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Misleading Advertising and Minimum Quality Standards

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Abstract

This paper examines the relationship between misinformation about product quality and quality standards, such as minimum quality standards and certification criteria, when products are vertically differentiated in their health/safety aspects. We investigate the welfare effect of regulating misinformation and strengthening MQSs. We find that when the amount of misinformation on both low- and high-quality products is small, regulating misinformation on low-quality products reduces welfare, although the strictness of an MQS influences its effect. On the other hand, regulating misinformation on high-quality products always improves welfare. We also find that a stricter MQS can harm welfare. This, in particular, is likely to occur when the difference between the perceived quality of the two types of products is large and when firms generate high degrees of misperceptions. Moreover, we extend the analysis by endogenizing quality investments and demonstrate that regulating misinformation on high-quality products may deteriorate their true quality and, thus, reduce welfare.

Keywords: Advertising, Minimum quality standards, Misinformation, Vertical differentiation.

JEL Code: L13, L15, M37, Q58.

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1 Introduction

In the past few decades, consumers—particularly in developed countries—have become more conscious of the health and safety aspects of the products they consume. For example, when purchasing food products, consumers want to know when, where, and how these products were produced. Being faced with the fact that the increasing number of persons suffers from diabetes and other chronic diseases, they also demand information on whether a product is healthy, such as the percentage of fat and number of calories in their food. Similarly, when parents purchase toys for their children, they may seek information not only on the safety of the toys but also on whether toxic substances were used in production processes.\(^1\) However, since consumers often respond fervently and rapidly to information on safety and health issues and place too much confidence in it, this trend may result in extreme reactions.\(^2\) Occasionally, they even purchase goods based on information that is not well-grounded.

There are two clear strands of responses by producers, such as food companies, toy companies, and farms to this growing trend in consumer behavior. Governments have reacted to both types of firm response.

First, since consumers are heterogeneous on the weight they place on these issues relative to other quality and price factors, firms vertically differentiate their own products from those of rivals. For example, some producers offer food free of genetically modified organisms and without irradiation, while normal low-priced food survives the competition with these high-quality products.\(^3\)

Consequently, governments have taken action to regulate product quality. They have

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\(^1\)See Grunert (2005) for more details on consumer behavior. In addition, many studies have evaluated consumers’ willingness to pay for higher-quality products according to safety and health factors (see Caswell and Joseph, 2007 for a survey of this literature). Although the magnitude of the willingness to pay varies across these studies, they show that consumers are prepared to pay a premium for health and safety factors. There is also literature that investigates the effect of safety information on demand by using objective data such as prices, news, and regulations (Chang and Kinnucan, 1991, Piggot and Marsh, 2004).

\(^2\)For example, it can often be observed in Japan that if a television program recommends a certain food as nourishing and healthy, sales of that particular food product soars.

\(^3\)See Caswell and Joseph (2007) on this point.
set two types of quality standards: minimum quality standards (MQSs), which are the standards for lower-quality products, and certification criteria, which are the standards for higher-quality products. In the U.S., the Food and Drug Administration (FDA) oversees food safety and sets strict standards to regulate various kinds of products, ingredients, and additives. In Japan, the Food Sanitation Act, originally enacted in 1947, regulates food safety, thereby protecting consumers’ health, as well as the use of additives and chemicals in toys and containers. This law has been amended many times and the standards have become increasingly strict over the years. Since all firms have to comply with these regulations, they can be considered as MQSs. In other words, governments assure consumers minimum health/safety levels in daily life.

They also set certification criteria for relatively high-quality products. In developed countries, organic food is usually certified by third parties and distributed with labels conveying that information to consumers. In certain cases, countries have labeling rules for pesticide-free agricultural produce, while in others, private companies and third parties have instituted voluntary labeling systems.

Second, firms conduct misleading advertising, which is referred to as “misinformation” in the field of Industrial Organization, causing consumers to misperceive the products’ quality. In particular, such misinformation can lead consumers to believe that a product is of a higher quality than it actually is. Moreover, because the image associated with a product can influence consumer perception, print advertising and television commercials

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4 See the FDA website (http://www.fda.gov/).
5 For example, in 2003, the upper limit on the percentage content of lead in cooking utensils was lowered from 10 percent to 0.1 percent. In addition, the standards on the content of cadmium and other chemicals in toys were revised. See the website of the Japanese Ministry of Health, Labor, and Welfare for information on food safety (http://www.mhlw.go.jp/english/topics/foodsafety/index.html).
6 In the case of toys, consumers in Japan can buy toys marked “ST” (“Safety Toy”), which means that they comply with a body of standards including those regulated under the Food Sanitation Act. Some foreign standards, such as part of the standards for “CE” (Conformité Européenne) certification, are also included in the inspection that authorizes the ST mark. A toy affixed with the CE mark complies with EU standards, some of which are stricter than those set by the Japanese government. About 70% of toys distributed in Japan carry the ST mark. Consumers believe that toys with the ST label are safer than those without; they may also observe the CE mark on certain toys sold in Japan and may know that EU standards are stricter than Japanese standards.
7 Glaeser and Ujhelyi (2010) and Hattori and Higashida (2011) provide several detailed examples.
often convey only positive information. For example, many kinds of diet food available are
often marketed with sales messages that read “our product is nutritious” or “low calorie,
low fat, and good for your health.” Certain food may be advertised in the media with the
sales message of “no chemical pesticides,” possibly making consumers to misperceive the
message to mean “organic.” In addition, a commercial for a fast food chain may advise
consumers that “your health depends on eating breakfast at our hamburger restaurant”;
therefore, certain consumers perceive that eating hamburgers for breakfast may be healthy
without contemplating the negative aspects of fast food.8 These simple informational
messages can create misperceptions among consumers who are concerned about eating
healthy.9

In response to this widespread misinformation, many governments began to regulate
advertising. The EU adopted the “Television without Frontiers” Directive in 1989, which
contains provisions that regulate advertising to protect consumers’ health and safety.
Moreover, Directive 2006/114/EC regulates misleading and comparative advertising to
control misinformation in the interest of consumers, competitors, and the general pub-
ic.10 In the U.S., the Federal Trade Commission is in charge of regulating misleading
advertising. It has a special division within the Bureau of Consumer Protection—the
Division of Advertising Practices—that sets advertising guidelines for several kinds of
products, including dietary supplements.11 In Japan, in addition to the Japan Fair Trade
Commission, the government has established the Consumer Affairs Agency in 2009, which
monitors misleading advertising on health, safety, and nutritional aspects of food prod-
ucts.12 Nevertheless, misleading advertising still often appears on television, in newspa-
pers, and other media, because of the difficulty faced in judging whether an advertise-

8Garde (2008) discusses the relationship between food advertising and obesity, particularly in relation
to overweight children.
9See Byrd-Bredbenner and Grasso (2001) on this point.
10See the website of this directive for details
11See the Division of Advertising Practices website (http://www.ftc.gov/bcp/bcpap.shtm).
violates the prescribed regulations.13

This paper focuses on the relationship between misinformation/misperceptions and MQSs when products are vertically differentiated in their health/safety aspects. Our model features a low-quality product and a high-quality product. This focus is important because misinformation and MQSs are closely related to each other in terms of theory and reality. We have two main purposes: to investigate the welfare effect of regulating misinformation in the presence of quality standards (an MQS and a certification criterion), and examine the welfare effect of a stricter MQS in the presence of misinformation. We consider MQSs to be more important than certification criteria in terms of health/safety issues, because MQSs guarantee minimum quality/safety and all firms that supply products to the market must abide by them. Therefore, we focus not on a certification criterion but on an MQS.

In the field of industrial organization, advertising is classified as either informative or persuasive. Informative advertising provides consumers with useful information that enables them to recognize more precisely the true quality or attribute of a product. Persuasive advertising, in contrast, appeals to consumers by sending only information on a product’s positive attributes, which is likely to cause consumers to perceive that a product is of a better quality than it truly is. The latter type of advertising can therefore be harmful to consumers and welfare. The advertising we deal with in this study comes under persuasive advertising.14

Significant literature exists on the economic analysis of advertising (Nelson 1974, Dixit and Norman 1978, Becker and Murphy 1993, Glaeser and Ujhelyi 2010).15 Our research is related to Dixit and Norman (1978) in terms of the demand-expansion effect of advertising. They utilize both pre- and post-advertising demands in evaluating welfare,

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14 Strictly speaking, there is one more kind of advertising, known as “complementary advertising.” See Bagwell (2007).

15 Bagwell (2007) provides a survey on the economics of advertising.
and demonstrate that the market equilibrium level of advertising is excessive under a monopoly, oligopoly, and monopolistic competition. Our research is also closely related to Glaeser and Ujhelyi (2010) in terms of their focus on the welfare effect of regulations on misinformation. In particular, they investigate several kinds of policies and derive conditions under which policies can increase welfare; however, both studies assume symmetric firms.

In contrast, we consider the situation in which products of two firms are vertically differentiated in terms of health/safety-related quality. In addition to regulations on misinformation, we examine the welfare effect of a stricter MQS (a quality standard), which has not yet been dealt with in previous studies. Consumers are usually heterogeneous in their evaluation of health/safety attributes and, accordingly, firms have incentives to vertically differentiate their products. Moreover, as noted above, in practice there are usually MQSs for lower-quality products and labeling schemes for higher-quality products. Thus, the relationship between misleading advertising and regulations/standards needs to be disentangled in the presence of vertical differentiation.

There is also a large amount of literature on markets with vertically differentiated products.\footnote{Ecchia et al. (2003) surveyed the regulation of vertically differentiated markets through MQSs, and Bacchiega et al. (2010) obtained important results on MQSs and market coverage.} For example, Schmalensee (1978) assumed quality differences and examined equilibria in terms of profits, advertising, and market shares of firms, but he did not investigate welfare effects. Moreover, a considerable number of studies have examined the effects of environmental policies, including MQSs and eco-certifications, in the presence of vertical differentiation (Motta and Thisse, 1999, Moraga-González and Padrón-Fumero, 2002, Hamilton and Zilberman, 2006, Toshimitsu, 2008). Our model is closely related to that of Moraga-González and Padrón-Fumero (2002) in terms of the framework, in particular, the shape of the utility function. In contrast to their study, we focus on the situation in which misinformation exists, and accordingly, perceived utility is different from true utility.
The model has two features to support the achievement of our goals. First, we consider the case where firms compete in price. In terms of theoretical clarity and simplicity, we choose the Bertrand competition, although the case of Cournot competition can be analyzed in a similar way. Second, we consider in the main analysis that true qualities are bound by quality standards, which implies that we exclude decision making on quality investments. It is generally costly for a firm to invest in quality. Therefore, if a quality standard is strict, it is likely that a firm does not have an incentive to improve the quality of its own product by any more than that required by the standard. In addition, it can be applied to the situation in which quality investments are long-term decisions, because it takes longer for firms to decide the level of investment than to choose a certain amount of misleading advertising. Therefore, except where firms are forced to change the true quality of their products because of changes in quality standards, they do not introduce these quality changes in the short run. However, a change in the quality of a high-quality product in response to a change in the MQS may have an important welfare effect in the long run. Therefore, as an extension we consider the case in which the quality of a high-quality product can vary.

We find that when the amount of misinformation on both low- and high-quality products is large, a small decrease in misinformation on a low-quality product improves welfare, although the strictness of an MQS influences its effect. On the other hand, a small decrease in the misinformation on the high-quality product always improves welfare. We also demonstrate that a more stringent MQS can harm welfare, which in particular is likely to occur when the difference between the perceived qualities of the two types of goods is large and when firms generate high degrees of misperceptions. Results of the MQS cannot be observed when there is no misperception among consumers. Thus, it becomes clear that misinformation plays an important role in the effect of a change in an MQS. Moreover, we extend the analysis by endogenizing the quality investment of a high-quality firm and demonstrate that regulating misinformation on high-quality products may deteriorate its

17 The results do not change essentially.
true quality, and therefore, reduce welfare.

For clarity, we choose not to deal with the following two factors. First, we do not consider the issue of the quality and credibility of advertising, which has been tackled in several articles. In contrast, following Glaeser and Ujhelyi (2010), we assume that misinformation can cause consumers to misperceive a product’s quality as intended by a firm. In other words, consumers are “naive” in the sense that they always believe misinformation. Then, we introduce the cost function that relates the cost of advertising to the degree of “misperception.”

Second, we do not delve into the credibility of the certification schemes that are standards for high-quality products. In reality, sometimes many kinds of labeling exist in one product category, which can confuse consumers and/or cause them not to believe the product labeling or third parties that award these labelings. Although this is also an important issue in vertically differentiated markets, we assume in this paper that the certification scheme for high-quality products is credible because our focus is on the distortion caused by misinformation and the effect of changes in the MQS.

The structure of the paper is as follows. Sections 2 and 3 describe the model and examine the effect of changes in the amount of misinformation on firm revenue and consumer surplus, respectively. Section 4 investigates the amount of misinformation in equilibrium and the effect of government intervention, such as sending counter-information and educating consumers. Section 5 investigates the effects of stricter MQSs on the amount of misinformation and, accordingly, on welfare. Section 6 extends the analysis to the case

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18For example, Mullainathan et al. (2008) investigated how senders of advertising can persuade receivers using the concepts of transference and framing. Anderson and Renault (2006) discriminated between price and quality information. Moreover, Kihlstrom and Riorden (1984) and Milgrom and Roberts (1986) examined the role of information sent by advertising as a signal.

19Although misleading advertising can benefit a firm in the short run, it may cause a loss in the long run by damaging the firm’s reputation. The assumption of naive consumers excludes the reputational effects of misleading advertising. In addition, we do not focus on branded products but on goods that relatively have a short life, such as diet foods. In the market for these products, new products enter the market quite often. Therefore, consumers are likely to be misled on repeated occasions. Our analysis also fits for situations in which it takes a long time for consumers to realize “misperception”.

20Mahenc (2009) examined this type of credibility problem and demonstrated that labeling may be wasteful if the third party is untrustworthy.
in which a firm producing a high-quality product chooses the amount of investment in improving the product’s true quality. Section 7 provides concluding remarks.

2 The Model

2.1 Demand

There exists a continuum of heterogeneous consumers who differ in their marginal valuations, $\theta$, of unhealthy/unsafe attribute of a product. To simplify, we assume that the valuation corresponding to a consumer is uniformly distributed in the market, $\theta \in [0, 1]$. We also assume that the market is partially covered, i.e., certain consumers are seriously concerned about the negative effects of the unhealthy/unsafe attribute of products and buy nothing.

Two types of products are supplied to the market: a low-quality (Type $l$) good, which is less healthy or safe, and a high-quality (Type $h$) good. Each consumer purchases either one or no units of the product. The perceived net surplus of consumer $\theta$ is as follows:\footnote{When “positive” attributes of vertically differentiated products are focused on, the utility is usually defined as $\hat{u} = \theta(s_k - e_k) - p_k$. Even if we assume this type of utility, similar results are obtained. However, we consider situations in which consumers who are very concerned about the attribute/quality do not buy the product. Thus, the utility defined in (1) fits real situations related to health/safety issues. The utility function as observed in (1) can be seen in the field of Environmental Economics (Moraga-González, José Luis and Noemi Padrón-Fumero (2002)).}

$$\hat{u} = v - \theta(s_k - e_k) - p_k, \quad k = l, h,$$

(1)

where subscripts $l$ and $h$ denote types $l$ and $h$, respectively. $v$ and $p_k$ denote the utility obtained from consuming a single unit of the product irrespective of the unhealthy/unsafe attribute and the price of the product of Type $k$ ($k = l, h$), respectively. In addition, $s_k$ and $e_k$ denote the true quality and degree of misperception for each type of product, respectively. Defining that $\hat{s}_k = s_k - e_k$ ($k = l, h$), we consider “naive” consumers, i.e., they are able to observe only $\hat{s}_k$.

Since firms create misperceptions by sending misinformation to consumers, it can be assumed that the degree of misperception is equivalent to the amount of misinformation. Thus, $e_k$ also represents the amount of misinformation sent by each firm. A greater $s_k$...
implies a less healthy/safe product. Thus, from the definition of types, \( s_l > s_h \) holds. We assume that the order of perceived qualities is not reversed by misinformation sent by each firm, i.e., \( \hat{s}_l > \hat{s}_h \) holds. We also assume that there is no externality on advertising, i.e., misinformation of one type of product does not directly affect the perceived quality of another type of product.\(^{22}\)

We derive the demand functions for these differentiated products. The index of the marginal consumer who is indifferent between the net surpluses given by purchasing the low- and the high-quality products is characterized by \( \tilde{\theta} = (p_h - p_l) / (\hat{s}_l - \hat{s}_h) \). The index of the marginal consumer who is indifferent between the net surpluses given by purchasing the high-quality product and nothing is \( \hat{\theta} = (v - p_h) / \hat{s}_h \).

Let \( q_l \) (resp. \( q_h \)) represent the quantity demanded of the low-quality (resp. high-quality) product. Assuming a uniform distribution, the demand functions are denoted as follows:

\[
q_l = \tilde{\theta} = \frac{p_h - p_l}{\hat{s}_l - \hat{s}_h}, \quad q_h = \hat{\theta} - \tilde{\theta} = \frac{\hat{s}_l (v - p_h) - \hat{s}_h (v - p_l)}{\hat{s}_h (\hat{s}_l - \hat{s}_h)}.
\]

Given perceived qualities \( (\hat{s}_k) \), the demand for each type of product decreases (resp. increases) in its own price (resp. in the price of the other type of product). Given the prices of both products, the demand for each type of product increases (resp. decreases) as its perceived quality (resp. the perceived quality of the other type of product) increases, i.e., \( \partial q_k / \partial \hat{s}_k < 0 \) and \( \partial q_k / \partial \hat{s}_i > 0 \), where \( i, k = l, h, i \neq k \).

2.2 Firms

Firm \( k \) \((k = l, h)\) supplies Type \( k \) good. Each firm sends misinformation to consumers, which is costly. The cost function is defined as follows:

\[
f_k(e_k) = \alpha_k e_k^\epsilon, \quad \alpha_k > 0, \quad k = l, h. \tag{2}
\]

We assume that \( f_k' > 0 \) (resp. \( f_k' < 0 \)) when \( e_k > 0 \) (resp. \( e_k < 0 \)) and \( f_k'' > 0 \). Note that \( f_k(0) = 0 \) and \( f_k'(0) = 0 \). For simplicity, it is assumed that costs depend on neither the

\(^{22}\)This externality is important when we consider the case in which products are horizontally differentiated. See Hattori and Higashida (2011).
production amounts nor true qualities. It is also costly to increase the true quality (to decrease $s_k$), and the cost function is defined as $F_k(s_k)$ ($F'_k < 0$, $F''_k > 0$, $k = l, h$). This cost is also assumed to depend on neither the production amount nor misinformation. Thus, profit functions are given as follows:

$$\pi_k = p_k q_k - f_k(e_k) - F_k(s_k), \quad k = l, h.$$  \hspace{1cm} (3)

Each firm’s objective is to maximize its own profit. Note that firms can observe the cost functions and misinformation of their own and rival firms. As mentioned before, we assume from Section 3 through Section 5 that both quality standards for Type $l$ and $h$ are binding, which means that firms do not freely choose the true qualities to maximize their own profits.

Thus, in the first stage, each firm chooses the amount of misinformation on its own product, $e_k$ ($k = l, h$), unless the government regulation on misinformation is binding. In the second stage, given the true quality of and the amount of misinformation (the degree of misperception) on both products, both firms compete in the market in price. The notion of equilibrium is a subgame perfect Nash equilibrium.

### 2.3 Government and Social Welfare

Quality standards exist for both types of products. The standard for a low-quality product is an MQS, and that for a high-quality product is a certification criterion conveyed to its consumers through labeling. In Stage 0, the government sets regulations on misinformation, an MQS, and a certification criterion; the policies are exogenous variables.

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23 Even in reality, it is difficult to verify the relationship between the cost of creating misperception and true quality (or the production amount), because there are no general correlations between them.

24 If both firms are symmetric, and firms can choose true qualities, it is possible that more than one equilibrium exist. However, we do not investigate the problem of multiple equilibria in detail. In the main analysis, those qualities are fixed. Moreover, even in Section 6 in which we consider quality investment, either one firm produces a low-quality product because firms compete in price, and the MQS is binding for the low-quality product. Therefore, the possibility that multiple equilibria exist does not affect our results.

25 The setting of the order of choices, which is “misinformation first, and price/quantity second,” is the same as in Glaeser and Ujhelyi (2010). In reality, although firms may set list prices and send misinformation simultaneously, they often change their prices, i.e., they discount list prices after shipping. Prices in our model are not the list prices but the prices that consumers actually pay when they purchase the products. Therefore, our setting of the stages is consistent with competition among firms in the real world.
in this model. This means that we do not delve into the government’s optimal policies and/or strategic behavior; rather, we conduct comparative statics to examine the effects of regulations and the MQS on firms’ behavior and welfare.

Consumers misperceive product quality because of the misinformation sent by firms, and maximize their utility based on perceived qualities, while the true surplus depends on true qualities. There are three types of consumers: (a) those who purchase the low-quality product, (b) those who purchase the high-quality product, and (c) those who buy nothing. Therefore, the aggregate consumer surplus can be represented as follows:

\[ CS = CS_l + CS_h = \int_{\hat{\theta}}^{\tilde{\theta}} (v - \theta s_l - p_l) d\theta + \int_{\hat{\theta}}^{\tilde{\theta}} (v - \theta s_h - p_h) d\theta, \]

where \( CS_l \) and \( CS_h \) denote the consumer surplus generated by the consumption of the low- and high-quality product, respectively. Note that misinformation \((e_k (k = l, h))\) is not directly included in the true surplus. Misinformation affects utility only through its effects on the consumption amounts \((\hat{\theta}, \tilde{\theta})\) and prices \((p_l, p_h)\). Thus, welfare is given by \( W = CS + \pi_l + \pi_h \).

### 3 Misinformation and Welfare

In this section, focusing on the second stage, we examine the effect of a change in misinformation on prices, outputs, and welfare. Given the degrees of misperception, the equilibrium prices and quantities in the second stage are given as follows:

\[ p^*_l(e_l, e_h; s_l, s_h) = \frac{(\hat{s}_l - \hat{s}_h)v}{4\hat{s}_l - \hat{s}_h}, \quad p^*_h(e_l, e_h; s_l, s_h) = \frac{2(\hat{s}_l - \hat{s}_h)v}{4\hat{s}_l - \hat{s}_h}, \quad (4) \]
\[ q^*_l(e_l, e_h; s_l, s_h) = \frac{v}{4\hat{s}_l - \hat{s}_h}, \quad q^*_h(e_l, e_h; s_l, s_h) = \frac{2\hat{s}_l v}{4\hat{s}_l - \hat{s}_h}. \quad (5) \]

It follows from (4) and (5) that (a) the price of Type \( k \) \((k = h, l)\) good is decreasing (resp. increasing) in misinformation on Type \( l \) (resp. Type \( h \)), (b) the quantity of Type \( k \) \((k = h, l)\) good is increasing (resp. decreasing) in misinformation on Type \( l \) (resp. Type \( h \)). Then, we obtain the effects of changes in the amount of misinformation on the
An increase in misinformation on Type $h$ good always decreases consumer surplus.

This implies that misinformation about Type $h$ good is always excessive for consumers.

Let us now examine the effect of a change in misinformation on welfare. First, assuming that the amount of misinformation on Type $h$ ($e_h$) is fixed, we examine the welfare effect of a change in the amount of misinformation on Type $l$ ($e_l$). From (6), (7), and (9), it is obtained that

\[
\frac{\partial (CS + R_l + R_h)}{\partial e_l} = \frac{48\hat{s}_l\hat{s}_h - 12\hat{s}_h^2 - 8s_l\hat{s}_h - 24\hat{s}_l s_h - 4s_h\hat{s}_h}{2\hat{s}_h (4\hat{s}_l - \hat{s}_h)^3}
\]
holds. If $s_k > 1.5\hat{s}_k$ \((k = l, h)\) and, accordingly, if $e_k > s_k/3$ for both types of products, (12) is negative. Moreover, it is costly for Firm \(l\) to increase the amount of misinformation. Thus, the following lemma holds.

**Lemma 2**

Suppose that the amount of misinformation on Type \(h\) ($e_h$) is fixed. If the misinformation amounts sent by both firms are greater than certain levels, i.e., if $e_k > s_k/3$ \((k = l, h)\), a small decrease in the misinformation on Type \(l\) good improves welfare.

The intuition is as follows. An increase in the misinformation on Type \(l\) good means that the price competition between firms becomes more intense because the two types of goods become more similar in the eyes of consumers. Thus, prices decrease and, accordingly, welfare improves in this respect. On the other hand, some of consumers who would have bought Type \(h\) good without an increase in the misinformation on Type \(l\) choose to buy Type \(l\) good. And, some of consumers who would have bought nothing without an increase in the misinformation on Type \(l\) choose to buy Type \(h\) good. Thus, when there is misinformation, the more intense the competition is, the greater the damage resulting from misperception is. When the amount of misinformation is significant, the increase in the damage resulting from misperception dominates the increase in the benefit derived from lower prices.

The possibility that a decrease in the amount of misinformation on Type \(l\) leads to lower welfare is noteworthy. As discussed in the intuition above, as price competition becomes more intense, a distortion of insufficient supply caused by an imperfectly competitive market is mitigated. Therefore, a small increase in misinformation may contribute to an improvement in welfare when the amount of misinformation is relatively small, and when the difference between the perceived qualities is large (see (12)).

It is also interesting to focus on the difference between the true qualities. Because $s_k > \hat{s}_k$ \((k = l, h)\), if the difference between the true quality of both types of products, $s_l$ and $s_h$, is small, it is likely that (12) is negative. A stricter MQS implies a smaller $s_l$, as long as the MQS is binding and, therefore, a stricter MQS creates a smaller difference
between the true qualities. Thus, it is clear that the strictness of the MQS influences the 
welfare effect of a change in misinformation on Type $l$ good; a stricter MQS implies that 
it is more likely that a small decrease in misinformation on Type $l$ good improves welfare. 

Second, assuming that the amount of misinformation on Type $l$ ($e_l$) is fixed, we examine 
the welfare effect of a change in the amount of misinformation on Type $h$ ($e_h$). From (6), 
(7), (10), and (11), the following is obtained:

$$
\frac{\partial (CS + R_l + R_h)}{\partial e_h} = -\frac{\hat{s}^3_h (s_l - \hat{s}_h) + 8\hat{s}_l \hat{s}_h^2 (\hat{s}_l - \hat{s}_h) + 2\hat{s}_l s_h (8\hat{s}_l^2 + 4\hat{s}_l \hat{s}_h + \hat{s}_h^2)}{\hat{s}_h^3 (4\hat{s}_l - \hat{s}_h)^3} \cdot v^2 < 0.
$$

This inequality means that the effect on consumer surplus, which is negative, dominates 
the effect on firm revenue, which is positive. Moreover, this inequality does not depend 
on the differences between true qualities and the qualities perceived by consumers. Since 
it is costly to increase misinformation, we record the following result.

**Lemma 3**

*Suppose that the amount of misinformation on Type $l$ ($e_l$) is fixed. Then, a small decrease 
in the misinformation on Type $h$ good always improves welfare.*

In this case, the effect on consumer surplus is in sharp contrast with the effect on rev-
enues. A small increase in misinformation on Type $h$ decreases intensity in competition. 
Accordingly, the prices of both products and revenues of both firms increase. Consumers 
then suffer not only from the price increases but also from greater damage from more 
 misinformation. Therefore, a small increase in $e_h$ necessarily decreases consumer surplus. 
In total, however, the effect on the consumer surplus always dominates the effect on firm 
revenues.

It follows from Lemmas 2 and 3 that misinformation on Type $h$ is always socially 
excessive, whereas it is possible that misinformation on Type $l$ is insufficient. Then, we 
obtain important policy implications related to our first purpose, which is to examine the 
welfare effect of regulating misinformation in the presence of an MQS and a certification 
criterion. Consider a situation where the government can directly regulate the amount
of misinformation/misperception. For example, the government may be able to set a maximum amount of misinformation so that it is binding.\textsuperscript{26} In this case, it is clear from Lemma 2 that a more stringent regulation on misinformation about Type \( l \) good improves welfare when the amount of misinformation is large. It can also be said from Lemma 3 that a stricter regulation on misinformation about Type \( h \) good always improves welfare. Moreover, the MQS complements the regulation on misinformation about Type \( l \) good in the sense that the stricter the MQS is, the more likely it is that a stricter regulation on misinformation about Type \( l \) good improves welfare. Consequently, from Lemmas 2 and 3, we obtain the following proposition.

**Proposition 1**

When the government can directly regulate the amount of misinformation/misperception, it can improve welfare by setting regulations on misinformation. The stricter regulation on misinformation about Type \( h \) good always improves welfare. On the other hand, the effect of stricter regulation on misinformation about Type \( l \) good depends on the strictness of the MQS. However, if \( e_k > s_k/3 \) \((k = l, h)\), then this also improves welfare irrespective of the strictness of the MQS.

4 

**4 Competition for “Misleading” Between Firms**

4.1 Misinformation in Equilibrium

Taking into consideration the effect of a change in the amount of misinformation on prices and quantities ((4) and (5)), each firm chooses the amount of misinformation on its own product in the first stage. The profit functions in the first stage \((\Pi_k \ (k=l, h))\) can be rewritten as follows:

\[
\Pi_k = R_k - f_k(e_k) - F_k(s_k), \quad k = l, h.
\]

The first-order condition (FOC) for each firm is denoted as follows:

\[
\frac{\partial \Pi_k}{\partial e_k} = \frac{\partial R_k}{\partial e_k} - f'_k = 0. \tag{13}
\]

\textsuperscript{26}We may recall that when consumers are naive, they know only \( \hat{s}_k \ (k = l, h) \) even under this type of policy.
We assume that the following second-order conditions (SOCs) hold.\(^{27}\)

**Assumption 1**

\[
\frac{\partial^2 R_k}{\partial e_k^2} - f_k'' < 0, \quad k = l, h, \quad \left| \frac{\partial^2 R_l}{\partial e_l \partial e_l} - f_l'' \right| = \Omega > 0.
\]

In particular, when \(\epsilon > 3\) (see Equation (2)), the SOCs always hold. See the Appendix for further details.

Solving the reaction functions for both firms ((13) for each type), we can characterize a subgame perfect Nash equilibrium of this game: \(e^N_k = e^N_k(s_l, s_h)\), and \(\hat{s}^N_k = s_k - e^N_k\). Moreover, we define \(R^N_k = p^N_k q^N_k = p^*_k(e^N_l, e^N_h, s_l, s_h)q^*_k(e^N_l, e^N_h, s_l, s_h)\) and \(CS^N = CS^N_l(e^N_l, e^N_h, s_l, s_h) + CS^N_h(e^N_l, e^N_h, s_l, s_h)\).

We also assume that the following condition holds.

**Assumption 2** \(4\hat{s}^N_l > 7\hat{s}^N_h\).

We recall from (6) that \(\partial R_l / \partial e_l = (4\hat{s}_l - 7\hat{s}_h)\theta^2 / (4\hat{s}_l^2 - \hat{s}_h^3)\). Thus, Assumption 2 implies that \(\partial R^N_l / \partial e_l > 0\) holds. In other words, because \(f'_k(0) = 0\), firms always have incentives to send misinformation so that consumers perceive that goods are healthier/safer than they actually are. Misinformation on Type \(l\) good has precisely two effects. First, by sending misinformation, Firm \(l\) can attract certain consumers who would have bought Type \(h\) goods if there were no such misinformation. Second, the two kinds of products may become more similar because of a small increase in the misinformation on Type \(l\) good, which means that price competition becomes more intense. When the difference in the perceived qualities is relatively large (i.e., \(4\hat{s}_l > 7\hat{s}_h\)), the former effect dominates the latter, and firms always have an incentive to send misinformation.\(^{28}\)

\(^{27}\)In detail, we have

\[
\frac{\partial^2 R_l}{\partial e_l^2} = \frac{16(2\hat{s}_l - 5\hat{s}_h)}{(4\hat{s}_l - \hat{s}_h)^2} \cdot \theta^2,
\]

\[
\frac{\partial^2 R_h}{\partial e_h^2} = \frac{8\hat{s}_l(16\hat{s}_l^2(\hat{s}_l - \hat{s}_h) + \hat{s}_h^2(6\hat{s}_l - 3\hat{s}_h))}{\hat{s}_l^3(4\hat{s}_l - \hat{s}_h)^2} \cdot \theta^2 > 0.
\]

\(^{28}\)When Assumption 2 does not hold, Firm \(l\) has an incentive to send misinformation so that consumers perceive that goods are unhealthier/less safe than they actually are to avoid fiercer price competition.
Moreover, strategic relationships should be noted. From (6), (7), the following is obtained:

$$\frac{\partial^2 \Pi_l}{\partial e_h \partial e_l} = \frac{\partial^2 R_l}{\partial e_h \partial e_l} = \frac{2(8\hat{s}_l + 7\hat{s}_h)}{(4\hat{s}_l - \hat{s}_h)^4} \cdot v^2 > 0, \quad (14)$$

$$\frac{\partial^2 \Pi_h}{\partial e_l \partial e_h} = \frac{\partial^2 R_h}{\partial e_l \partial e_h} = \frac{4(8\hat{s}_l^2 + \hat{s}_h^2 + 12\hat{s}_h\hat{s}_l)}{4(\hat{s}_l - \hat{s}_h)^4} \cdot v^2 > 0. \quad (15)$$

Assuming that the SOCs are assumed to hold, we obtain the following lemma.

**Lemma 4**

*The amount of misinformation sent by Firm l and that sent by Firm h are strategic complements.*

This relationship holds because firms want to avoid intense price competition during the second stage. This implies that the smaller the amount of misinformation on one type of product, the less misinformation is sent by the firm producing the other type of product.

### 4.2 Government Intervention

In the previous section, we considered the effects of changes in the amount of misinformation (the degree of misperception) that can be applied when the government directly controls misinformation. In reality, however, the government often indirectly reduces misinformation/misperceptions by increasing the cost of creating misperceptions by (a) sending counter-information, (b) educating consumers, and/or (c) taxing advertising. For example, the FDA is responsible for advancing public health by helping the public receive accurate, scientifically based information required when consuming medicines and food that maintain and improve their health. The greater the counter-information/education generated by the government, the more difficult it is for firms to persuade consumers to misperceive the quality of their products. In other words, firms have to pay higher costs to create a certain degree of misperception among consumers. In the case of taxation on advertising, it is clear that an increase in the tax rate raises the cost of creating a

---

Although this situation is theoretically possible, it is different from real world issues we focus on. Thus, we exclude the case of this type of “negative” misinformation from our analysis.
certain degree of misinformation. It follows from the analysis in the previous section that misinformation is likely to reduce welfare. Therefore, in such a case, the government has an incentive to increase the firms’ costs for creating misperceptions to improve welfare. In this subsection, we examine this type of government intervention. An increase in the cost of creating a certain degree of misperception is captured by an increase in $\alpha_k$ in (2).

First, suppose that $\alpha_l = \alpha_k = \alpha$. We totally differentiate the FOCs for both firms ((13)) to obtain the following:

$$
\begin{align*}
&\begin{pmatrix}
\frac{\partial^2 R_N^l}{\partial e_l^2} - f_l'' & \frac{\partial^2 R_N^h}{\partial e_l \partial e_h} \\
\frac{\partial^2 R_N^l}{\partial e_l \partial e_h} & \frac{\partial^2 R_N^h}{\partial e_h^2} - f_h''
\end{pmatrix}
\begin{pmatrix}
\frac{de_N^l}{de_l} \\
\frac{de_N^h}{de_h}
\end{pmatrix}
= \begin{pmatrix}
\epsilon \cdot (e_N^l)^{\epsilon-1} \\
\epsilon \cdot (e_N^h)^{\epsilon-1}
\end{pmatrix}.
\end{align*}
$$

We may recall that the SOCs are assumed to hold (see Appendix). In addition, $\epsilon e_k^{\epsilon-1} > 0$ when $e_k$ is positive. Thus, from Lemma 4 and (16), the following result is obtained.

**Lemma 5**

Suppose that $\alpha_l = \alpha_k = \alpha$. Then, counter-information/education provided by the government and/or taxation on advertising reduces the amount of misinformation about both types of products.

Next, consider the situation in which $\alpha_l \neq \alpha_k$ and the government increases either one of two firms’ costs for creating misinformation. For example, the government may warn consumers about the unhealthy attributes of “lower quality” (Type $l$), or may need to undeceive consumers when they put too much confidence in the quality of Type $h$ good.

Similar to (16), the former case can be analyzed by the total differentiation of the FOCs:

$$
\begin{align*}
&\begin{pmatrix}
\frac{\partial^2 R_N^l}{\partial e_l^2} - f_l'' & \frac{\partial^2 R_N^h}{\partial e_l \partial e_k} \\
\frac{\partial^2 R_N^l}{\partial e_l \partial e_k} & \frac{\partial^2 R_N^h}{\partial e_k^2} - f_h''
\end{pmatrix}
\begin{pmatrix}
\frac{de_N^l}{de_l} \\
\frac{de_N^h}{de_k}
\end{pmatrix}
= \begin{pmatrix}
\epsilon \cdot (e_N^l)^{\epsilon-1} \\
0
\end{pmatrix}.
\end{align*}
$$

The latter case can be analyzed in a similar way. Consequently, the following result is established.

**Lemma 6**

Suppose that $\alpha_l \neq \alpha_k$. When the government sends counter-information and/or educates consumers on either type of product, misinformation amounts on both products decrease.
Counter-information/education on Type $k$ ($k = l, h$) directly reduces misinformation on Type $k$. Moreover, since misinformation amounts are strategic complements (Lemma 4), the firm that produces the other type of good responds to the decrease in misinformation on Type $k$ by decreasing the misinformation on its own product.

Thus, from the analysis in Section 3, this type of effort by the government is likely to lead to a higher sum of revenues and consumer surplus. The increase can be observed particularly with a large amount of misinformation. Consequently, from Lemmas 2, 3, 5, and 6, we obtain the following proposition.

**Proposition 2**

The government can improve welfare by mitigating the competition of misleading between firms, unless policy implementation costs are not very high. In particular, when the amount of misinformation is large ($e_k^N > s_k/3$ ($k = l, h$)), government intervention works in favor of welfare.

5 **Minimum Quality Standards**

Our second purpose is to investigate the effect of a stricter MQS on welfare in the presence of misinformation. The MQS directly affects the true quality of Type $l$ good: a stricter MQS implies a smaller $s_l$. Since a change in $s_l$ affects the amounts of misinformation, the total effect of a change in MQS on welfare is denoted as follows:

$$
\frac{dW^N}{ds_l} = \frac{\partial W^N}{\partial s_l} + \frac{\partial W^N}{\partial e_l} \frac{de_l^N}{ds_l} + \frac{\partial W^N}{\partial e_h} \frac{de_h^N}{ds_l}.
$$

The first term represents the direct effect, while the second and third terms represent the indirect effects. Note that $W^N = CS^N + \pi_l^N(e_l^N, e_h^N, s_l, s_h) + \pi_h^N(e_l^N, e_h^N, s_l, s_h)$, where

$$
\pi_k^N = R_k^N - f(e_k^N) - F(s_k) \ (k = l, h).
$$

---

29 We do not consider explicitly the cost of implementing these policies. The result, however, essentially does not change even if we take into consideration the implementation cost.

30 We may recall that our purpose is to conduct comparative statics excluding the repeatedness of consumption. Although we use the word “change,” it does not mean that consumers purchase goods several times or change their behavior after the MQS becomes stricter.
First, we examine the direct effect. Since \( \hat{s}_l = s_l - e_l \) holds, from Assumption 2 and (6) and (7), we obtain the following:

\[
\frac{\partial R_l^N}{\partial s_l} = - \frac{\partial R_l^N}{\partial e_l} = \frac{(-4\hat{s}_l^N + 7\hat{s}_h^N)v^2}{(4\hat{s}_l^N - \hat{s}_h^N)^3} < 0, \tag{17}
\]

\[
\frac{\partial R_h^N}{\partial s_l} = - \frac{\partial R_h^N}{\partial e_l} = \frac{4(2\hat{s}_l^N + \hat{s}_h^N)v^2}{(4\hat{s}_l^N - \hat{s}_h^N)^3} > 0. \tag{18}
\]

Although the effects of a change in the MQS on both firms’ revenues conflict with each other, the effect on total revenue is as follows:

\[
\frac{\partial (R_l^N + R_h^N)}{\partial s_l} = (4\hat{s}_l^N + 11\hat{s}_h^N)v^2 \frac{(4\hat{s}_l^N - \hat{s}_h^N)^3}{(4\hat{s}_l^N - \hat{s}_h^N)^3} > 0. \tag{19}
\]

From (8), we derive the following:

\[
\frac{\partial CS_l^N}{\partial s_l} = \frac{v^2}{2} \cdot \frac{5(4\hat{s}_l^N - \hat{s}_h^N) - 8(6\hat{s}_l^N - s_l)}{(4\hat{s}_l^N - \hat{s}_h^N)^3}, \tag{20}
\]

\[
\frac{\partial CS_h^N}{\partial s_l} = \frac{v^2}{2\hat{s}_h^N} \cdot \frac{4s_h(6\hat{s}_l^N + \hat{s}_h^N) - 4\hat{s}_h^N(8\hat{s}_l^N + \hat{s}_h^N)}{(4\hat{s}_l^N - \hat{s}_h^N)^3}. \tag{21}
\]

From (19) through (21), the following is obtained:

\[
\frac{\partial CS^N + R_l^N + R_h^N}{\partial s_l} = \frac{v^2}{2\hat{s}_h^N} \cdot \frac{24s_h\hat{s}_l^N + 8s_h^N s_l - 52\hat{s}_h^N \hat{s}_l^N + 13\hat{s}_h^N^2 + 4s_h\hat{s}_l^N}{(4\hat{s}_l^N - \hat{s}_h^N)^3}.
\]

Thus, if \( e_k^N > 5/13 \cdot s_k \) (\( k = l, h \)), then \( \partial (CS^N + R_l^N + R_h^N)/\partial s_l > 0 \) always holds. Consequently, because it is costly to decrease \( s_l \), we obtain the following lemma.\(^{31}\)

**Lemma 7**

The greater the amount of misinformation, the more likely it is that the direct effect of a stricter MQS (a small decrease in \( s_l \)) works to reduce welfare, i.e., \( \partial W/\partial s_l > 0 \). In particular, if \( e_k^N > 5s_k/13 \) for \( k = l, h \), then \( \partial W/\partial s_l > 0 \) always holds.

We can apply the intuition to Lemma 2 to understand Lemma 7. A decrease in \( s_l \) means that price competition between firms becomes more intense because the two types of goods increase in similarity. Thus, the price decreases and, accordingly, welfare improves in this respect. However, some of consumers who would have bought Type \( h \) good if there

\(^{31}\)Note that this lemma also applies to the case in which the regulations on misinformation are binding.
were no change in the MQS now choose to buy Type $l$ good. The larger the amount of misinformation about Type $l$ good, the greater will be the damage resulting from misperception when goods are purchased. Moreover, some of consumers who would have bought nothing if there were no change in the MQS now choose to buy Type $h$ product. The larger the amount of misinformation on Type $h$ good, the greater will be the damage resulting from misperception by “new” consumers. When misinformation amounts are large, the increase in the damage dominates the increase in the benefit derived from lower prices.

Second, we examine the indirect effects from changes in the amount of misinformation on both types of goods. From Lemmas 2 and 3, we have already obtained the results on $\partial W^N/\partial e_k$ ($k = l, h$). Therefore, we focus on the effect of a change in the MQS ($s_l$) on the amount of misinformation sent by both firms.

As noted in Section 3, each firm chooses the amount of misinformation in the first stage to maximize its own profit given the quality standards. We totally differentiate the FOCs ((13)) for both firms to obtain the following:

$$
\begin{bmatrix}
\frac{\partial^2 R^N_{l}}{\partial e^2_{l}} - f''_l \\
\frac{\partial^2 R^N_{h}}{\partial e^2_{h}} - f''_h
\end{bmatrix}
\begin{bmatrix}
\frac{\partial e^N_{l}}{\partial s_l} \\
\frac{\partial e^N_{h}}{\partial s_l}
\end{bmatrix}
= -
\begin{bmatrix}
\frac{\partial^2 R^N_{l}}{\partial s_l \partial e_l} \\
\frac{\partial^2 R^N_{h}}{\partial s_l \partial e_h}
\end{bmatrix}
.$$

From (14) and (15), and the facts that $\frac{\partial^2 R^N_{l}}{\partial s_l \partial e_l} = -\frac{\partial^2 R^N_{l}}{\partial e^2_{l}}$ and $\frac{\partial^2 R^N_{h}}{\partial s_l \partial e_h} = -\frac{\partial^2 R^N_{h}}{\partial e^2_{h}}$, the following lemma holds.

**Lemma 8**

A stricter MQS (a smaller $s_l$) always leads to a larger amount of misinformation on Type $h$ good. Moreover, if $\hat{s}^N_{l} > 5\hat{s}^N_{h} / 2$, a stricter MQS leads to a larger amount of misinformation on Type $l$ good.

A stricter MQS, given misinformation, makes the two types of products more similar in terms of consumers’ perceptions. Thus, Firm $h$ has an incentive to send a larger amount of misinformation to differentiate more clearly its own product from Type $l$ good, and avoid intense price competition. On the other hand, two effects work in terms of Firm $l$’s incentive, which conflict with each other. First, a small increase in $e_l$ and a small decrease
in $s_l$ have the same meaning in terms of consumer perception: the smaller is $s_l$, the less misinformation Firm $l$ needs to send to create a certain level of perceived quality ($\hat{s}_l$). Second, misinformation amounts sent by both firms are strategic complements. In this respect, Firm $l$ responds to an increase in $e_h$ by increasing its own misinformation. When the difference between the perceived qualities of the two types of goods is large, the latter effect dominates the former effect, i.e., facing a greater $e_h$ as a result of a stricter MQS, Firm $l$ can increase its profit by increasing its own misinformation.

Consequently, from Lemmas 2, 3, 7, and 8, we obtain the following proposition.

**Proposition 3**

A stricter MQS can harm welfare because of the existence of misinformation. In particular, (a) the larger misinformation amounts sent by firms and (b) the greater the difference between the perceived qualities of the two types of goods, the more likely it is that a stricter MQS reduces welfare.

From the definition of profits of firms ((3)), it is costly for Firm $l$ to increase the quality of its own product. A stricter MQS implies higher fixed costs, and welfare is reduced in this respect, even in the absence of misinformation. However, in the present case, misinformation may itself hinder the government from improving welfare by setting a stricter MQS: this means that $\partial(C^N + R^N_l + R^N_h)/\partial s_l > 0$ may hold in the presence of misinformation.

To clarify the role of misinformation, we consider the case of no misinformation where $\hat{s}_k = s_k$ ($k = l, h$) holds.\textsuperscript{32} From the definition ((8)), we obtain the following consumer surplus:

$$ CS^{NM} = CS^{NM}_{l} + CS^{NM}_{h} = \frac{5s_l v^2}{2(4s_l - s_h)^2} + \frac{4s_l^2 v^2}{2s_h (4s_l - s_h)^2}, $$

where $NM$ denotes the equilibrium in the case of no misinformation. The effect of a change in the MQS on consumer surplus is as follows:

$$ \frac{\partial CS^{NM}}{\partial s_l} = -\frac{(28s_l + 5s_h)v^2}{2(4s_l - s_h)^3} < 0 \quad (22) $$

\textsuperscript{32}Complete prohibition of misinformation fits this situation.
When the MQS becomes stricter, the qualities of the two types of products increase in similarity, which leads to lower prices. In contrast with the case in the presence of misinformation, there is no damage resulting from misperception. Thus, a stricter MQS results in greater consumer surplus. In other words, a stricter MQS removes excess differentiation.

Moreover, similar to (17) and (18), the effects of a change in the MQS on firm revenues are given as follows:

\[
\frac{\partial R_{NM}^l}{\partial s_l} = \frac{(-4s_l + 7s_h)v^2}{(4s_l - s_h)^3} < 0, \quad \frac{\partial R_{NM}^h}{\partial s_l} = \frac{4(2s_l + s_h)v^2}{(4s_l - s_h)^3} > 0.
\]

Thus, the effect on total revenue is as follows:

\[
\frac{\partial (R_{NM}^l + R_{NM}^h)}{\partial s_l} = \frac{(4s_l + 11s_h)v^2}{(4s_l - s_h)^3} > 0.
\] (23)

A small increase in \(s_l\) makes Type \(l\) good less healthy. Thus, certain consumers buy Type \(h\) instead of Type \(l\). Moreover, because the difference between the two types of products increases, price competition becomes less intense. Therefore, the revenue of Firm \(h\) increases. On the other hand, the change in consumers’ behavior decreases Firm \(l\)’s revenue, whereas the price change increases it. In total, when Assumption 2 holds, the former effect dominates the latter effect in terms of the revenue of Firm \(l\). Thus, the revenue of Firm \(l\) decreases.

In total, from (22) and (23), the effect on consumer surplus dominates the effect on revenues: \(\partial (C^N + R_{NM}^l + R_{NM}^h)/\partial s_l\) is always negative.

**Proposition 4**

*Suppose that there is no misinformation. Then, unless there is a large increase in fixed costs to make Type \(l\) product healthier/safer, the stricter the MQS, the greater will be the welfare.*

Since it is costly to improve the quality of Type \(l\) good \((F_l' < 0)\), Firm \(l\) may not have an incentive to improve the quality of its own product. However, a quality improvement would increase consumer surplus. Thus, a stricter MQS may improve welfare. It can be
emphasized that, in contrast with the case in the presence of misinformation, a change in consumers’ purchasing behavior in response to a change in the MQS does not hinder government intervention of raising welfare by making the MQS stricter.

6 Extension: Investment in Quality by the Producer of the High-quality Product

To this point, we have assumed that both the MQS and the certification criterion are binding, by which we extracted clearly the relationship between quality standards and misinformation. However, it may be possible, particularly in the long run, that firms producing high-quality products choose not only a certain amount of misleading advertising but also the quality of their own products. In this section, we extend our analysis to the case where the certification criterion is not binding. As in the previous sections, the MQS is assumed to be binding.

First, we examine the partial effect of a change in $s_h$ given the MQS and the amount of misinformation. Because $\hat{s}_h = s_h - e_h$ holds, we obtain the following:

$$\frac{\partial R_l}{\partial s_h} = - \frac{\partial R_l}{\partial e_h} = - \frac{2\hat{s}_l + \hat{s}_h}{(4\hat{s}_l - \hat{s}_h)^3} \cdot v^2 < 0,$$

$$\frac{\partial R_h}{\partial s_h} = \frac{\partial R_h}{\partial e_h} = - \frac{4\hat{s}_l(4\hat{s}_l^2 - 3\hat{s}_h\hat{s}_l + 2\hat{s}_h^2)}{s_h^3(4\hat{s}_l - \hat{s}_h)^3} \cdot v^2 < 0.$$

On the other hand, we obtain the following from (8):

$$\frac{\partial CS_l}{\partial s_h} = \frac{6\hat{s}_l - s_l}{(4\hat{s}_l - \hat{s}_h)^3} \cdot v^2 > 0,$$

$$\frac{\partial CS_h}{\partial s_h} = \frac{2(-12\hat{s}_l^3\hat{s}_h + 3\hat{s}_l^2\hat{s}_h^2 + 3\hat{s}_l\hat{s}_h^3 + 8\hat{s}_l^3s_h - 3\hat{s}_l\hat{s}_h^2s_h)}{s_h^3(4\hat{s}_l - \hat{s}_h)^3} \cdot v^2 < 0.$$

From (24) through (27), $\partial(CS + R_l + R_h)/\partial s_h < 0$ holds. Thus, we have the following result.

**Lemma 9**

Given the MQS and the misinformation amounts on both types of products, a small improvement in the quality of the high-quality product increases the sum of consumer surplus and firms’ revenues.
Next, we consider the choice of true quality by Firm $h$ in response to a change in the amount of misinformation. Different from the structure of the game in the previous sections, we assume that the amount of misinformation and MQS are determined in Stage 0, and accordingly, they are binding for firms. Then, Firm $h$ chooses quality in the first stage. The FOC for Firm $h$ is given by $\partial R_h / \partial s_h - F'_h = 0$. It is also assumed that the SOC holds: $\partial^2 R_h / \partial s_h^2 - F''_h < 0$.

First, consider a small decrease in $e_h$, which can be applied to the case where the government sets a stricter regulation on misinformation of Type $h$ good. As noted above, $\hat{s}_h = s_h - e_h$ holds. Thus, on the analogy of $\partial^2 R_h / \partial e_h^2 > 0$ (See footnote 28), $\partial^2 R_h / \partial e_h \partial s_h < 0$ holds. This implies that the marginal benefit of improving quality for Firm $h$ becomes greater as the amount of misinformation on its own product becomes larger. Thus, $ds_h^{NT} / de_h < 0$ holds, where $NT$ denotes the equilibrium in the case where Firm $h$ chooses true quality. Second, consider a small decrease in $e_l$. It follows from (15) that $\partial^2 R_h / \partial e_l \partial s_h = -\partial^2 R_h / \partial e_l \partial e_h < 0$. This implies that the marginal benefit of improving quality for Firm $h$ becomes greater as the amount of misinformation on the rival product increases. Thus, $ds_h^{NT} / de_l < 0$ holds. Consequently, the following result is obtained.

**Lemma 10**

Suppose that both the regulations on misinformation on both types of products and MQS are binding. Then, a small decrease in $e_k$ ($k = l, h$), which means stricter regulation on misinformation of Type $k$ goods, deteriorates the true quality of the high-quality product.

When the MQS and regulations on misinformation for both types of products are binding, a small decrease in $e_l$ may improve welfare (Lemma 2), while a small decrease in $e_h$ always improves welfare (Lemma 3). However, these changes indirectly reduce welfare through a change in the true quality of the high-quality product (Lemmas 9 and 10). Thus, we obtain the following proposition.

**Proposition 5**

Suppose that both the regulations on misinformation on both types of products and the
MQS are binding. Then, the welfare-improving effect of a small decrease in $e_k$ ($k = l, h$) is weakened by deterioration of the true quality of the high-quality product.

In particular, the result on the effect of a change in $e_h$ is in sharp contrast to Lemma 3. Whether a decrease in $e_h$ harms welfare depends on the degree of the change in $s_h$ in response to the decrease in $e_h$, which depends on the shapes of the cost function, $F(\cdot)$.

Now, let us turn to a situation in which Firm $h$ chooses the amount of misinformation and true quality simultaneously in the first stage, given $e_l$ and $s_l$. We set the following assumption for simplicity.

Assumption 3 Firm $h$ chooses both the true quality ($s_h$) and amount of misinformation ($e_h$) in the first stage, when neither is binding.

Then, the FOCs are $\partial R_h/\partial e_h - f'_h = 0$ and $\partial R_h/\partial s_h - F'_h = 0$. From the definition of perceived utility ((1)), it is clear that the true quality and amount of misinformation are symmetric for Firm $h$. In other words, perceived quality matters for a firm’s profit. Thus, Firm $h$ chooses both the true quality and the misinformation so that

$$\frac{\partial R_h}{\partial e_h} = -\frac{\partial R_h}{\partial s_h} = f'_h = -F'_h$$

holds, when neither of them is binding. We assume that the following SOCs are satisfied.

Assumption 4

$$\frac{\partial^2 \pi_h}{\partial s_h^2} < 0, \quad \frac{\partial^2 \pi_h}{\partial e_h^2} < 0, \quad \text{and} \quad \left| \frac{\partial^2 R_h}{\partial e_h \partial s_h} = f''_h \right| \frac{\partial^2 R_h}{\partial s_h^2} - F''_h > 0$$

Assume that the regulation on the misinformation of the low-quality product is binding and consider a small decrease in $e_l$. Note that since $s_l$ matters for Firm $h$ in choosing $e_h$ and $s_h$, a small increase in $s_l$, which means a laxer MQS, also gives rise to the same result.

\footnote{Strictly speaking, a situation in which $e_l$, $e_h$, and $s_h$ are determined simultaneously should be described. However, we describe the situation in which $e_l$ and $s_l$ are exogenous to make the presentation as simple as possible. We will briefly discuss the case in which three variables are determined simultaneously.}
We totally differentiate the FOCs for Firm $h$ to obtain:

$$\left(\frac{\partial^2 R_{NT}^h}{\partial e_h^2} - f''_h \right) \left( \frac{\partial^2 R_{NT}^h}{\partial s_h^2} - \frac{\partial^2 R_{NT}^h}{\partial e_h \partial s_h} \right) - F''_h = \left( \begin{array}{c} \frac{d e_{NT}^h}{d e_l} \\ \frac{d s_{NT}^h}{d e_l} \end{array} \right).$$

(29)

Recalling that $\frac{\partial^2 R_h^h}{\partial e_h \partial s_h} < 0$ (footnote 28), and that $\frac{\partial^2 R_h^h}{\partial e_h \partial e_l} = -\frac{\partial^2 R_h^h}{\partial s_h \partial e_l} > 0$ ((15)), we obtain that $d e_{NT}^h / d e_l > 0$ and $d s_{NT}^h / d e_l < 0$.

**Lemma 11**

Suppose that both the regulation on misinformation on the low-quality product and the MQS are binding. Then, a small decrease in $e_l$ (resp. a small increase in $s_l$), which means a stricter regulation on misinformation of the low-quality product (resp. a laxer MQS), deteriorates the true quality of the high-quality product.

Consequently, from Lemmas 2, 9 and 11, we obtain the following proposition.

**Proposition 6**

Suppose that both the regulation on misinformation on the low-quality product and the MQS are binding. Then, the improvement in welfare via a stricter regulation on misinformation on the low-quality product may be weakened because of deterioration of the true quality of the high-quality product.

We recall that misinformation amounts are strategic complements in the first stage (Lemma 4). Since a decrease in $e_l$ or an increase in $s_l$ implies that the perceived quality of Type $l$ good decreases (an increase in $s_l$), Firm $h$ responds to this change in the regulation or standard by lowering the perceived quality of its own product. Thus, Firm $h$ not only decreases the misinformation but also deteriorates the true quality.

When focusing on the effect of a stricter MQS, it follows from Lemma 11 that misinformation on Type $h$ good increases and the true quality of the good improves. In terms of the effect on welfare, two effects can be considered to occur. First, compared with the case in the absence of the choice over true quality, it is likely that the increase in misinformation on Type $h$ product in response to a change in the MQS is smaller. This is because some of the outlay on advertising is substituted by a quality investment. Then,
the improvement in the true quality of Type \( h \) product mitigates the undesirable effect of an increase in misinformation.

Second, it is easier for Firm \( h \) to differentiate its own product from the rival’s low-quality product when it can choose not only the amount of misinformation but also the level of the true quality than when it can choose only the amount of misinformation. Therefore, the decreases in prices because of intense competition also become smaller. In this respect, an improvement in the true quality of Type \( h \) may damage the desirable effect that mitigates insufficient supply.

Even when Firm \( l \) can choose the amount of misinformation, which means that the regulation on misinformation of Type \( l \) is not binding, similar results are obtained on the effect of a stricter MQS, as far as a stricter MQS leads to a smaller \( \hat{s}_l \). In particular, from Lemma 8, they hold when the difference between the perceived quality of the two types of products is large.

In total, the effect of an improvement in true quality and that of an increase in misinformation conflict with each other in terms of welfare, while they have the same effect on firm profits. Therefore, the additional means for Firm \( h \), which is the choice of true quality, can work against welfare when considering the effect of regulations on misinformation and quality standards. It should be emphasized that this undesirable effect in relation to the MQS is specific to the situations in which misinformation exists.

7 Conclusion

This paper focuses on the relationship between misinformation/misperceptions and MQSs when products are vertically differentiated in their health/safety aspects. In particular, we investigated the welfare effect of regulating misinformation in the presence of an MQS and a certification criterion. We also examined the welfare effect of a change in an MQS (a stricter MQS) in the presence of misinformation.

The important results are as follows. First, when both the MQS and the certification criterion are binding, a small decrease in misinformation on the low-quality product leads
to greater welfare if the amount of misinformation on both low- and high-quality products is large, while a small decrease in misinformation on the high-quality product always leads to greater welfare. In determining whether or not regulation improves welfare, two conflicting factors play key roles: the distortion of insufficient supply because of imperfect competition, and the difference between true and perceived qualities. We also demonstrated that the government’s intervention to increase the cost of creating misperception, such as sending counter-information, educating consumers, and/or taxing advertising, can improve welfare.

Second, when the certification criterion is binding, a stricter MQS can harm welfare because of the presence of misinformation. In particular, when the amount of misinformation is large, the direct effect of a stricter MQS (a small decrease in $s_l$) works against welfare. This is because a stricter MQS intensifies price competition and, accordingly, increases total consumption. Then, the disutility from purchasing a product according to misperception of the product’s quality is large. Moreover, when the difference between the perceived qualities of the two types of products is large, a stricter MQS is likely to increase the amount of misinformation on both products. Thus, when these two preconditions are satisfied, a stricter MQS reduces welfare. This relationship between a stricter MQS and welfare cannot be observed in the absence of misinformation.

Third, we extended the analysis to the case in which Firm $h$ can choose both the true quality and the amount of misinformation in the first stage. We demonstrate that the possibility of changes in the true quality of the high-quality product may weaken the positive effects of regulations and standards on welfare.

The results obtained in this paper clarify the relationship between misleading advertising and quality standards which is very important when consumers misperceive product qualities. In the presence of misinformation, stricter quality standards can work against welfare even if we exclude the increase in fixed costs of quality investment.

Although our results are clear and suggestive in terms of policies on quality standards and advertising regulations, we did not deal with the following interesting points. First,
the effect of a change in certification criteria for the high-quality product was not exam-
ined, although the effect and credibility of certification criteria occasionally draw attention
in reality, particularly when many types of criteria coexist. Second, we did not consider
the optimal policies, and it is also important to investigate whether equilibrium amounts
of misinformation are excessive or insufficient. The investigation of these points is for
future research.

Appendix: The Second-order Conditions for Assump-
tion 1

First, we prove that the second-order conditions are satisfied. In such a case, it must be
the case that
\[
\frac{\partial^2 \pi_l}{\partial e_l^2} = \frac{16v^2(2\hat{s}_l - 5\hat{s}_h)}{(4\hat{s}_l - \hat{s}_h)^4} - f_l''' < 0, \text{ and}
\]
\[
\frac{\partial^2 \pi_h}{\partial e_h^2} = \frac{8v^2\hat{s}_l(16\hat{s}_l^2\hat{s}_l - \hat{s}_h) + \hat{s}_h^2(6\hat{s}_l - 3\hat{s}_h))}{\hat{s}_h^4(4\hat{s}_l - \hat{s}_h)^4} - f_h''' < 0. \tag{31}
\]
Using Euler’s theorem \((e_k f_k'' = (\epsilon - 1)f_k')\), (30) can be rewritten as:
\[
\frac{\partial^2 \pi_l}{\partial e_l^2} = \frac{16v^2(2\hat{s}_l - 5\hat{s}_h)}{(4\hat{s}_l - \hat{s}_h)^4} - \frac{\epsilon - 1 (4\hat{s}_l - 7\hat{s}_h)v^2}{e_l (\hat{s}_l - \hat{s}_h)^3} \\
= \frac{v^2(16e_l(2\hat{s}_l - 5\hat{s}_h) - (\epsilon - 1)(4\hat{s}_l - 7\hat{s}_h)(6\hat{s}_l - 3\hat{s}_h))}{e_l(4\hat{s}_l - \hat{s}_h)^4}
\]
Because \(\hat{s}_l > e_l\), it is verified that if \(\epsilon > 3\), \(\partial^2 \pi_l/\partial e_l^2 < 0\). In a similar way, (31) can be
rewritten as:
\[
\frac{\partial^2 \pi_h}{\partial e_h^2} = \frac{8v^2\hat{s}_l(16\hat{s}_l^2\hat{s}_l - \hat{s}_h) + \hat{s}_h^2(6\hat{s}_l - 3\hat{s}_h))}{\hat{s}_h^4(4\hat{s}_l - \hat{s}_h)^4} - \frac{\epsilon - 1 4\hat{s}_l(4\hat{s}_l^2 - 3\hat{s}_h\hat{s}_l + 2\hat{s}_h^2)v^2}{e_h \hat{s}_h^4(4\hat{s}_l^2 - \hat{s}_h)^3} \\
< 0
\]
Because \(\hat{s}_h > e_h\), the inequality always holds.

Next, we check if \(\Omega > 0\). Using Euler’s theorem, we obtain:
\[
\frac{\partial^2 R_i}{\partial e_i \partial \hat{s}_i} \cdot \hat{s}_i + \frac{\partial^2 R_i}{\partial e_i \partial \hat{s}_j} \cdot \hat{s}_j = -2f_i', \text{ } i, j = h, l, \text{ } i \neq j.
\]
Then, $\Omega$ can be rewritten as:

$$
\Omega = \frac{\partial^2 R_l}{\partial e_l \partial \hat{s}_l} \cdot \hat{s}_h - \frac{2 f'_l}{\hat{s}_l} - f''_l \left\{ \frac{\partial^2 R_h}{\partial e_h \partial \hat{s}_h} \cdot \hat{s}_l + \frac{2 f'_h}{\hat{s}_h} - f''_h \right\} - \frac{\partial^2 R_l}{\partial e_l \partial e_l} \frac{\partial^2 R_h}{\partial e_h \partial e_l}
$$

Recall that

$$
\frac{\partial^2 R_i}{\partial e_i \partial \hat{s}_j} = -\frac{\partial^2 R_i}{\partial e_i \partial e_j} < 0, \quad i, j = h, f, \quad i \neq j.
$$

Thus, if $\Gamma = 2 f'_i / \hat{s}_i - f''_i < 0$, $\Omega > 0$ holds. It holds that

$$
\Gamma = f'_i \cdot \left\{ \frac{2}{\hat{s}_i} - \frac{\epsilon - 1}{e_l} \right\}.
$$

Because $\hat{s}_i > e_i$, the sufficient condition for $\Gamma$ to be negative is $\epsilon > 3$. Note that even if $\epsilon \leq 3$, $\Gamma < 0$ may hold.

Finally, we check that both firms obtain positive profits excluding the investment cost in true qualities in equilibrium. Using Euler’s theorem, we obtain that:

$$
\pi^N_l + F_l = -\frac{\partial R^N_l}{\partial \hat{s}^N_l} \cdot \hat{s}_l - \frac{\partial R^N_h}{\partial \hat{s}^N_h} \cdot \hat{s}_l - \frac{e^N_l}{\epsilon} \cdot \frac{\partial R^N_l}{\partial e_l}
$$

Under Assumption 1, $\partial R^N_l / \partial e_l = -\partial R^N_h / \partial \hat{s}_l > 0$. Moreover, $\partial R^N_h / \partial e_h = -\partial R^N_h / \partial \hat{s}_h > 0$ holds. Therefore, if $\epsilon \hat{s}^N_l > e^N_l$, the profit of firm $l$ is positive in equilibrium. Similarly,

$$
\pi^N_h + F_h = -\frac{\partial R^N_l}{\partial \hat{s}^N_h} \cdot \hat{s}_l - \frac{\partial R^N_h}{\partial \hat{s}^N_h} \cdot \hat{s}_h - \frac{e^N_h}{\epsilon} \cdot \frac{\partial R^N_h}{\partial e_h}.
$$

Therefore, if $\epsilon \hat{s}^N_h > e^N_h$, the profit of firm $h$ is positive in equilibrium. And, if $\epsilon > 1$, $\epsilon \hat{s}^N_k > e^N_k, k = l, h$ always holds.

References


