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Losses from Competition in a Dynamic Game Model of a Renewable Resource Oligopoly

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Abstract

This paper develops a dynamic game model of an asymmetric oligopoly with a renewable resource to reconsider welfare effects of increases in the number of firms. We show that increasing not only the number of *inefficient* firms but also that of *efficient* firms reduces welfare, which sharply contrasts to a static outcome. It is discussed that the closed-loop property of feedback strategies plays a decisive role in this finding.

JEL classification: C73; L13; Q20.

Keywords: Differential game, Asymmetric oligopoly, Feedback strategy.

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1 Introduction

Is increasing competition, i.e., an exogenous increase in the number of firms, beneficial to social welfare in an oligopoly? This is one of the primary interests in economics and there is a considerable literature based on static Cournot-Nash models. When all the oligopolistic firms have an identically constant marginal cost and no fixed cost, increasing the number of firms benefits welfare. However, it is stringent whether welfare improves as a result of increasing competition under asymmetric costs among firms. In a seminal work, Lahiri and Ono (1988, Proposition 2, p. 1201) find that 'national welfare increases if a firm with a sufficiently low share is removed from the market.' This result has long had a great influence on the policymaking of competition.

Are these results still valid even in a resource oligopoly as well? To give an answer, this paper constructs a differential game model of a renewable and open access resource oligopoly. A typical example is a transboundary fishery. Suppose that efficient Northern firms and inefficient Southern firms compete in not only the world output market but also global fishery. In such a world, there is no world government and thus extraction is completely decentralized by private firms, the number of which is fixed even though the resource has open access. Within this framework, we prove that an increase in the number of *efficient* firms harms welfare as is opposed to the static result. Therefore, it straightforwardly from this result that 'helping *any* firms reduces welfare.' What is worth noting is that our results need no assumption on the initial market share of efficient firms.[1] It is the closed-loop property of feedback strategies that plays a central role in our arguments. While closed-loop effects are a priori absent in any static analysis, they are quite relevant in dynamic environments, particularly in dynamic games. Our result is an example where the closed-loop effects can dominate the static effects, which yields a sharp contrast between the static and dynamic outcomes.

We are not the first to identify the role of closed-loop properties of feedback strategies in dynamic games. Constructing a differential game model of a renewable resource duopoly, Benchekroun (2003) demonstrates that a unilateral production restriction on

[1] The Lahiri-Ono (1988) proposition requires that the initial market share of the efficient firms be large enough.

a firm can decrease the resource stock. Allowing for an arbitrary number of (symmetric) firms in the same model, Benchekroun (2008) proves that increasing the number of firms reduces the resource stock and the industry output in the long-run. Lohoues (2006) introduces heterogeneity in marginal cost and the number of firms in the Benchekroun (2008) model and characterizes the feedback Nash equilibrium. One of Lohoues' (2006) notable results is that the low-cost firm's feedback strategy exhibits jumps in the presence of asymmetric costs. However, he is mainly interested in characterizing the equilibrium, leaving comparative statics/dynamics out.

The paper is organized as follows. Section 2 presents the model and Section 3 characterizes the feedback Nash equilibria. Section 4 states and discusses the main results. Section 5 concludes the paper. The appendices prove the results in the main text.

2 A model

Consider an oligopoly consisting of $m \geq 1$ efficient firms with zero marginal cost and $n \geq 1$ inefficient firms with a positive marginal cost c. Fixed costs are assumed away. During production, firms extract a renewable resource with the following dynamics:

$$\dot{S} = kS - \sum_{i=1}^{m} x_i - \sum_{j=1}^{n} x_j, \quad S(0) : \text{given}, \quad k > 0, \tag{1}$$

where S is a stock of the resource, x_i represents an efficient firm's output and x_j represents an inefficient firm's output.[2] The parameter k denotes a natural growth rate of the resource. Assuming linear inverse demand $p = a - \sum_{i=1}^{m} x_i - \sum_{j=1}^{n} x_j, a > c$, where p is the price, each firm's profit maximization problem is formulated as

$$\begin{aligned}
&\max_{x_i} \quad \int_0^\infty e^{-rt} \left(a - \sum_{i=1}^{m} x_i - \sum_{j=1}^{n} x_j \right) x_i dt \\
&\max_{x_j} \quad \int_0^\infty e^{-rt} \left(a - c - \sum_{i=1}^{m} x_i - \sum_{j=1}^{n} x_j \right) x_j dt \\
&\text{subject to} \quad (1),
\end{aligned}$$

[2]Benchekroun (2008, 2003) and Lohoues (2006) assume an inverted-V shaped dynamics of resource accumulation, namely, the resource decreases if its stock is sufficiently large. On the other hand, some previous studies, e.g., Tornell and Velasco (1992), Benchekroun and Long (2002) and Long and Wang (2009), use linear resource dynamics.

where $r > 0$ denotes a constant rate of discount.

3 Feedback Nash equilibria

We seek stationary feedback strategies of this dynamic game. Stationary feedback strategies are a decision rule such that each firm's extraction is a function of the current resource stock only: $x_i = x_i(S)$ and $x_j = x_j(S)$ with $x_i(S) \geq 0$ and $x_j(S) \geq 0$ for any $S \geq 0$, and $x_i(0) = x_j(0) = 0$. An $n+m$-tuple of strategies constitutes a feedback Nash equilibrium if it solves the problem defined above for $i = 1, \cdots, m$ and $j = 1, \cdots, n$.[3]

We employ a technique by Tsutsui and Mino (1990) and Shimomura (1991) to derive the feedback Nash equilibrium.[4] It begins by defining firm i's Hamilton-Jacobi-Bellman (HJB) equation:

$$rV_i(S) = \max_{x_i} \left\{ \left[a - x_i - \sum_{l=1, l\neq i}^{m} x_l(S) - \sum_{j=1}^{n} x_j(S) \right] x_i + V_i'(S) \left[kS - x_i - \sum_{l=1, l\neq i}^{m} x_l(S) - \sum_{j=1}^{n} x_j(S) \right] \right\}, \tag{2}$$

where $V_i(\cdot)$ is a value function of firm i:

$$\begin{aligned} V_i(S) \equiv \max_{x_i} \Bigg\{ & \int_t^{\infty} e^{-r(\tau - t)} \left[a - x_i - \sum_{l=1, l\neq i}^{m} x_l(S) - \sum_{j=1}^{n} x_j(S) \right] x_i d\tau \ \Bigg| \\ & \dot{S} = kS - x_i - \sum_{l=1, l\neq i}^{m} x_l(S) - \sum_{j=1}^{n} x_j(S), \ S(t) : \text{given} \Bigg\}. \end{aligned}$$

Maximizing the right-hand side of (2) and assuming that all the efficient (resp. inefficient) firms choose $x_i(S)$ (resp. $x_j(S)$), we have the first-order condition of the efficient firms: $V_i'(S) = a - (m+1)x_i(S) - nx_j(S)$. Substituting this into (2) yields an identity in S:

$$rV_i(S) = \left[a - mx_i(S) - nx_j(S) \right] x_i(S) + \left[a - (m+1)x_i(S) - nx_j(S) \right] \left[kS - mx_i(S) - nx_j(S) \right],$$

[3] These definitions of strategies and equilibrium owe to Benchekroun (2008) and Lohoues (2006). Note that any stationary feedback Nash equilibrium is subgame perfect in an autonomous game defined above (see Dockner et al. 2000, p. 105).

[4] The reason for choosing the Tsutsui-Mino-Shimomura technique for deriving feedback strategies is simply that it is more convenient for our purpose than guessing quadratic value functions. Appendix A briefly derives the same solution through the latter approach.

in view of the symmetry in each group of firms. Differentiating both sides with respect to S and rearranging terms, we have an auxiliary differential equation:

$$
\begin{aligned}
& \left[2m^2 x_i(S) + 2mnx_j(S) - (m-1)a - (m+1)kS\right] x_i'(S) \\
& + \left[2mx_i(S) + 2nx_j(S) - a - kS\right] nx_j'(S) \\
= \; & (k-r)\left[(m+1)x_i(S) + nx_j(S) - a\right]. \qquad (3)
\end{aligned}
$$

Applying the same procedure to the inefficient firms' problem, we have a counterpart of firm j:

$$
\begin{aligned}
& \left[2mx_i(S) + 2nx_j(S) - (a-c) - kS\right] mx_i'(S) \\
& + \left[2mnx_i(S) + 2n^2 x_j(S) - (n-1)(a-c) - (n+1)kS\right] x_j'(S) \\
= \; & (k-r)\left[mx_i(S) + (n+1)x_j(S) - (a-c)\right]. \qquad (4)
\end{aligned}
$$

Feedback Nash equilibrium strategies are determined by solving the system of differential equations (3) and (4).[5] However, since it is almost impossible to explicitly solve the system, we focus on linear strategies: $x_i(S) = \alpha_i S + \beta_i$ and $x_j(S) = \alpha_j S + \beta_j$, where $\alpha_i, \alpha_j, \beta_j$ and β_j are undetermined coefficients. Under these specifications, (3) and (4) become

$$
\begin{aligned}
& \left[2m^2(\alpha_i S + \beta_i) + 2mn(\alpha_j S + \beta_j) - (m-1)a - (m+1)kS\right] \alpha_i \\
& + \left[2m(\alpha_i S + \beta_i) + 2n(\alpha_j S + \beta_j) - a - kS\right] n\alpha_j \\
= \; & (k-r)\left[(m+1)(\alpha_i S + \beta_i) + n(\alpha_j S + \beta_j) - a\right] \qquad (5)
\end{aligned}
$$

$$
\begin{aligned}
& \left[2m(\alpha_i S + \beta_i) + 2n(\alpha_j S + \beta_j) - (a-c) - kS\right] m\alpha_i \\
& + \left[2mn(\alpha_i S + \beta_i) + 2n^2(\alpha_j S + \beta_j) - (n-1)(a-c) - (n+1)kS\right] \alpha_j \\
= \; & (k-r)\left[m(\alpha_i S + \beta_i) + (n+1)(\alpha_j S + \beta_j) - (a-c)\right], \qquad (6)
\end{aligned}
$$

which are rewritten as

$$
\Delta_i S + \left[2m^2\alpha_i + 2mn\alpha_j - (m+1)(k-r)\right] \beta_i
$$

[5] See Dockner et al. (2000, pp. 96-97).

$$+\left[2m\alpha_i + 2n\alpha_j - k + r\right] n\beta_j + \left[k - r - (m-1)\alpha_i - n\alpha_j\right] a = 0 \tag{7}$$

$$\begin{aligned}
&\Delta_j S + \left[2m\alpha_i + 2n\alpha_j - k + r\right] m\beta_i \\
&+ \left[2mn\alpha_i + 2n^2\alpha_j - (n+1)(k-r)\right] \beta_j + \left[k - r - m\alpha_i - (n-1)\alpha_j\right](a-c) = 0
\end{aligned} \tag{8}$$

$$\begin{aligned}
\Delta_i &\equiv \left[2m^2\alpha_i + 2mn\alpha_j - (m+1)(2k-r)\right]\alpha_i + \left(2m\alpha_i + 2n\alpha_j - 2k + r\right) n\alpha_j \\
\Delta_j &\equiv \left(2m\alpha_i + 2n\alpha_j - 2k + r\right) m\alpha_i + \left[2mn\alpha_i + 2n^2\alpha_j - (n+1)(2k-r)\right]\alpha_j .
\end{aligned}$$

The four coefficients are determined as follows. First, α_i and α_j are determined so that the terms multiplied by S are zero, i.e., $\Delta_i = \Delta_j = 0$. Subtracting Δ_j from Δ_i yields $\Delta_i - \Delta_j = -(2k-r)(\alpha_i - \alpha_j) = 0$, from which we see that $\alpha_i = \alpha_j = \alpha$. Hence, substituting $\alpha_i = \alpha_j = \alpha$ into either $\Delta_i = 0$ or $\Delta_j = 0$ and solving the resulting equation for α, we obtain

$$\alpha = 0, \frac{(2k-r)(m+n+1)}{2(m+n)^2}. \tag{9}$$

On the other hand, β_i and β_j are determined through the system of equations obtained by setting the constant terms in (7) and (8) to zero:

$$\begin{aligned}
&\begin{bmatrix} 2m^2\alpha_i + 2mn\alpha_j - (m+1)(k-r) & \left[2m\alpha_i + 2n\alpha_j - k + r\right] n \\ \left[2m\alpha_i + 2n\alpha_j - k + r\right] m & 2mn\alpha_i + 2n^2\alpha_j - (n+1)(k-r) \end{bmatrix} \begin{bmatrix} \beta_i \\ \beta_j \end{bmatrix} \\
= &\begin{bmatrix} -\left[k - r - (m-1)\alpha_i - n\alpha_j\right] a \\ -\left[k - r - m\alpha_i - (n-1)\alpha_j\right](a-c) \end{bmatrix}.
\end{aligned}$$

The solution to this system is

$$\beta_i = \frac{\left[k - r - (m+n-1)\alpha\right]\left\{(k-r)a + \left[k - r - 2(m+n)\alpha\right] nc\right\}}{(k-r)\left[(m+n+1)(k-r) - 2(m+n)^2\alpha\right]} \tag{10}$$

$$\beta_j = \frac{\left[k - r - (m+n-1)\alpha\right]\left\{(k-r)a - \left[(m+1)(k-r) - 2m(m+n)\alpha\right] c\right\}}{(k-r)\left[(m+n+1)(k-r) - 2(m+n)^2\alpha\right]}. \tag{11}$$

Substituting (9) into (10) and (11), the closed form of β_i and β_j is computed. Note here that $\alpha = 0$ corresponds to the static Cournot-Nash outcome because (10) and (11) show that

$$\begin{aligned}
x_i &= \left(\alpha S + \beta_i\right)_{\alpha=0} = \frac{a + nc}{m+n+1} \\
x_j &= \left(\alpha S + \beta_j\right)_{\alpha=0} = \frac{a - (m+1)c}{m+n+1}.
\end{aligned}$$

In the subsequent arguments, we restrict attention to the case where both types of firms employ an interior feedback strategy since there is nothing new in the case where static outputs are chosen. This is justified by assuming that the initial resource stock belongs to $[-\beta_j/\alpha, [a-(m+1)c-(m+n+1)\beta_j]/[(m+n+1)\alpha]$.[6][7] In addition, we assume $r \to 0$ since it is too difficult to facilitate analysis for an arbitrary r.[8] Under these restrictions, the coefficients obtained above are

$$\alpha = \frac{k(m+n+1)}{(m+n)^2} > 0 \tag{12}$$

$$\beta_i = \frac{-(m+n)a + n(m+n+2)c}{(m+n)^3(m+n+1)} \tag{13}$$

$$\beta_j = \frac{-(m+n)a - [2m + (m+n)(m-1)]c}{(m+n)^3(m+n+1)} < 0. \tag{14}$$

Assuming that a is sufficiently larger compared to c to ensure $\beta_i < 0$, substitution of (12)-(14) into $x_i(S) = \alpha S + \beta_i$ and $x_j(S) = \alpha S + \beta_j$ yields the feedback strategy of each firm as follows.[9]

$$x_i(S) = \begin{cases} 0 & \text{if} \quad S < \frac{-\beta_j}{\alpha} \\ \alpha S + \beta_i & \text{if} \quad \frac{-\beta_j}{\alpha} \leq S < \frac{a-(m+1)c-(m+n+1)\beta_j}{(m+n+1)\alpha} \\ \frac{a+nc}{m+n+1} & \text{if} \quad S \geq \frac{a-(m+1)c-(m+n+1)\beta_j}{(m+n+1)\alpha} \end{cases} \tag{15}$$

$$x_j(S) = \begin{cases} 0 & \text{if} \quad S < \frac{-\beta_j}{\alpha} \\ \alpha S + \beta_j & \text{if} \quad \frac{-\beta_j}{\alpha} \leq S < \frac{a-(m+1)c-(m+n+1)\beta_j}{(m+n+1)\alpha} \\ \frac{a-(m+1)c}{m+n+1} & \text{if} \quad S \geq \frac{a-(m+1)c-(m+n+1)\beta_j}{\alpha} \end{cases}. \tag{16}$$

Roughly speaking, (15) and (16) state that feedback strategy outputs are zero (resp. static output) if S is sufficiently small (resp. large) and linearly increasing in S if it is in a certain closed interval.

We draw two notes on the equilibrium strategies above. First, each firm voluntarily stops extraction if the resource stock is low enough. That is, 'firms prefer to let the asset grow and refrain from any exploitation as long as the stock has not reached its maturity

[6]See Tsutsui and Mino (1990) and Dockner and Long (1993). More recent examples to impose the same assumptions are found in Itaya and Shimomura (2001) and Rubio and Casino (2002).

[7]'Interior' here means that α is non-zero.

[8]Setting $r \to 0$ is often observed in the existing literature as well, e.g., Fershtman and Kamien (1987), Tsutsui and Mino (1990) and Dockner and Long (1993).

[9]See also Proposition 1 of Benchekroun (2003, 2008) and of Lohoues (2006).

threshold.' (Benchekroun, 2008, p. 242) Second, the efficient firms' strategy exhibits jumps at $S = -\beta_j/\alpha$ and $S = [a - (m+1)c - (m+n+1)\beta_j]/\alpha$ whereas the inefficient firms' strategies are globally continuous.

The intuitions behind these jumps in the efficient firms' equilibrium output are as follows. Suppose first that S is small enough to satisfy $S < -\beta_j/\alpha$. Then, the efficient firms find it more profitable to wait for resource accumulation than to begin producing. If they produce positive output at such a small resource, they immediately run out of the resource and can not produce any more since feedback strategies require that $x_i(0) = 0$. Thus, the efficient firms voluntarily quit production until S exceed $-\beta_j/\alpha$. On the other hand, the jump at $S = [a-(m+1)c-(m+n+1)\beta_j]/[(m+n+1)\alpha]$ is explained as follows. When the resource stock reaches this level, the inefficient firms switch their output from $\alpha S + \beta_j$ to the static Cournot level. Given this, it is more profitable for the efficient firms to choose a larger output between $\alpha S + \beta_i$ and static Cournot output. Accordingly, the efficient firms also abandon $\alpha S + \beta_i$ and choose static Cournot output. For these reasons, the efficient firms' equilibrium output exhibits two jumps.

4 The main results

This section establishes the main results. Let us first consider the effect of an increase in the number of efficient firms on steady state welfare. For this purpose, define steady state welfare U as follows.

$$\begin{aligned}
U &= CS + m\pi_i + n\pi_j \\
&= \frac{(mx_i + nx_j)^2}{2} + m(a - mx_i - nx_j)x_i + n(a - c - mx_i - nx_j)x_j \\
&= \frac{(kS)^2}{2} + (a - kS)mx_i + (a - c - kS)nx_j \qquad (17) \\
&= \frac{(kS)^2}{2} + (a - kS)(mx_i + nx_j) - ncx_j \\
&= \frac{(kS)^2}{2} + (a - kS)kS - ncx_j \\
&= \frac{kS(2a - kS)}{2} - ncx_j, \qquad (18)
\end{aligned}$$

where CS denotes consumer surplus, π_i is each efficient firm's profit and π_j is each inefficient firm's profit. Rearrangements after (17) use the steady state condition that $\dot{S} = kS - mx_i - nx_j = 0$, namely, $mx_i + nx_j = kS$.

Based on preliminary analyses so far, we can state a striking effect of increasing the number of *efficient* firms:

Proposition 1. *An increase in the number of efficient firms reduces steady state welfare.*

Proof. See Appendix B.

In order to understand implications of Proposition 1 better, we prove a useful result.

Lemma 1. *The steady state in the feedback Nash equilibrium involves over-exploitation of the resource as compared to social optimum.*

Proof. Social optimum in our context is defined by the solution to the following welfare-maximizing problem:

$$\begin{aligned} \max_{x_i, x_j} \quad & \int_0^\infty e^{-rt} U(x_i, x_j) dt \\ \text{subject to} \quad & \dot{S} = kS - mx_i - nx_j, \quad S(0) : \text{given}, \end{aligned}$$

where $U(x_i, x_j)$ is the sum of consumer surplus and aggregate profits:

$$U(x_i, x_j) \equiv \frac{(mx_i + nx_j)^2}{2} + m(a - mx_i - nx_j)x_i + n(a - c - mx_i - nx_j)x_j.$$

Since this is a single agent's optimal control problem, it can be solved with a Hamiltonian: $H = U(x_i, x_j) + \lambda(kS - mx_i - nx_j)$, where λ is a costate variable. The maximum principle gives the following first-order conditions:

$$m(a - mx_i - nx_j - \lambda) \leq 0, \;\; x_i \geq 0, \;\; mx_i(a - mx_i - nx_j - \lambda) = 0 \tag{19}$$

$$n(a - c - mx_i - nx_j - \lambda) \leq 0, \;\; x_j \geq 0, \;\; nx_j(a - c - mx_i - nx_j - \lambda) = 0 \tag{20}$$

$$\dot{\lambda} = (r - k)\lambda \tag{21}$$

$$\dot{S} = kS - mx_i - nx_j \tag{22}$$

$$\lim_{t\to\infty} e^{-rt}\lambda S = 0.$$

It naturally follow from (19) and (20) that $x_j = 0$, i.e., the social optimum drives inefficient firms out of the market and that $mx_i = a - \lambda$. Substituting this into (22), the equilibrium system reduces to

$$\begin{bmatrix} \dot{\lambda} \\ \dot{S} \end{bmatrix} = \begin{bmatrix} r-k & 0 \\ 1 & k \end{bmatrix} \begin{bmatrix} \lambda \\ S \end{bmatrix} + \begin{bmatrix} 0 \\ -a \end{bmatrix}.$$

It is easy to confirm that the steady state is a stable saddle point under $k > r$ and that the steady state level of S becomes $S^O = a/k$, where superscript O stands for social optimum.[10] Comparing S^E in (30) with S^O, we have

$$S^O - S^E = \frac{a}{k} - \frac{(m+n)a - nc}{k(m+n)(m+n+1)} = \frac{(m+n)^2 a + nc}{k(m+n)(m+n+1)} > 0.$$

That is, the steady state resource stock in the oligopoly equilibrium is less compared to the socially optimal level.

Lemma 1 states that over-exploitation or the tragedy of the commons survives our model. This is simply because the resource has open access.[11]

We now discuss what causes Proposition 1, which sharply contrasts to conventional wisdom that increasing the number of efficient firms benefits welfare. The effects of an increase in m are depicted in Figure 2.

(Figure 1 around here)

Our model admits two effects of an increase in m on welfare. On the one hand, it expands industry output and lowers price, which has an incremental effect on consumer surplus and welfare. This effect can be called a static effect. Note further that an increase in m improves welfare by mitigating the distortion that comes from the presence of high-cost firms. On the other hand, an increase in m decreases the resource stock by

[10] In the limiting case where $r \to 0$, it is trivially satisfied that $k > r$.

[11] 'Open access' means an inability to restrict access to the resource, which is distinguished from a 'common property' referring to the resource collectively owned by a group of owners (Bulte et al., 1995, and Brander and Taylor, 1997).

accelerating over-exploitation.[12] Taking into account this decrease in S, all firms reduce output since feedback strategies are increasing in S (recall that $x_i'(S) = x_j'(S) = \alpha > 0$). This can be called a closed-loop effect. Because all firms reduce output, the industry output and consumer surplus will inevitably decrease.

Since the static effect positively affects welfare and the closed-loop effect negatively affects welfare, the total effect seems ambiguous. Nevertheless, Proposition 1 states that the latter effect necessarily dominates the former effect, which unambiguously results in welfare losses. In other words, the intertemporally strategic interaction through the change in the resource stock plays a dominant role and outweighs the potentially beneficial effect.[13]

Proposition 1 straightforwardly leads to:

Corollary 1. *An increase in the number of inefficient firms reduces steady state welfare.*

A remark is attached to this result. In Lahiri and Ono's (1988) argument, increasing the number of inefficient firms has two competing effects. One is a procompetitive effect and the other is a profit-shifting effect from efficient firms to inefficient firms. Their conclusion that 'helping minor firms reduces welfare' rests on an additional assumption that the efficient firms' share is initially large enough. Unless this is satisfied, increasing the number of inefficient firms still benefits welfare. In contrast, both Proposition 1 and Corollary 1 need no such assumption. What is relevant is that increasing competition reduces the total output through the closed-loop effects of feedback strategies.

At this stage, someone may be interested in what would happen if static Cournot outputs were to be chosen.[14] This confirms the conventional wisdom:

[12] Even in the absence of oligopolistic interactions in the output market, over-exploitation easily occurs. See Chapter 12 of Docknet et al. (2000).

[13] It is possible to prove a result closely related to Proposition 1, which asserts that an increase in c reduces welfare, i.e., $dU/dc < 0$. While detailed calculations are omitted, the final outcome is

$$\frac{dU}{dc} = \frac{-n\left\{(m+n)^2(2m+2n+1)a - \left[2m^3 + 2(3n+2)m^2 + (6n^2+5n+2)m + n^2(2n+1)\right]c\right\}}{(m+n)^3(m+n+1)^2} < 0.$$

A rise in c directly yields a larger market share of efficient firms but lowers welfare. This parallels Proposition 1.

[14] As shown in Benchekroun (2003, 2008) in a context of symmetric oligopoly, the steady state associ-

Proposition 2. *If static Cournot outputs were to be chosen, an increase in the number of efficient firms raises steady state welfare.*

Proof. See Appendix C.

This result clearly convinces us that a result parallel to Proposition 1 would no longer be the case if static outputs were to be chosen. This is because static outputs have no closed-loop property, i.e., no firm adjusts output to a change in S. In this case, only the static effect prevails and thus increasing the number of efficient firms unambiguously improves welfare.

5 Concluding remarks

Extending Benchekroun's (2008) model of a productive asset oligopoly to an asymmetric oligopoly, we have proved that 'helping major firms reduces welfare.' This yields a natural corollary that 'helping any firm reduces welfare.' These results have theoretically and practically important implications. From a theoretical point of view, there does exist a case in which predictions of the static theory are completely reversed. In our context, the adverse effect through the closed-loop property of feedback strategies dominates the favorable effect coming from the static effect.

On the other hand, our results cast a serious doubt on practical competition policies. It has been tacitly believed that increasing the number of efficient firms is always welfare-improving. However, we have demonstrated that such recognition is too hasty. If we allow for intertemporal interactions among firms, *reducing* the number of firms can be beneficial.

Nonetheless, we have admittedly rested on numerous simplifying assumptions some of which are stricter than Benchekroun (2008) and Lohoues (2006). For instance, (i) dynamics of the resource is linearly increasing, (ii) the analysis is restricted to steady states and (iii) the resource is open access and is not private. While this paper has not

ated with static Cournot outputs is asymptotically unstable, which is also true of our model.

pursued generality, it is our future research agenda to explore the validity and generality of our results by relaxing these assumptions.

Appendix A. Feedback strategies through the guessing method

While the main text employs the Tsutsui-Mino-Shimomura approach, this appendix derives the feedback strategy by assuming quadratic value functions. Maximizing the right-hand side of firm i's HJB equation with respect to x_i and using the symmetry in each group of firms, we obtain the first-order condition of efficient firms:

$$a - (m+1)x_i - nx_j - V_i'(S) = 0. \tag{23}$$

Inefficient firms' counterpart is

$$a - c - mx_i - (n+1)x_j - V_j'(S) = 0. \tag{24}$$

From (23) and (24), feedback strategies are solved as

$$\begin{aligned} x_i(S) &= \frac{a + nc - (n+1)V_i'(S) + nV_j'(S)}{m+n+1} \\ x_j(S) &= \frac{a - (m+1)c + mV_i'(S) - (m+1)V_j'(S)}{m+n+1}. \end{aligned}$$

Supposing that each firm's value function is quadratic in S so that $V_i(S) = A_iS^2/2+B_iS+C_i$ and $V_j(S) = A_jS^2/2 + B_jS + C_j$, we have $V_i'(S) = A_iS + B_i$ and $V_j'(S) = A_jS + B_j$, and (23) and (24) become

$$x_i(S) = \frac{[-(n+1)A_i + nA_j]S - (n+1)B_i + nB_j + a + nc}{m+n+1} \tag{25}$$

$$x_j(S) = \frac{[mA_i - (m+1)A_j]S + mB_i - (m+1)B_j + a - (m+1)c}{m+n+1}. \tag{26}$$

Substituting these into the original HJB equations, we have an identity of efficient firms

$$\begin{aligned} & r\left(\frac{A_i}{2}S^2 + B_iS + C_i\right) \\ = & \left[a + \frac{(mA_i + nA_j)S + mB_i + nB_j - (m+n)a + nc}{m+n+1}\right] \end{aligned}$$

$$
\begin{aligned}
&\times\frac{[-(n+1)A_i+nA_j]S-(n+1)B_i+nB_j+a+nc}{m+n+1}\\
&+(A_iS+B_i)\left[kS+\frac{(mA_i+nA_j)S+mB_i+nB_j-(m+n)a+nc}{m+n+1}\right],
\end{aligned}
$$

and of inefficient firms

$$
\begin{aligned}
&r\left(\frac{A_j}{2}S^2+B_jS+C_j\right)\\
=\;&\left[a-c+\frac{(mA_i+nA_j)S+mB_i+nB_j-(m+n)a+nc}{m+n+1}\right]\\
&\times\frac{[mA_i-(m+1)A_j]S+mB_i-(m+1)B_j+a-(m+1)c}{m+n+1}\\
&+(A_jS+B_j)\left[kS+\frac{(mA_i+nA_j)S+mB_i+nB_j-(m+n)a+nc}{m+n+1}\right].
\end{aligned}
$$

Equating the terms multiplied by S^2 in both sides of these identities, we can get $A_i = A_j = A = 0, -(2k-r)(m+n+1)^2/2(m+n)^2 < 0$. Analogously, B_i and B_j satisfy the system of equations

$$
\begin{aligned}
&\begin{bmatrix} 2m(m+n)A+(k-r)(m+n+1)^2 & 2n(m+n)A \\ 2m(m+n)A & 2n(m+n)A+(k-r)(m+n+1)^2 \end{bmatrix}\begin{bmatrix} B_i \\ B_j \end{bmatrix}\\
=\;&\begin{bmatrix} [(m+n)^2+1]\,a-2n(m+n)c \\ [(m+n)^2+1]\,a+(m^2-n^2-1)c \end{bmatrix}A.
\end{aligned}
$$

by equating the terms multiplied by S to zero. Finally, C_i and C_j are obtained from the constant terms in the HJB equations. Substituting these back into (25) and (26), feedback strategies are derived as in (15) and (16). Note that $A_i = A_j = A = 0$ corresponds to the static Cournot-Nash outputs.

Appendix B. Proof of Proposition 1

Differentiating (18) with respect to m, we have

$$
\frac{dU}{dm} = k(a-kS)\frac{dS}{dm} - nc\frac{dx_j}{dm}. \tag{27}
$$

The steady state in which $\dot{S} = kS - m(\alpha S+\beta_i) - n(\alpha S+\beta_j) = 0$ involves

$$
S = \frac{m\beta_i+n\beta_j}{k-(m+n)\alpha}. \tag{28}
$$

Substituting this into $x_j(S) = \alpha S + \beta_j$, an inefficient firm's steady state output is

$$x_j = \alpha S + \beta_j = \alpha \cdot \frac{m\beta_i + n\beta_j}{k - (m+n)\alpha} + \beta_j = \frac{m\alpha(\beta_i - \beta_j) + k\beta_j}{k - (m+n)\alpha}. \tag{29}$$

Substituting (12)-(14) into (28) and (29), the closed-form of S and x_j in the steady state is

$$\begin{aligned} S^E &= \frac{(m+n)a - nc}{k(m+n)(m+n+1)} \\ x_j^E &= \frac{(m+n)^2 a - \left[m + (m+n)^2\right] c}{(m+n)^3(m+n+1)}, \end{aligned} \tag{30}$$

where superscript E denotes the Nash equilibrium.

Thus, differentiating these with respect to m yields

$$\begin{aligned} \frac{dS^E}{dm} &= \frac{-(m+n)^2 a + n(2m+2n+1)c}{k(m+n)^2(m+n+1)^2} < 0 \\ \frac{dx_j^E}{dm} &= \frac{-(m+n)^2(2m+2n+1)a + \left[2(m+n)^3 + (4m-1)(m+n) + 3m\right] c}{(m+n)^4(m+n+1)^2} < 0. \end{aligned}$$

Applying these results to (27), rearranging terms and defining $N \equiv m + n$, dU/dm becomes

$$\begin{aligned} \frac{dU}{dm} &= \frac{\Gamma}{kN^4(N+1)^3} \\ \Gamma &= -N^5 a^2 + N^2 n \left[2N^2 + k(N+1)(2N+1)\right] ac \\ &\quad + n\left\{N(2N+1)n - k(N+1)\left[2N^3 + (4m-1)N + 3m\right]\right\} c^2 < 0. \end{aligned}$$

Consequently, we have concluded that $dU/dm < 0$.

Appendix C. Proof of Proposition 2

It is convenient to slightly rewrite (18) as follows.

$$U = \frac{X(2a - X)}{2} - ncx_j,$$

where $X \equiv mx_i + nx_j$. Therefore, a change in m induces

$$\frac{dU}{dm} = (a - X)\frac{dX}{dm} - nc\frac{dx_j}{dm}. \tag{31}$$

Static Cournot-Nash outcomes give

$$X = \frac{(m+n)a - nc}{m+n+1}, \quad x_j = \frac{a-(m+1)c}{m+n+1}$$
$$\frac{dX}{dm} = \frac{a+nc}{(m+n+1)^2}, \quad \frac{dx_j}{dm} = \frac{-a-nc}{(m+n+1)^2}.$$

Substituting these into (31), a welfare change associated with a change in m is

$$\frac{dU}{dm} = \frac{[a+n(m+n+2)c](a+nc)}{(m+n+1)^2} > 0,$$

that is, increasing the number of efficient firms benefits welfare.

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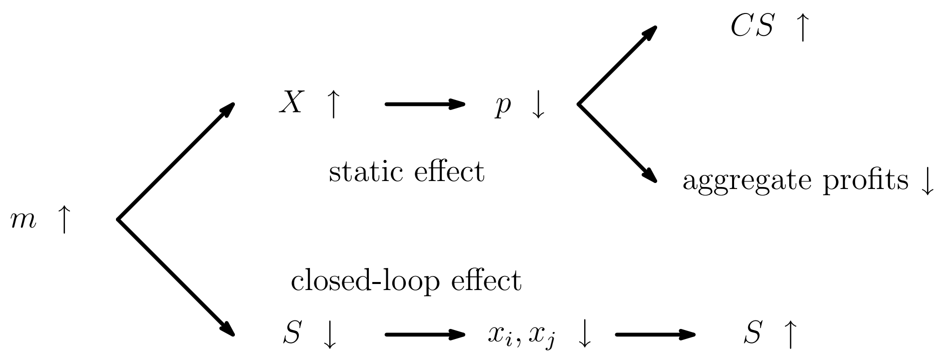

Figure 1: Effects of an increase in m