sector (i.e., $w^4_H > w^3_H$ and $w^4_L < w^3_L$). However, our numerical exercises suggest that this case rarely occurs under plausible sets of parameter values.

These results indicate that when the economy initially lies in equilibrium $E_2$, the increase in the skilled ratio from $h_M$ to $h_H$ induces an organizational change and increases the skill premium, as the skill premium at $E_4$ is larger than the skill premium at $E_2$. Moreover, even when the economy initially lies in the equilibrium $E_3$ and an organizational change has already occurred, the increase in the skilled ratio from $h_M$ to $h_H$ raises the skill premium further under a relatively wider and plausible set of parameter values. In sum, the increase in the skilled ratio induces the skill-biased technological change and raises the skill premium. A corresponding enhancement of the delegation of authority in firms complements the appreciation of the skill premium. Carolli and Van Reenen (2001) show that although organizational changes and technological changes are complementary to the supply of skilled labor, organizational changes have a positive effect that is independent of the effect of technological change on the demand for skilled labor. Moreover, Görlich and Snower (2010) show that the recent organizational changes raise the skill premium.22 Our theoretical results are partly consistent with these empirical findings.

### 4.3 Within-group Inequality

In this subsection, we examine how within-group inequality responds to organizational changes that are caused by changes in the skilled ratio in the population. Empirical studies of wage inequality in the U.S. have argued that within-group inequality is a major component of the increased dispersion in overall wage inequality (Autor et al., 2008). These studies have documented that the rise in within-group inequality

\[21\] From (21), in the case of the D-mode equilibrium, the amount of the variety of intermediate goods $A$ is expressed as a function of $w_H$: $A = \frac{w_H}{hM}$. Therefore, because $w^3_H > w^4_H$ in Figure 6, we find that $\frac{A_4}{hM} > \frac{A_3}{hM}$. However, we cannot generally confirm that $\frac{A_4}{hM} \geq \frac{A_3}{hM}$ holds because $h_H > h_M$.

\[22\] Görlich and Snower (2010) focus on the changes in workers’ spans of competence, which are defined in terms of the breadth of their portfolios of tasks.
appears to be largely above the median of the residual wage distribution (i.e., in the upper tail of the distribution, which mainly includes college-educated workers). In our model, the heterogeneity among the skilled individuals arises on account of the two occupations that are available to them: skilled workers in the final goods sector and managers in the intermediate goods sector. An organizational change in firms at the intermediate goods sector triggers the dispersion of wages among skilled individuals.

Let us first examine within-group inequality at $E_1$, $E_2$, $E_3$ and $E_4$. Under the C-mode equilibrium at $E_1$ and $E_2$, the manager’s wage equals the skilled wage $w_H$. Therefore, within-group inequality does not appear at $E_1$ and $E_2$. However, under the D-mode equilibrium at $E_3$ and $E_4$, within-group inequality does appear. When (IC) is binding, the manager’s wage $\frac{q(1)}{\Delta q} (g + b)$ is higher than the skilled wage $w_H$, as the manager has an informational advantage over the owner. When (IR) is binding, the manager’s wage $w_H + g$ is also higher than the skilled wage because the owner needs to compensate for the manager’s action cost $g$. As a result, within-group inequality arises at $E_3$ and $E_4$.

These results indicate that the increment in the skilled ratio from $h_L$ to $h_M$ increases the within-group inequality if it induces an organizational change. This can be confirmed by the fact that within-group inequality appears at $E_3$, but it does not appear at $E_1$ and $E_2$. In addition, supposing that the economy initially lies in the equilibrium at $E_2$, the increase in the skilled ratio from $h_M$ to $h_H$ will induce an organizational change and will increase the within-group inequality. We can also confirm this from the fact that within-group inequality appears at $E_4$, but it does not appear at $E_2$.

Next, we consider the case where the economy initially lies in equilibrium $E_3$ and the skilled ratio increases from $h_M$ to $h_H$. In this case, if both $E_3$ and $E_4$ lie in the regions where (IR) is binding (i.e., $\omega_H \leq w^3_H < w^4_H$), the within-group inequality remains constant because wage differentials at $E_3$ and $E_4$ are simply given by $g$. However, if $E_3$ lies in regions where (IC) is binding and $E_4$ lies in regions where (IR) is binding (i.e., $w^3_H < \omega_H$), the within-group inequality declines slightly as the wage differentials at $E_3$ (i.e., $\frac{q(1)}{\Delta q} g - w^3_H$) are larger than they are at $E_4$ (i.e., $g$). This discrepancy results from $w^3_H < \omega_H \equiv \frac{q(1)}{\Delta q} g - g$. Supposing that both $E_3$ and $E_4$ lie in the regions where (IC) is binding (i.e., $w^3_H < w^4_H \leq \omega_H$), the within-group inequality will also decline slightly because the wage differentials at $E_3$ (i.e., $\frac{q(1)}{\Delta q} g - w^3_H$) are larger than they are at $E_4$ (i.e., $\frac{q(1)}{\Delta q} g - w^4_H$). This discrepancy results from $w^3_H < w^4_H$.

[Figure 7]

Figure 7 depicts a typical example of the relationship between the skilled ratio and within-group in-
equality. At a sufficiently low value of \( h \), only the C-mode equilibrium is realized; therefore, within-group inequality does not exist. However, at the intermediate range of values of \( h \in [h_L, h_R] \), an organizational change occurs and the D-mode equilibrium is realized. Figure 7 shows the case where the organizational change occurs at \( h' \in [h_L, h_R] \), and the skilled wage at \( h' \) lies in the regions where (IC) is binding (i.e., \( w_{H}^{h=h'} < \overline{w}_{H} \)). In this case, within-group inequality at \( h = h' \) is represented by \( \frac{q(1)}{\Delta q}(g + b) - w_{H}^{h=h'} \). However, the skilled wage increases as the skilled ratio increases, and therefore, within-group inequality \( \frac{q(1)}{\Delta q}(g + b) - w_{H} \) declines slightly. Supposing that the skilled wage reaches the value of \( \overline{w}_{H} \), the economy will enter the regions where (IR) is binding (i.e., \( \overline{w}_{H} \leq w_{H} \)). Therefore, the within-group inequality will have a constant value of \( g \).

These results indicate that the increase in the skilled ratio enhances the delegation of authority in firms, which increases within-group inequality. Further, the increase in the skilled labor ratio enables firm owners to reduce the rent for the delegated manager, which negatively affects within-group inequality. However, within-group inequality does not vanish, as it is necessary to induce managerial effort from the delegated manager.\(^{23}\)

5 Concluding Remarks

In this paper, we developed a simple general equilibrium model of monopolistic competition with moral hazard contracting to examine the interactions among skill-biased technological change, equilibrium organizational modes and wage inequality. Analyzing this model, We demonstrate that the increase in the relative supply of skilled labor leads to a skill-biased technological change that is represented by an increase in the variety of intermediate goods. This improves the productivity of skilled labor and leads to an organizational change from a centralized mode, in which an owner maintains the authority, to a decentralized mode, in which an owner delegates the authority to a manager. This organizational change results in the widening of within-group skilled labor wage inequality, and it increases skilled and unskilled labor wage inequality. Moreover, we show that there are multiple equilibria where the centralized and decentralized modes simultaneously emerge at the intermediate values of the relative skilled labor supply. The fact that there are multiple equilibria implies that the proportion of firms that adopt the decentralized mode can differ even among countries that have similar skilled labor ratios and technological conditions. Our results could provide an explanation for the recent empirical findings on

\(^{23}\)As an empirical study, Görlich and Snower (2010) find that recent organizational changes lead to a skill premium and within-group inequality.
the interactions among skill-biased technological change, organizational changes, and wage inequality.

Appendix A

By differentiating (22) with respect to $w_H$, we find

$$
\frac{\partial G(w_H, k; h)}{\partial w_H} = \pi \left[ (1-h)N\left[ \left( \frac{w_H}{\beta B} \right)^{\frac{1}{1-\beta}} \right] \right]^{\frac{\beta - 1}{\beta - 1}} \left( \frac{w_H}{\beta B} \right)^{\frac{\beta - 1}{\beta - 1}} \pi \left[ (1-\rho)k \right] \beta B - \frac{1}{\tilde{w} + k} (1-k) \varphi^{\frac{1}{1-\rho}} \{ \right].
$$

Supposing $\rho < \frac{1}{2}$, $\lim_{w_H \to 0} \frac{\partial G(w_H, k; h)}{\partial w_H} = \infty$, $\lim_{w_H \to \infty} \frac{\partial G(w_H, k; h)}{\partial w_H} = 0$ and $\frac{\partial}{\partial w_H} \left( \frac{\partial G(w_H, k; h)}{\partial w_H} \right) < 0$ hold.

There will be unique $\tilde{w}_H$ values that can satisfy (23).

Appendix B

This section analyzes the stability of equilibria. We mainly discuss the properties of the D-mode equilibrium. An analogous discussion could be applied to the properties of the C-mode equilibrium.

From (6), we obtain $H_O(w_H) = (1-h)N\left[ \left( \frac{w_H}{\beta B} \right)^{\frac{1}{1-\rho}} \right]^{\frac{\beta - 1}{\beta - 1}}$. We also obtain $w_H(A) = \tilde{w} + k (1-k) \varphi^{\frac{1}{1-\rho}} A$ from (20). Substituting $w_H(A)$ into $H_O(w_H)$, we obtain $H_O(w_H(A)) = (1-h)N\left[ \left( \frac{w_H(A)}{\beta B} \right)^{\frac{1}{1-\rho}} \right]^{\frac{\beta - 1}{\beta - 1}}$.

Further, by substituting (22), $w_H(A)$, and $H_O(w_H(A))$ into (12), we can express the owner’s expected payoff under the D-mode $\hat{\pi}_D$ as a function of $A$ as follows:

$$
\hat{\pi}_D(w_H(A), A) = R(w_H(A), A) - \max\left\{ \frac{g(1)}{\Delta g} (g+b), w_H(A) + g \right\},
$$

where $R(w_H(A), A) \equiv \pi \left[ hN - H_O(w_H(A)) - A \right]$. Note that the number of firms $A^*$ at a stable equilibrium must satisfy the following property: $\frac{d\hat{\pi}_D(w_H(A^*), A^*)}{dA} \leq 0$. An intuitive explanation of this property is as follows. Suppose that the economy is in equilibrium with $A^*$, which satisfies $\frac{d\hat{\pi}_D(w_H(A^*), A^*)}{dA} > 0$, and assume that the number of firms that enter the intermediate goods market has increased (resp. decreased) slightly from $A^*$ to $A^* + \epsilon$ (resp. $A^* - \epsilon$) for exogenous reasons. In this case, because $\frac{d\hat{\pi}_D(w_H(A^*), A^*)}{dA} > 0$, the relation $\hat{\pi}_D(w_H(A^* + \epsilon), A^* + \epsilon) > 0$ (resp. $\hat{\pi}_D(w_H(A^* - \epsilon), A^* - \epsilon) < 0$) must hold, and more firms have incentives to enter (resp. exit from) the intermediate goods market. Hence, the number of firms increases (resp. decreases) from $A^*$, and the equilibrium with $A^*$ that satisfies $\frac{d\hat{\pi}_D(w_H(A^*), A^*)}{dA} > 0$ is unstable, as the economy cannot be returned to the original equilibrium once it deviates for exogenous reasons.

By differentiating $\hat{\pi}_D(w_H(A), A)$ with $A$, we obtain

$$
\frac{d\hat{\pi}_D(w_H(A), A)}{dA} = \begin{cases} 
\frac{1}{2\Delta g} \left[ \frac{\partial R}{\partial w_H} + \frac{\partial R}{\partial A} \right] - \frac{1}{\Delta g} \left[ \frac{\partial w_H}{\partial A} \right], & \text{if } w_H \geq \frac{g(1)}{2\Delta g} (g+b) - g, \\
\frac{1}{2\Delta g} \left[ \frac{\partial R}{\partial w_H} + \frac{\partial R}{\partial A} \right] + \frac{\partial R}{\partial A}, & \text{if } w_H < \frac{g(1)}{2\Delta g} (g+b) - g,
\end{cases}
$$

23
\[
\frac{d\pi_D(w_H(A), A)}{dA} = \begin{cases} 
-\pi \frac{\partial H_O}{\partial w_H} - 1 & \text{if } w_H \geq \frac{q(1)}{\Delta q}(g + b) - g, \\
-\pi \frac{\partial H_O}{\partial w_H} - \pi & \text{if } w_H < \frac{q(1)}{\Delta q}(g + b) - g.
\end{cases}
\]

A simple calculation leads to the following conditions, which are satisfied at the stable equilibrium:

\[
\frac{\partial \pi_D(w_H(A), A)}{\partial A} \leq 0 \Rightarrow \begin{cases} 
\pi \left( \frac{\partial H_O}{\partial w_H} - \frac{1}{w_H} \right) < 1 & \text{if } w_H \geq \frac{q(1)}{\Delta q}(g + b) - g, \\
\pi \left( \frac{\partial H_O}{\partial w_H} - \frac{1}{w_H} \right) < 0 & \text{if } w_H < \frac{q(1)}{\Delta q}(g + b) - g.
\end{cases}
\]

On the contrary, from (16) and (22), the conditions that the slope of the \( F_D(w_H) \) line is not smaller than that of the \( G(w_H, k; h) \) line are expressed as follows:

\[
\frac{\partial G}{\partial w_H} \leq \frac{\partial F_D}{\partial w_H} \Rightarrow \begin{cases} 
\pi \left( \frac{\partial H_O}{\partial w_H} - \frac{1}{w_H} \right) \leq 1 & \text{if } w_H \geq \frac{q(1)}{\Delta q}(g + b) - g, \\
\pi \left( \frac{\partial H_O}{\partial w_H} - \frac{1}{w_H} \right) \leq 0 & \text{if } w_H < \frac{q(1)}{\Delta q}(g + b) - g.
\end{cases}
\]

Hence, the D-mode equilibrium is stable when the slope of the \( F_D(w_H) \) line is not smaller than the slope of the \( G(w_H, k; h) \) line (i.e., \( \frac{\partial G}{\partial w_H} \leq \frac{\partial F_D}{\partial w_H} \)).

**Appendix C**

This section examines the parameter conditions for which the both of the D-mode and the C-mode equilibria exist.

First, we consider the necessary parameter conditions for which the stable D-mode equilibrium exists when \( h = 1 \). From Figures 3 to 5, the D-mode equilibrium is more likely to emerge when the skilled ratio \( h \) is high. Here, we consider the case where \( h = 1 \) in order to obtain necessary parameter conditions.

When \( h = 1 \), because \( H_O(w_H; 1) = 0 \), the gross profit under the D-mode of (22) is represented as

\[
R = \pi[N - A(w_H, k)] \equiv G(w_H, k; 1).
\]

Because the \( G(w_H, k; 1) \) line is downward sloping in \( w_H \), \( G(w_H, k; 1) \) and the free-entry condition of \( F(w_H) \) in (18) have a unique intersection. Hence, noting \( k = 1 \), the equilibrium for skilled wages when (IC) is binding under the D-mode satisfies the following equality \( \pi[hN - \frac{w_H}{w}] = \frac{q(1)}{\Delta q}(g + b) \) or

\[
w_H = \frac{\pi hN}{\pi} - \frac{q(1)}{\Delta q}(g + b) \frac{1}{\pi}.
\]

On the contrary, from (18), the D-mode equilibrium is realized when \( \frac{q(1)}{\Delta q}(g + b) \phi^{-\frac{\pi}{\Delta q}} - c \leq w_H \).

Therefore, supposing that the condition

\[
\frac{q(1)}{\Delta q}(g + b) \phi^{-\frac{\pi}{\Delta q}} - c < \pi hN - \frac{q(1)}{\Delta q}(g + b) \frac{1}{\pi}.
\]

(29)
holds, the stable D-mode equilibrium will exist when $h = 1$. Note that the sufficiently large value of the population size $N$ ensures the above inequality.

Next, we examine the parameter conditions for which the stable C-mode equilibrium exists. For clarity of explanation, we consider the Cobb-Douglas production function (i.e., $\rho \to 0$) in the old technology: $Y_O = BH_O^\alpha L_O^{1-\beta}$. From Appendix B, supposing that the condition

$$\frac{\partial G}{\partial w_H} \leq \frac{\partial F_C}{\partial w_H} \Leftrightarrow \pi\left(-\frac{\partial H_O}{\partial w_H} - \frac{1}{\pi(k + (1 - k)\varphi^{-\frac{1}{1-\beta}})}\right) \leq \varphi^{-\frac{1}{1-\beta}}$$

holds, the C-mode equilibrium will be stable. Because $k = 0$ under the C-mode and $H_O(w_H; h) = (1 - h)N(\beta B)^{\frac{1}{1-\beta}}w_H^{\frac{1}{1-\beta}}$ under the Cobb-Douglas specification, this stability condition is represented as

$$\left[\frac{\pi(\beta B)^{\frac{1}{1-\beta}}(1 - h)N}{(1 + \alpha)(1 - \beta)\varphi^{\frac{1}{1-\beta}}}\right]^{\frac{1}{1-\beta}} \leq w_H.$$ 

On the contrary, from (18), the C-mode equilibrium is realized when $\frac{q(1)}{\Delta q}(g + b)\varphi^{-\frac{1}{1-\beta}} - c \geq w_H$. Therefore, supposing that the condition

$$\left[\frac{\pi(\beta B)^{\frac{1}{1-\beta}}(1 - h)N}{(1 + \alpha)(1 - \beta)\varphi^{\frac{1}{1-\beta}}}\right]^{\frac{1}{1-\beta}} \leq \frac{q(1)}{\Delta q}(g + b)\varphi^{-\frac{1}{1-\beta}} - c \quad (30)$$

holds, the stable C-mode equilibrium will exist. Note that the sufficiently small values of the old economy technology parameter $B$ and the skilled ratio $h$ ensure the above inequality.

As a result, under the parameter conditions under which (29) and (30) are simultaneously satisfied, the equilibrium organizational mode changes from the C-mode equilibrium to the D-mode equilibrium as the skilled ratio increases.

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Figure 1: The Optimal Organizational Modes

Figure 2: Free Entry Conditions
Figure 3: C-mode Equilibrium $E_C: h = h_C$

Figure 4: D-mode Equilibrium $E_D: h = h_D$
Figure 5: Multiple Equilibrium $E_C$, $E_D$, and $E_M$: $h = h_M$

Figure 6: Skilled and Unskilled Wages
Figure 7: Within-Group Inequality