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The endogenous timing of tariff policy

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Abstract
Using the Hamilton–Slutsky extended endogenous timing game of observable delay framework, we analyze the endogenous timing of tariff policy in the presence of a time lag between production and trade decisions. We particularly focus on the strategic relationships between the government of the importing country and an exporting monopoly firm. We show that a natural Stackelberg situation exists in which the importing country government as first mover determines the tariff rate and the exporting monopoly firm as second mover determines the production level. We also find this equilibrium is Pareto superior to both the Nash and alternative Stackelberg equilibria. This implies that commitment to a given tariff policy before the production decision is made is optimal for affected parties.

JEL Classification Number: F13
Keywords: endogenous timing game, Nash equilibrium, Stackelberg equilibrium, tariff policy, monopoly

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1. Introduction

In the field of international trade and industrial policy, many studies have analyzed the timing of strategic trade policy (including export subsidies, tariffs, quotas, and countervailing duties) between importing and exporting countries in the presence of international oligopolies using multistage games (Arvan, 1991; Collie, 1991, 1994; Syropoulos, 1994; Hayashibara, 2002; Supasri and Tawada, 2007; and others). At least some of these assume uncertainty (Shivakumar, 1993; Wong and Chow, 1997). For the most part, these analyses have also dealt with games in which the governments of the importing and exporting countries are players. However, by focusing on the strategic relationship between the government in the importing country and the private sector in the exporting country, and on the presence of a time lag between production and trade decisions, we instead consider the optimal timing of tariff policy.

Using Hamilton and Slutsky’s (1990) extended endogenous timing game of observable delay framework, we consider an endogenous timing game between the government of the importing country (hereafter, the home government) in determining the tariff rate and a monopoly firm in the exporting country (hereafter, the foreign monopoly) determining its production given a time lag between the trade and production decisions. In other words, we analyze whether the home government should decide the tariff rate prior to the production level decision under perfect information. Further, we show whether a Stackelberg equilibrium, under which the home government is the leader and the foreign monopoly is the follower, is preferable for both players to any other possible equilibria.

Incidentally, it sometimes occurs that in the presence of a time lag, the home government may renege on its tariff policy ex post, i.e., a time consistency problem. In other words, the ex ante optimal tariff rate determined by the home government is not credible to the foreign
monopoly. With regard to this issue, we discuss the possibility that the optimal timing of tariff policy does not change if there are sufficiently large political costs involved in its revision.

The remainder of the paper is structured as follows. In Section 2, we present a simple model. In Section 3, we first derive the reaction functions of the two players, i.e., the home (importing) government and the foreign (exporting) monopoly, and show the Nash equilibrium in a simultaneous move game. We then respectively derive the Stackelberg equilibrium in two sequential move games: namely, where the home government is the leader (follower) and the foreign monopoly is the follower (leader).

In Section 4, we use the extended endogenous timing game of observable delay framework in Hamilton and Slutsky (1990) to introduce a zero-stage in which the players choose the move of the action and consider the optimal timing of tariff policy. Using this, we illustrate that a natural Stackelberg equilibrium exists that is Pareto superior to any of the other equilibria. Finally, in Section 5, we summarize our discussion. We clarify the characteristics of the endogenous timing game with regard to the tariff rate and the production level decisions in the Appendix after assuming general demand and cost functions.

2. A Simple Model

2.1 Setup

Exploiting the simple setting presented by Toshimitsu (1997), we assume that there are no domestic firms in the importing country, with only a firm in the exporting country (*) supplying a product to both the home and foreign markets. As we assume the foreign firm is a monopoly, we refer to it hereafter as the foreign monopoly.

To justify this simple setup, examples include natural resources industries such as crude
oil, natural gas, rare metals, and other mineral resources along with agricultural products industries such as corn, wheat, and other grains. We can characterize these industries using the following two key features: first, market control through monopolistic power, and second, a time lag between production (capacity) and trade (consumption) decisions. In terms of the first key feature, oligopolies control many natural resources markets. With oil, these include Shell, BP, and the Organization for Petroleum Exporting Countries along with various state-owned companies. With agricultural products, they include large multinational agriculture companies such as Cargill and Continental Grain, among others. In terms of the second key feature, we easily appreciate that a time lag exists between extracting crude oil and producing agricultural products and their supply to international markets.

Let us assume the linear inverse demand functions for the segmented home and foreign markets as:

\[ P = a - \frac{q}{s} \quad \text{and} \quad P^* = a - q^*, \]

where \( P \) (\( P^* \)) is the price and \( q \) (\( q^* \)) is the quantity supplied in the home (foreign) markets and \( s \) represents the market size of the home country.

The players in the following games are the foreign monopoly and the home government. We present the objective (i.e., payoff) functions of the players as follows. The profit function of the foreign monopoly is expressed by:

\[ \Pi^* = (P - t)q + P^* q^* - c^* X^*, \]

where \( q + q^* = X^*, \ X^* \) denotes the production level. \( t(\geq 0) \) is a per unit import tariff and \((a >)c^* (\geq 0) \) is the marginal cost of production.

The welfare of the home country is expressed by:
Thus, the home government decides the tariff rate to maximize the welfare given by (3). The welfare of the foreign country is given by:

\[ W^* = \int_0^q P^*(z)dz - P^* q^* + \Pi^* = \frac{1}{2} q^*^2 + \Pi^*. \]  

(4)

2.2 The trade decision in the final stage

As analyzed in the following sections, we consider the Stackelberg situation in two sequential move games, each comprising three stages. Note, however, that the final stage in each of the two games is common to both games in which the foreign monopoly decides to allocate the product to the home and foreign markets, i.e., \( q \) and \( q^* \), given the production level, \( X^* > 0 \), and the tariff rate, \( t \geq 0 \). Thus, we derive the equilibrium in the final stage as:

\[ q(X^*, t) = \frac{S}{S+1} X^* - \frac{S}{2(S+1)} t \quad \text{and} \quad q^*(X^*, t) = \frac{1}{S+1} X^* + \frac{S}{2(S+1)} t, \]  

(5)

where \( \bar{t} \equiv 2X^* > t \geq 0 \), such that \( \bar{t} \) is a prohibitive tariff level.

3. Tariff Rate and Production Level Decision Game

3.1 Reaction functions and the Nash equilibrium

Before considering the Stackelberg games, we analyze the Nash equilibrium in a simultaneous move game to clarify the reaction and the payoff functions of the two players. First, bearing in mind (1), (2), and (5), and the first-order condition (FOC) to maximize the
foreign monopoly’s profit, given \( t(\geq 0) \), the production level is:

\[
X^* = \frac{s + 1}{2} A - \frac{s}{2} t, \tag{6}
\]

where \( A = a - c^* > 0 \). Equation (6) is the reaction function of the foreign firm. In particular, a strategic substitute relationship holds between the production level and the tariff rate. Thus, an increase in the tariff rate reduces the production level, and this in turn decreases the profit of the foreign firm, i.e., \( \frac{\partial \Pi^*}{\partial t} < 0 \). Second, the home government decides the tariff rate, given \( X^* (> 0) \). Taking (5) into account, the welfare function of the home government in (3) can be expressed by:

\[
W(X^*, t) = \frac{1}{2s} (q(X^*, t))^2 + tq(X^*, t). \tag{3'}
\]

From the FOC with respect to the maximization of (3’), we derive:

\[
t = \frac{2(2s + 1)}{4s + 3} X^*, \tag{7}
\]

where (7) is the reaction function of the home government. It is clear here that a strategic complement relationship holds between the production level and the tariff rate. Thus, an increase in the production level increases the tariff rate, and this in turn increases the welfare of the home government, i.e., \( \frac{\partial W}{\partial X^*} > 0 \).

Figure 1 depicts the reaction curves using (6) and (7). As shown, the reaction curve of the foreign monopoly is downward sloping and that of the home government is upward sloping. Therefore, we obtain the Nash equilibrium as:

\[
t^N = \frac{(s + 1)(2s + 1)}{2s^2 + 5s + 3} A \quad \text{and} \quad X^{*N} = \frac{(s + 1)(4s + 3)}{2(2s^2 + 5s + 3)} A. \tag{8}
\]
where superscript $N$ denotes the Nash equilibrium. See point $N$ in Figure 1, $\{X^N, X^{*N}\}$.

Substituting (8) into (5), and taking (2) and (3) into account, we have the payoffs of the players, i.e., the home government’s welfare and the foreign monopoly’s profit, in the Nash equilibrium as:

$$W^N = \frac{s(s + 1)^2(5s + 2)}{2(2s^2 + 5s + 3)^2} A^2$$
$$\Pi^* = \frac{(2s^2 + 5s + 3)^2 + 4s(2 - s)(s + 1)^2}{4(2s^2 + 5s + 3)^2} A^2, \quad (9)$$

Figure 1 depicts the iso-profit and iso-welfare curves, i.e., $\Pi^N$ and $W^N$, crossing at $N$. For the following analysis, we obtain the foreign country’s welfare as:

$$W^{*N} = \frac{3(2s^2 + 5s + 3)^2 + 8s(2 - s)(s + 1)^2}{8(2s^2 + 5s + 3)^2} A^2.$$  

3.2 Two Stackelberg equilibria

In this subsection, we consider the Stackelberg equilibrium in two sequential move games in which the home government and then the foreign monopoly is the leader.

3.2.1 The home government as leader

Substituting (6) into (5), the allocation of the product to the home and foreign markets is given by:

$$q^{hL}(t) = \frac{s}{2}(A - t) \quad \text{and} \quad q^{*hL} = \frac{1}{2} A, \quad (10)$$

where superscript $hL$ indicates the case the home government is the leader. Using (3) and (10), the home country’s welfare function is expressed as:

$$W^{hL}(t) = \frac{1}{2s} (q^{hL}(t))^2 + tq^{hL}(t). \quad (11)$$

From the FOC to maximize the home country’s welfare given by (11), we derive the ex ante
optimal tariff rate:

\[ t_{hL} = \frac{A}{3}. \]  

(12)

Substituting (12) into (6), we have the production level as:

\[ X_{hL}^* = \frac{2s + 3}{6} A. \]  

(13)

In this case, point \( S_{hL}^* \) in Figure 1 denotes the Stackelberg equilibrium in which the home government is the leader and the foreign monopoly is the follower. Thus, the welfare of the home country and the profit of the foreign firm are respectively given by:

\[ W_{hL}^* = \frac{s}{6} A^2 \quad \text{and} \quad \Pi_{hL}^* = \frac{4s + 9}{36} A^2. \]  

(14)

We depict the iso-profit and iso-welfare curves, i.e., \( \Pi_{hL}^* \) and \( W_{hL}^* \), in Figure 1. The foreign country’s welfare is given by \( W_{hL}^* = \frac{8s + 27}{72} A^2 \).

3.2.2 The foreign firm as a leader

Substituting (7) and (5), we derive the following:

\[ q_{fL}^*(X^*) = \frac{2s}{4s + 3} X^* \quad \text{and} \quad q_{fL}^*(X^*) = \frac{2s + 3}{4s + 3} X^*, \]  

(15)

where superscript \( fL \) indicates the foreign firm is the leader.

Using (1), (2), and (15), the foreign monopoly decides on the production level to maximize the profit: \( \Pi_{fL}^*(X^*) = X^* \left\{ A - \frac{12s^2 + 20s + 9}{(4s + 3)^2} X^* \right\} \). That is, from the FOC, we obtain the production level as:

\[ X_{fL}^* = \frac{(4s + 3)^2}{2(12s^2 + 20s + 9)} A. \]  

(16)
Substituting (16) into (7), the ex post optimal tariff rate is given by:

\[
t^*_{FL} = \frac{(2s + 1)(4s + 3)}{12s^2 + 20s + 9} A.
\]  

(17)

Hence, point \( S^*_{FL} \) in Figure 1 denotes the Stackelberg equilibrium in which the foreign monopoly is the leader and the home government is the follower. Thus, the welfare of the home government and the profit of the foreign firm are respectively given by:

\[
W^*_{FL} = \frac{s(5s + 2)^2}{2(12s^2 + 20s + 9)^2} A^2 \quad \text{and} \quad \Pi^*_{FL} = \frac{(4s + 3)^2}{4(12s^2 + 20s + 9)} A^2.
\]  

(18)

We similarly depict the iso-profit and iso-welfare curves, i.e., \( \Pi^*_{FL} \) and \( W^*_{FL} \), in Figure 1. The foreign country’s welfare is given by

\[
W^*_{FL} = \frac{(4s + 3)^2 (28s^2 + 52s + 27)}{8(12s^2 + 20s + 9)^2} A^2.
\]

3.3 Comparison between the equilibrium outcomes and a free-trade policy

Taking (8), (12), (13), (16), and (17) into account, with regard to the tariff rate and the production level in each equilibrium, the following relationships holds:

\[
t^hL < t^*_{FL} < t^N \quad \text{and} \quad X^*_{hL} > X^*_{FL} > X^*N.
\]  

(19)

Given (19), \( t^hL < t^*_{FL} \) holds. As pointed out by Lapan (1988) in the case of perfect competition, we reconfirm that the ex post optimal tariff rate is higher than the ex ante optimal tariff rate in the presence of an exporting monopoly. Further, it holds that \( X^*_{hL} > X^*_{FL} \). That is, to avoid a heavy tariff burden, the foreign monopoly reduces its production level compared with the situation in which the home government is the leader.

In the following analysis, we sequentially compare the payoffs of the home government and the foreign monopoly in the Nash equilibrium and the two Stackelberg equilibria. Taking (9), (14), and (18) into account, the following relationships hold:
Based on (20) and (21), it is clear that the home government prefers to be a leader and the foreign monopoly a follower.

Here we compare the equilibrium outcomes derived above with a free-trade policy in which the home government can commit to zero tariffs, i.e., \( t = 0 \). Hence, from (6), the production level is given by

\[
X^*_{\text{free}} = \frac{s + 1}{2} A > X^*_{\text{FL}},
\]

where subscript \( \text{free} \) indicates the free-trade policy. Accordingly, because \( q^f = \frac{s}{2} A \) holds, we obtain \( W_{\text{free}} = \frac{s}{8} A^2 \). Hence, in view of (14), we obtain easily \( W^{hL} > W_{\text{free}} \). As shown by Brander and Spencer (1984, Proposition 1), the ex ante optimal tariff policy is superior to the free-trade policy for the importing country. That is, the tariff policy reduces imports and thus reduces consumer surplus, but increases tariff revenues by capturing the rent of the foreign monopoly. As the latter (positive) effect outweighs the former (negative) effect, the ex ante optimal tariff policy improves the importing country’s welfare relative to a free-trade policy.

In view of (18), and with respect to the ex post tariff policy and the free-trade policy, it holds that \( W^{FL} \geq (<) W_{\text{free}} \leftrightarrow \hat{s} \geq (<) s \), where \( \hat{s} = 0.564 \). As Toshimitsu (1997) argues, if the market size of the home country is larger (smaller) than a particular level, i.e., \( s > (<) \hat{s} \), the home government prefers a free-trade (the ex post tariff) policy. For example, most developed economies are large markets. Hence, the larger the market size, the larger the
production level. This, in turn, leads to an increase in the home country’s welfare. Thus, because the effect of a decrease in consumer surplus outweighs that of an increase in tariff revenue through rent capture with the tariff policy, governments in developed economies prefer to a free-trade policy.

4. The Endogenous Timing Game and the Natural Stackelberg Situation

4.1 The optimal timing of tariff policy

Based on the results derived in the previous section, i.e., (20) and (21), and employing the extended endogenous timing game of observable delay framework in Hamilton and Slutsky (1990), we now consider the optimal timing of tariff policy. That is, the players (the home government and the foreign firm) determine the timing of the action as well as the action itself, i.e., both the tariff rate and the production level. If the players choose the actions at different times, then the player choosing a later time observes the action chosen by the player playing first, giving rise to a sequential play subgame, and thus a Stackelberg equilibrium holds in the game. If the players instead choose actions at the same time, then a simultaneous play subgame occurs, and thus a Nash equilibrium holds in the game. Consequently, by using an extended endogenous timing game of observable delay, we provide comparisons between the payoffs in the simultaneous play game and the two sequential play games.

We introduce a zero-stage game, in which the players choose in advance the timing of the action, into the three-stage games discussed earlier. That is, each player moves first, $F$, or second, $S$. Let us define the payoffs as:


(22)

$$ \Pi^*[F, S] = \Pi^{*hL}, \quad \Pi^*[S, F] = \Pi^{*fL}, \quad \text{and} \quad \Pi^*[F, F] = \Pi^*[S, S] = \Pi^*N. $$

(23)
where the two terms in parentheses, [ , ], denote the home government’s move and the foreign firm’s move, respectively. In this case, we obtain the payoff matrix with respect to the endogenous timing game (see Table 1).

Taking (20), (21), (22), and (23) into account, it is clear that making the first move, i.e., being a leader, is a dominant strategy for the home government. Similarly, the foreign monopoly moves second (first) when the home government moves first (second). However, the foreign monopoly has to make the second move, because it expects under perfect information that the home government always makes the first move.

Therefore, based on Hamilton and Slutsky (1990, Theorem V (B)) and Albaek (1990, Definition), a natural Stackelberg situation is attained in which the home government is a leader and the foreign firm is a follower. In addition, a natural Stackelberg equilibrium exists in the Pareto superior set (see point $S^{hl}$ in Figure 1). Further, it is preferable for the foreign government that the natural Stackelberg situation continues, because $W^{hl} > W^{fL} > W^{*N}$ holds.

We summarize the following proposition with respect to the optimal timing of tariff policy.

Proposition 1

*The Pareto optimal timing is that the home government first determines the ex ante optimal tariff rate and the foreign monopoly then determines the production level.*
4.2 Discussion: the possibility of reneging on the tariff rate and credibility

When considering the situation where the home government is a leader and the foreign monopoly is a follower, we implicitly assume that the foreign monopoly believes that the home government executes the ex ante optimal tariff policy after the foreign monopoly makes the production decision. In other words, the home government commits to the tariff policy and never reneges on the tariff rate. However, in the presence of a time lag between the production and trade decisions, if there are no commitment devices in relation to the determination of the tariff rate, the ex ante optimal tariff policy of the home government may not be credible to the foreign monopoly. As mentioned, the production level (or the investment needed to increase the capacity) decision is irreversible, whereas it is not perfectly impossible for the home government to revise the tariff policy. That is, after the foreign monopoly decides the production level, the home government has an incentive to revise the tariff rate. We can easily derive

\[ W^R = W(X^{*hL}, t_i^{hL}) > W^{hL} = W(X^{*hL}, t_i^{hL}) = W[S, F], \]

where superscript \( R \) indicates where the tariff rate is revised. Consequently, the natural Stackelberg situation may not be credible to the foreign monopoly.

However, we can observe that a government must incur some heavy costs to revise its tariff policy. For example, a government must input various resources and time to change its public policies, and it also incurs costs in relation to the damage to its reputation. In particular, once a revision occurs, other governments and economic agencies (including domestic and foreign firms and domestic consumers) no longer have faith in the government’s policies. This implies opportunity costs for the government. Moreover, we expect costs associated with retaliation by the foreign government.

Therefore, if we assume a sufficiently large political cost, \( g \), involved in revising the tariff policy, which satisfies \( g > W^R - W^{hL} (> 0) \), then the home government does not have
an incentive to renege on the ex ante tariff rate. In this case, Proposition 1 is unaltered.

5. Concluding Remarks

To justify our model, we have taken the example of a game between the government of a resource-poor country and a resource-exporting monopoly. We could propose another example as follows. When there is a time lag between research and development (R&D) investment and production by big companies in advanced countries, even if there are domestic manufacturing industries in some developing countries, the domestic firms do not have any power in the international oligopolistic market. In this case, the governments of developing countries should also consider the optimal timing of intervention along with an optimal trade policy.

Regardless, we do not deny the specificity of our model, e.g. with no domestic firms in the importing country, no trade policy of the government in the exporting country, monopolistic market, and so on. Thus, we would need to relax these strong assumptions to simplify our analysis. However, based on a simple model, we have presented some interesting results as follows.

First, we have shown that a natural Stackelberg situation exists in which the government of the importing country is a leader and a monopoly in the exporting country is a follower. In other words, the optimal timing is that the government first determines the ex ante optimal tariff rate before the monopoly determines the production level. Furthermore, the natural Stackelberg equilibrium is Pareto superior to any of the other equilibria.

Second, we have discussed that if the government has to incur a sufficiently large political cost in revising its tariff policy, then the government does not have an incentive to renege on
the tariff rate ex post. However, we have assumed political costs as exogenously given. This assumption limits our model, so that we must further investigate the economic implications of the political cost.

For example, Toshimitsu (2012) employs a similar setup as the current analysis, but in a different context, and analyzes the role of international organizations as commitment devices to a free-trade policy in the presence of time inconsistency. The results show that if the expected loss caused by reneging of policy is sufficiently large, the government of the importing country never has an incentive to change from a free-trade policy. In our model, we could interpret this as meaning that the expected loss is equal to the political cost.

Appendix: A General Model

Assuming general inverse demand and production cost functions, we first derive the reaction functions of the players, i.e., the home government and the foreign monopoly, in the tariff rate and production level decision game. We then illustrate the properties of the reaction and the payoff functions. We also illustrate the Pareto superior sets in which a natural Stackelberg equilibrium exists.

(1) Setup and the equilibrium in the final stage

We assume general inverse demand functions as follows.

\[ P = P(q), \quad P^* = P^*(q^*), \quad \text{and} \quad q + q^* = X^*, \]  

(A.1)

where \( P'(q) < 0 \) and \( P''(q^*) < 0 \). With respect to the production cost function, we provide:

\[ C^* = C^*(X^*), \]  

(A.2)
where \( C^* (X^*) > 0 \) and \( C^{**} (X^*) \geq 0 \). Thus, the profit function of the foreign monopoly is expressed by:

\[
\Pi^* = \{P(q) - t\}q + P^* (q^*)q^* - C^* (X^*).
\]  
(A.3)

As given by (3) in the text, the objective function of the home government is given by:

\[
W = \int_0^q P(z)dz - P(q)q + tq.
\]  
(A.4)

Let us derive the equilibrium for the trade decision in the final stage, in which the foreign monopoly determines the allocation of the product for the home and the foreign markets, given the tariff rate, \( t \geq 0 \), and the production level, \( X^* > 0 \). The first-order condition (FOC) is given by:

\[
P(q) - t + P'(q)q - \left\{ P^* (q^*) + P^* (q^*)q^* \right\} = 0,
\]  
(A.5)

where \( q + q^* = X^* \). That is, (A.5) implies that the marginal revenue in the home market is equal to that in the foreign market, given the production level. Thus, in view of (A.5), we express the allocation of the product for both the home and foreign markets as depending on the production level and the tax rate as:

\[
q = q(t, X^*) \quad \text{and} \quad q^* = q^* (t, X^*) = X^* - q(t, X^*).
\]  
(A.6)

By totally differentiating the equation systems of (A.5), we obtain:

\[
\frac{\partial q}{\partial t} = q_t = \frac{1}{\Delta} < 0,
\]  
(A.7.1)

\[
\frac{\partial q^*}{\partial t} = q^*_t = -\frac{1}{\Delta} > 0,
\]  
(A.7.2)

\[
\frac{\partial q}{\partial X^*} = q_{X^*} = \frac{\delta}{\Delta} > 0,
\]  
(A.8.1)
where by assuming the second-order conditions, \( \delta = 2P^* q + P^* q^* < 0 \) and \( \Delta = 2P + P^* q + 2P^* q^* < 0 \) hold.

For the following analysis, we derive the second-order effects of the tariff rate as:

\[
\frac{\partial^2 q}{\partial t^2} = -\left( \frac{1}{\Delta^2} \right) \left( \frac{\partial \Delta}{\partial t} \right)\tag{A.9.1}
\]

\[
\frac{\partial^2 q^*}{\partial t^2} = -q_{tt}, \tag{A.9.2}
\]

where \( \frac{\partial \Delta}{\partial t} = \left\{ 3P^* + P^* q \right\} q^* \).

Similarly, for the second-order effects of the production level, we have:

\[
\frac{\partial^2 q}{\partial x^2} = q_{xx} = \left( \frac{1}{\Delta^2} \right) \left( \frac{\partial \delta}{\partial x} \right) - \left( \frac{\partial \Delta}{\partial x} \right) \delta \tag{A.10.1}
\]

\[
\frac{\partial^2 q^*}{\partial x^2} = -q_{xx} = -q_{xx}, \tag{A.10.2}
\]

where \( \frac{\partial \delta}{\partial x} = \left\{ 3P^* + P^* q^* \right\} q^* \) and \( \frac{\partial \Delta}{\partial x} = \left\{ 3P^* + P^* q \right\} q^* + \left( 3P^* + P^* q \right) q^* \).

With regard to the cross effects, we obtain:

\[
\frac{\partial^2 q}{\partial x^2 t} = q_{xt} = -\left( \frac{1}{\Delta^2} \right) \left( \frac{\partial \Delta}{\partial x} \right) \tag{A.11.1}
\]

\[
\frac{\partial^2 q^*}{\partial x^2 t} = -q_{xt} = -q_{xt}. \tag{A.11.2}
\]

In view of equations (A.9.1) to (A.11.2), the sign of these effects is not unidirectional.

Thus, we assume that the second and third derivatives of the inverse demand functions are
insignificant as follows.

Assumption 1

\[3P^* + P^*q = 0 \quad \text{and} \quad 3P^{**} + P^{**}q^* = 0.\]

Therefore, under Assumption 1, the second-order and cross effects of the tariff rate and production level on the product for both the home and foreign markets are negligible, i.e.,

\[q_{u*} = -q^* u \approx 0, \quad q_{XX^*} = -q^* X^* \approx 0, \quad \text{and} \quad q_{X^*t} = -q^* X^*t \approx 0.\]

(2) The properties of the reaction functions

We proceed to consider the reaction functions in the second-stage game in which the home government decides the tariff rate and the foreign monopoly decides the production level. More particularly, taking (A.6) into account, we consider the Nash game as follows.

First, given the tariff rate, the foreign monopoly decides the production level to maximize the profit given by (A.3). That is, we obtain the FOC as follows:

\[
\frac{\partial \Pi^*}{\partial X^*} = \frac{\partial \Pi^*}{\partial q} \frac{\partial q}{\partial X^*} + \frac{\partial \Pi^*}{\partial q^*} \frac{\partial q^*}{\partial X^*} + \frac{\partial \Pi^*}{\partial \Pi^*}
\]

\[
= \frac{\partial \Pi^*}{\partial X^*} = P^* + P^{**} q^* - C^* = 0.
\]  

(A.12)

The second-order properties are given by:

\[
\frac{\partial^2 \Pi^*}{\partial X^{**2}} = \left\{2P^* + P^{**} q^* \right\} q^* X^* - C^{**} < 0,
\]  

(A.13)

and

\[
\frac{\partial^2 \Pi^*}{\partial X^* \partial t} = \left\{2P^{**} + P^* q^* \right\} q^* t < 0.
\]  

(A.14)
The effect of an increase in the tariff rate on profit is given by:

$$\frac{d\Pi^*}{dt} = \frac{\partial \Pi^*}{\partial q} \frac{dq}{dt} + \frac{\partial \Pi^*}{\partial q^*} \frac{dq^*}{dt} + \frac{\partial \Pi^*}{\partial t}$$

(A.15)

$$= \frac{\partial \Pi^*}{\partial t} = -q < 0.$$

Therefore, taking (A.13), (A.14), and (A.15) into account, we present the following lemma.

Lemma 1

The reaction function of the foreign monopoly is given by $X^* = X^*(t)$, where $X^*(t) < 0$.

Thus, the strategic substitute relationship holds between the production level and the tariff rate. Further, an increase in the tariff rate reduces the profit of the foreign monopoly.

Second, given the production level, the home government decides the tariff rate to maximize welfare given by (A.4). That is, the FOC is given by:

$$\frac{dW^*}{dt} = \frac{\partial W^*}{\partial q} \frac{dq}{dt} + \frac{\partial W^*}{\partial q^*} \frac{dq^*}{dt} + \frac{\partial W^*}{\partial t}$$

(A.16)

Equation (A.16) details an optimal tariff rate that depends on the production level. Given Assumption 1, the second-order properties are given by:

$$\frac{d^2W}{dt^2} = -\left(P' + P'' q\right)\frac{dq}{dt} + 2q_{tt} + \left(-P' q + t\right)\frac{dq}{dt}^2$$

(A.17)

$$\approx q_{tt}^2 - \left(P' + P'' q\right)\frac{dq}{dt} < 0,$$

$$\frac{d^2W}{\partial t\partial X^*} = -\left(P' + P'' q\right)\frac{dq}{dt} + q_{XX^*} + \left(-P' q + t\right)\frac{dq}{dt}$$

(A.18)

$$\approx q_{XX^*} \left[ -\left(P' + P'' q\right)\frac{dq}{dt} \right] > 0.$$
The effect of an increase in the production level on the welfare is then:

\[
\frac{dW}{dX} = q X^* \left\{ -P' q + t \right\} > 0.
\]  

(A.19)

By taking (A.17), (A.18), and (A.19) into account, we present the following lemma.

Lemma 2

*The reaction function of the home government is given by \( t = t(X^*) \) where \( t'(X^*) > 0 \). Thus, the strategic complement relationship holds between the production level and the tariff rate. Further, an increase in the production level increases the welfare of the home government.*

Taking Lemmas 1 and 2 into account, we can show that a Nash equilibrium in the tariff rate and production level decision game exists between the home government and the foreign monopoly, i.e., point \( N \) in Figure 2, \( \{ P_N, X^*N \} \)

(Figure 2)

(3) Pareto superior sets and the natural Stackelberg situation

Finally, based on Lemmas 1 and 2, we illustrate the Pareto superior sets (see Figure 2). As shown, the iso-profit curve \( \Pi^N \) and the iso-welfare curve \( W^N \) cross at the Nash equilibrium, \( N \). Here the profit of the foreign monopoly improves more in the left-hand area of the iso-profit curve \( \Pi^N \). Conversely, the welfare of the home government enhances more in the upper area of the iso-welfare curve \( W^N \). In this case we illustrate the Pareto sets dominating the Nash equilibrium for both players, i.e., the area illustrated by the dashed lines in Figure 2. Thus, based on Theorem V (B) presented by Hamilton and Slutsky (1990), and
taking Lemmas 1 and 2 into account, because the reaction function of the foreign monopoly enters the Pareto superior set, the home government moves first and the foreign monopoly moves second. In other words, a Stackelberg equilibrium exists in the Pareto superior set in which the home government is the leader and the foreign monopoly is the follower. In the main body of the paper, we refer to this Stackelberg equilibrium as the natural Stackelberg situation. Further, in our model, the natural Stackelberg situation is superior to the other Stackelberg equilibrium in which the home government is the follower and the foreign monopoly is the leader.
References


Toshimitsu, T. (1997) Should a large country sustain laissez faire in the long-run? School of Economics, Kwansei Gakuin University, Discussion Paper Series No.18


Figure 1 The Stackelberg equilibria and the Nash equilibrium
Table 1 The payoff matrix in the endogenous timing game of observable delay

<table>
<thead>
<tr>
<th>The Home government</th>
<th>$F$ (First move)</th>
<th>$S$ (Second move)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$ (First move)</td>
<td>$\Pi^N$, $W^N$</td>
<td>$\Pi^fL$, $W^{fL}$</td>
</tr>
<tr>
<td>$S$ (Second move)</td>
<td>$\Pi^hL$, $W^{hL}$</td>
<td>$\Pi^N$, $W^N$</td>
</tr>
</tbody>
</table>
Figure 2 Reaction curves and Pareto superior sets