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Analysis of child care policies with an uncertainty model

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Abstract

The purpose of this paper is examination of how fertility is determined using a model with income uncertainty. Income uncertainty here is related to unemployment. The analysis is based on a model that differs from perfect foresight, with a situation in which income cannot be earned because of unemployment, but where future unemployment is not known in advance. In a small open economy, both a child allowance and unemployment benefits have the effect of raising fertility rates. Further analysis conducted with a closed economy has yielded interesting results. The presence of unemployment benefits ultimately leads to a lower fertility rate through lower accumulation of capital stock. However, it was demonstrated that a higher level of unemployment benefits engenders a higher fertility rate than in the absence of benefits.

Keywords: Child allowance, Fertility, Uncertainty, Unemployment

JEL: H51, H55, J14

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1. Introduction

The purpose of this paper is examination of how fertility is determined in a model where income uncertainty exists. Income uncertainty here is related to unemployment. The analysis is based on a model that differs from one with perfect foresight, with a situation in which income cannot be earned because of unemployment, but future unemployment is not known in advance.

Although earlier studies have already analyzed small open economies, this paper differs from earlier studies in that it analyses a closed economy. Moreover, it uses a model that incorporates endogenous unemployment, which can be characterized as an original point.

The results of the analysis show that, in the case of a small open economy, both child benefit payments and unemployment benefits have the effect of raising fertility rates. In addition, although the analyses in this paper are based on a logarithmic utility function, results demonstrated that the same result is obtainable with a Constant Relative Risk Averse (CRRA) utility function. Further analysis of closed economy, however, yields interesting results. The presence of unemployment benefits engenders lower wage rates through lower capital stock accumulation. The results demonstrate that the presence of unemployment benefits alleviates income shocks caused by uncertainty about future income. Therefore, unemployment benefits increase the fertility rate, but the lower wage rate attributable to the closed economy ultimately reduces the fertility rate. It can also be shown that higher amounts of unemployment benefits would lead to a higher fertility rate than in the absence of benefits.

We undertake additional analyses with the unemployment rate as endogenized. Results indicate that the model is fundamentally the same as the model for an exogenous unemployment rate. This result means that, for analyses that take the unemployment rate into account, no difficulty arises with an exogenously given unemployment rate.

Herein, we explain reports of related work in the literature. Although various fertility endogenization models exist, the basic models are listed hereinafter: Galor and Weil (1996) model child caring using parental time, whereas van Groezen, Leers and Meijdam (2003) model child caring using goods and services. The latter model is based on the use of goods and services to provide childcare. They show that, in the case of parental time, an increase in the wage rate reduces fertility because it is an opportunity cost, whereas childcare using goods and services increases fertility with an increase in the wage rate. However, when the price of childcare services is wage-proportional rather than fixed, as described by Yasuoka and Miyake (2010), an increase in the wage rate does not necessarily engender an increase in the fertility rate. Other models in which fertility is determined by goods and services and parental time include those of Hirazawa and Yakita (2009) and of Yasuoka (2014).

Consequently, there are various models of fertility endogenization models. Furthermore, childcare support policies have been analyzed using various models. For example, van Groezen, Leers and Meijdam (2003) show that child allowances raise fertility. Also, Apps and Rees (2004) demonstrate that subsidies for childcare services raise fertility. However, these models are analyzed as deterministic models, where uncertainty does not exist. In reality, uncertainty about future income, including the risk of unemployment, is likely to be an important concern. The existence of uncertainty about future income motivates precautionary savings, which reduces consumption in the present. Similarly, fertility is analyzed by Yasuoka (2021) as reducing child care expenditures. In a model with such unemployment exists, this point is analyzed by Gori and Fanti (2007). They show that a child allowance cannot raise fertility. However, that study uses expected income, not expected utility. In such a setting, there are no precautionary savings. Therefore, an analytical model such as the one used for the analyses described in this paper is necessary to incorporate precautionary savings. The paper is structured as follows: section 2 presents examination of how unemployment benefits and a child allowance affect fertility in a model of a small open economy; section 3 relaxes the assumption of a logarithmic utility function and considers it in a CRRA-type utility function; section 4 presents analyses of a closed economy; section 5 analyzes it in a model of endogenous unemployment; and section 6 concludes the discussion.

2. Basic Model

This section presents explanation of the model described by Yasuoka (2021). Individuals in households exist in two periods: a young period and an adult period. During the young period, individuals pay for consumption and for care for their children. During the adult period, they pay only for consumption. In both periods, they work to obtain labor income. However, during the adult period, some people become unemployed and become unable to obtain labor income. We assume the probabilities of q and $1-q$ respectively as employment and unemployment. The utility function of individuals U_t is assumed as presented below:

$$U_t = \alpha ln n_t + \beta ln c_t + q(1-\alpha-\beta) ln c_{t+1}^g + (1-q)(1-\alpha-\beta) ln c_{t+1}^b, \quad 0<\alpha<1, 0<\beta<1, \alpha+\beta<1, 0<q<1. \tag{1}$$

Therein, n_t and c_t respectively stand for the number of children (fertility) and consumption in the young period. If the adult people continue working, then consumption during the adult period is given as c_{t+1}^g. Otherwise, if the adult people become unemployed, then consumption is given as c_{t+1}^b. Also, t denotes the period. During the young period, they decide the allocation for consumption and saving. Subsequently, they consider the expected utility function because of uncertainty.

The budget constraint in young period is shown as

$$(z-x)n_t + c_t + s_t = (1-\varepsilon)w_t. \tag{2}$$

Therein, z and x respectively denote the childcare cost and child allowance for a child. The child allowance is financed by the income tax at rate ε. In addition, s_t and w_t respectively represent the savings for consumption during the adult period and wage rate.

Budget constraints of two kinds exist for the adult period: one for the case of employment and the other for the case of unemployment. In the case of employment, the budget constraint for the adult period is as presented below.

$$c_{t+1}^{g} = (1 + r_{t+1})s_t + (1 - \tau)w_{t+1}. \qquad (3)$$

Therein, r_{t+1} stands for the interest rate. The benefit for unemployment is financed by the income tax at the tax rate τ. The budget constraint in the case of unemployment is

$$c_{t+1}^{b} = (1 + r_{t+1})s_t + b_{t+1}. \qquad (4)$$

In that equation, b_{t+1} denotes the unemployment benefit.

2.1 Probability of employment

We next derive the optimal household allocations. We consider maximization of utility function (1) subject to the budget constraint in (2)–(4). Saving s_t and fertility n_t are given below:

$$\frac{\alpha + \beta}{(1 - \varepsilon)w_t - s_t} = \frac{q(1 - \alpha - \beta)(1 + r_{t+1})}{(1 + r_{t+1})s_t + (1 - \tau)w_{t+1}} + \frac{(1 - q)(1 - \alpha - \beta)(1 + r_{t+1})}{(1 + r_{t+1})s_t + b_{t+1}}. \qquad (5)$$

$$n_t = \frac{\alpha}{\alpha + \beta}\frac{(1 - \varepsilon)w_t - s_t}{z - x}. \qquad (6)$$

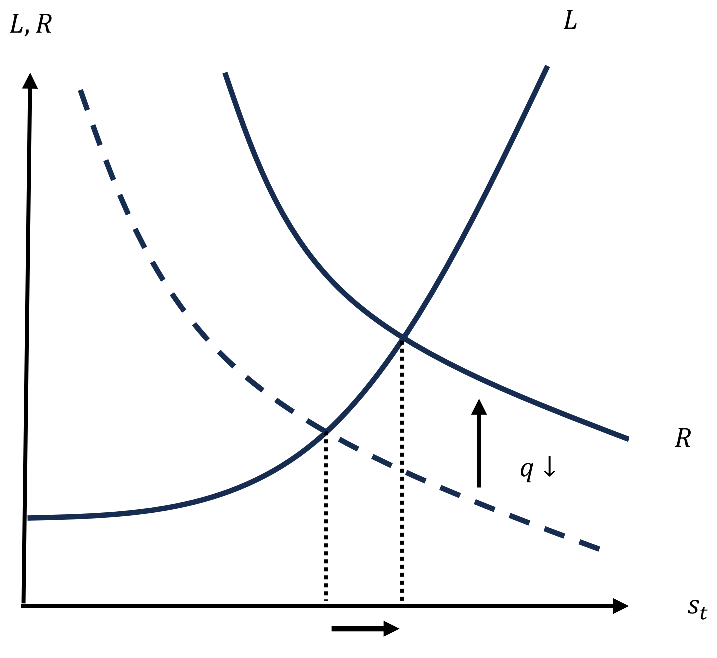

Fig. 1: Decision of saving s_t.

Defining the left-hand-side and the right-hand-side of (5) respectively as L and R, we can produce

Fig. 1, showing that saving s_t is uniquely determined. With $(1-\tau)w_{t+1} > b_{t+1}$, a decrease in q, which is the probability of employment, increases saving s_t. This result is shown in earlier work reported by Yasuoka (2021).

2.2 Effect of child allowance

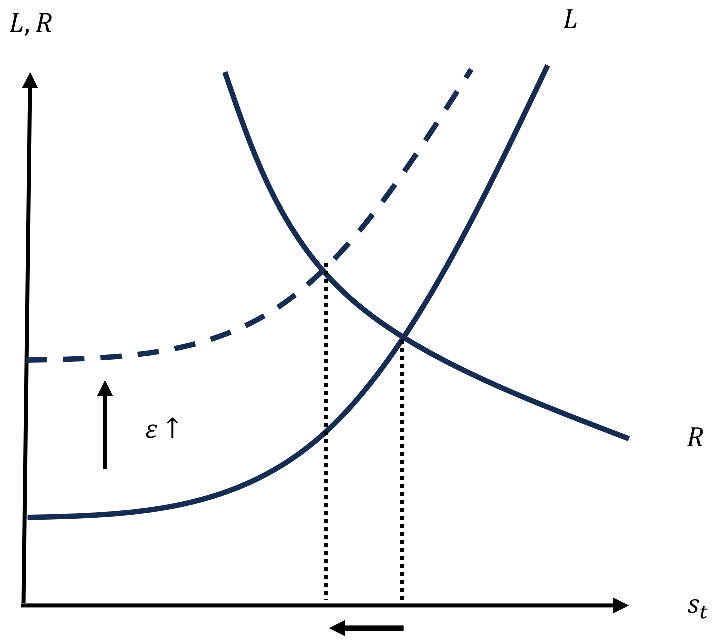

Fig. 2: Effect of child allowance on saving.

If the child allowance is provided by the balanced government budget constraint, then, we show the following government budget constraint of the child allowance.

$$xn_t = \varepsilon w_t. \tag{7}$$

By substituting (7) into (6), one can obtain the following for fertility.

$$n_t = \frac{\alpha(w_t - s_t)}{(\alpha+\beta)z - \beta x}. \tag{8}$$

The equation of saving s_t does not change. An increase in tax rate ε pulls up L in Fig. 2. Then, saving s_t decreases. The effect of a decrease in s_t has a positive effect on fertility n_t. In addition, by virtue of an increase in the child allowance, fertility n_t raises. Then, the following proposition can be established.

Proposition 1

An increase in the child allowance can raise the fertility.

This result differs from that described by Gori and Fanti (2007). They consider the probability of unemployment and infer that an increase in the child allowance reduces fertility because of an increased tax burden.

2.3 Unemployment benefit

We consider the case in which the government gives benefits for unemployed people. With the balanced budget, the unemployment benefit is shown as

$$q\tau w_{t+1} = (1-q)b_{t+1} \rightarrow b_{t+1} = \frac{q\tau w_{t+1}}{1-q}. \quad (9)$$

By substituting (9) into (5) and calculating $\frac{dR}{d\tau}$, we can obtain the following result.

$$\frac{dR}{d\tau} = (1+r)q(1-\alpha-\beta)w_{t+1}\left(\frac{1}{\left((1+r)s_t+(1-\tau)w_{t+1}\right)^2} - \frac{1}{\left((1+r)s_t+\frac{q\tau w_{t+1}}{1-q}\right)^2}\right). \quad (10)$$

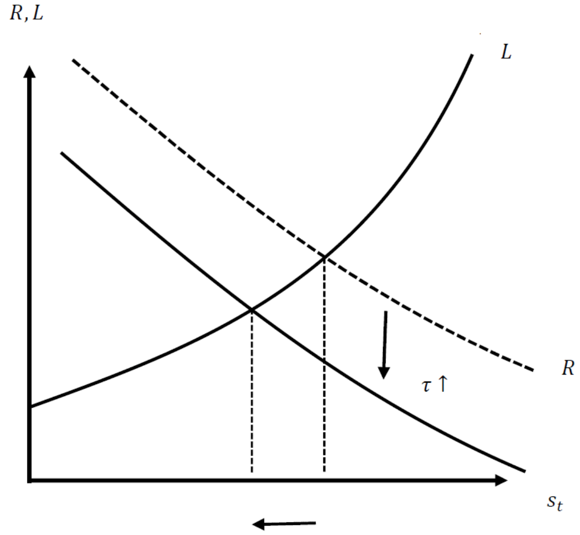

Fig. 3: Increase in unemployment benefits.

The condition to have $\frac{dR}{d\tau} < 0$ can be presented as shown below.

$$\tau < 1 - q\,. \quad (11)$$

As shown in Fig. 3, an increase in unemployment benefit reduces saving. Then, fertility increases. This result is the same as that reported by Yasuoka (2021).

In the following section, based on the model of Yasuoka (2021), we examine a different case of model setting.

3. CRRA Model

For the basic model, we assume a logarithmic utility function. However, the logarithmic utility function is a case of Constant Relative Risk Averse (CRRA) utility function, that is, the elasticity of substitution is 1. We examine how fertility and saving are determined in the case of CRRA utility function. Therefore, we assume the following utility function:

$$U_t = \alpha \frac{n_t^{1-\gamma}}{1-\gamma} + \beta \frac{c_t^{1-\gamma}}{1-\gamma} + q(1-\alpha-\beta)\frac{\left(c_t^g\right)^{1-\gamma}}{1-\gamma} + (1-q)(1-\alpha-\beta)\frac{\left(c_t^b\right)^{1-\gamma}}{1-\gamma}, \gamma < 1. \tag{12}$$

The budget constraint is given by (2)–(4). Then, the household optimal saving s_t and fertility n_t are given such that the following equation holds.

$$\left(\frac{(1-\varepsilon)w_t - s_t}{\alpha^{\frac{1}{\gamma}}(z-x)^{1-\frac{1}{\gamma}} + \beta^{-\frac{1}{\gamma}}}\right)^{-\gamma} = \frac{q(1-\alpha-\beta)(1+r_{t+1})}{\left((1+r_{t+1})s_t + (1-\tau)w_{t+1}\right)^{\gamma}} + \frac{(1-q)(1-\alpha-\beta)(1+r_{t+1})}{\left((1+r_{t+1})s_t + b_{t+1}\right)^{\gamma}}. \tag{13}$$

$$n_t = \frac{(1-\varepsilon)w_t - s_t}{z - x + \beta^{-\frac{1}{\gamma}}}. \tag{14}$$

As the substitution case, we consider the case of $0 < \gamma < 1$. As shown by the equations above, an increase in unemployment benefits reduces saving; also, fertility increases. This result is the same as that obtained for the case using a logarithmic utility function. However, the effects of a child allowance on saving and fertility can differ from those when using a logarithmic utility function.

4. Closed economy model

For the discussion presented in this section, we set the model with endogenous capital accumulation in the case of the logarithmic utility function. Defining $k_t = \frac{K_t}{L_t}$ as the capital stock labor ratio, we can obtain the following dynamics equation of capital stock labor ratio k_t. In the equation, K_t and L_t respectively denote the aggregate capital stock and labor input.

$$k_{t+1} = \frac{s_t}{l_{t+1} n_t}. \tag{15}$$

l_{t+1} denotes the labor supply per capita and $l_{t+1} = q$. Assuming a Cobb–Douglas function $Y_t = K_t^{\theta} L_t^{1-\theta}$, where $0 < \theta < 1$. The product function per capita is

$$y_t = k_t^{\theta}. \tag{16}$$

We note especially that $L_t = qN_t$, where N_t denotes the population size. The population growth is given by $n_t = \frac{N_{t+1}}{N_t}$.

With a perfectly competitive market, the interest rate and wage rate are given as

$$1 + r_t = \theta k_t^{\theta-1}. \tag{17}$$

$$w_t = (1-\theta)k_t^{\theta}. \tag{18}$$

To derive the effects of the policies with numerical examples, we set the following parameters.

Table 1: Parameter settings

θ	0.3
z	0.1669
α	0.5509
β	0.1496
$1-\alpha-\beta$	0.2994
q	0.98

Particularly, θ denotes the share of capital income. Recently, in OECD countries, the share of capital income is about 0.3. Considering work reported by de la Croix and Doepke (2003), we set $1-\alpha-\beta$=0.2994. Oshio (2001) sets the parameters to reflect a preference for consumption and fertility in the young period as equal, that is 0.5. Then, we set α and β as shown in Table 1. Here, q denotes the employment rate. Therefore, $1-q$ necessarily represents the unemployment rate. The recent unemployment rate in Japan is about 2%. Using these parameters, we set parameter z such that $n=1$ holds.

The results of numerical examples are presented as shown below.

Table 2: Results of Numerical Examples

	s	n	w	$1+r$	k	b	x
no policy	0.028472	1	0.24067	3.6227	0.028472	0	0
ε =0.01	0.027887	1.004217	0.240327	3.634766	0.028337	0	0.002393
τ =0.01	0.02722	0.998136	0.239022	3.681235	0.027827	0.117121	0
τ =0.1	0.02873	1.006541	0.242317	3.565518	0.029126	1.187351	0
ε =0.6209	0.006657	0.868288	0.163349	8.948403	0.007823	0	0.116809

Table 2 presents endogenous variables in the steady state. No policy represents the case of $\varepsilon=\tau=0$. The case of $\varepsilon=0.01$ shows the policy of child allowance that is financed by the income tax when the tax rate is 1%. The case of $\tau=0.01$ shows the policy of unemployment benefits financed by an income tax at the tax rate of 1%.

We can obtain interesting results. In the case of a small open economy, the unemployment benefit can raise the fertility rate. However, in the case of a closed economy, the unemployment benefit reduces fertility. Even if the unemployment benefit reduces negative effects of a decrease in wage income attributable to uncertainty and even if fertility is raised in the case of small open economy, fertility is still reduced overall because of decreased wage income in the closed economy.

However, considering the case of $\tau = 0.1$, one can obtain the result that fertility increases compared with the case of no policy. Consequently, we have the following proposition.

Proposition 2

In a small open economy, an increase in unemployment benefits can always raise fertility for the inequality (11) because of precautionary saving. However, in a closed economy, an increase in unemployment benefits does not necessarily raise the fertility rate even if precautionary saving decreases because of decreased capital stock accumulation.

Table 2 shows the case of a child allowance with aggregate benefit that is equal to the case of $\tau = 0.01$. In this case, child allowance reduces the fertility rate. This result shows a decrease in capital stock accumulation. Moreover, the result that child allowance reduces capital stock accumulation and has the negative effect on the fertility rate is obtained by van Groezen and Meijdam (2008).

5. Endogenous Unemployment Model

As presented in this section, we try examining endogenous unemployment. Based on a report by Ono (2010), we consider the following objective function of labor union v_t.

$$v_t = q_t(1-\tau)w_t + (1-q_t)b_t. \tag{19}$$

q_t denotes the endogenous employment rate. Then, subject to the constraint (18) and maximization of (18), the unemployment rate (= 1 - unemployment rate) can be derived as

$$q_t = \frac{(1-\theta)(1-\tau)}{1-\theta+\theta\tau}. \tag{20}$$

We consider three cases: no policy (no unemployment), 2% unemployment rate, and 5% unemployment rate. Table 3 presents the cases using numerical examples.

Table 3: Results of Numerical Examples

	s	n	w	$1+r$	k	b	q
no policy	0.028472	1.000001	0.24067	3.622697	0.028472	0	1
τ =0.0141	0.027119	0.997555	0.238798	3.689296	0.02774	0.164805	0.98
τ =0.0355	0.026913	1.004215	0.240005	3.646148	0.02821	0.417877	0.95

Then, the following proposition can be established.

Proposition 3

An increase in unemployment benefit necessarily increases the unemployment rate. However, the fertility rate is not necessarily reduced. A high level of unemployment benefit raises the fertility rate compared with the case of no policy.

Even if we consider endogenous unemployment, the results of an increase in unemployment benefit are the same with exogenous unemployment.

6. Conclusions

The purpose of this paper is to examine how fertility is determined in a model where income uncertainty exists. Income uncertainty here is regarded as unemployment.

The results of the analysis are as follows. In the case of a small open economy, both child allowance and unemployment benefits have the effect of raising fertility. In addition, although the analysis in this paper is based on a logarithmic utility function, findings demonstrated that the same result is obtainable with a Constant Relative Risk Averse (CRRA) utility function. Furthermore, the analysis for closed economy yields interesting results. The presence of unemployment benefits engenders a lower wage rate through a lower accumulation of capital stock. Results indicate that the presence of unemployment alleviates income shocks caused by uncertainty about future income and therefore increases fertility. However, the lower wage rate caused by the closed economy ultimately reduces fertility. It can also be shown that a higher level of unemployment benefits would engender a higher fertility rate than in the absence of benefits.

The analysis undertaken for this study is rich in policy implications. Unemployment benefits, rather than a child allowance, can increase fertility and can have the same effect as policies to support childcare. This result cannot be derived using a deterministic model in which income uncertainty does not exist, which is the salient contribution of this paper.

References

Apps P. & Rees R. (2004). Fertility, taxation and family policy. Scandinavian Journal of Economics, 106(4), 745-763.

de la Croix D. & Doepke M. (2003). Inequality and Growth: Why Differential Fertility Matters, American Economic Review, 93(4), 1091-1113.

Galor O. & Weil D. N. (1996). The gender gap, fertility, and growth, American Economic Review, 86(3), 374-387.

Hirazawa M. & Yakita A. (2009). Fertility, child care outside the home, and pay-as-you-go social security, Journal of Population Economics, 22(3), 565-583.

Gori L. & Fanti L. (2007). Income taxation, child-rearing policies, fertility and unemployment, Economics Bulletin, 10(19), 1-10.

Oshio T. (2001). Child-care Support, Pension Reform, and Fertility, (Ikujishien・Nenkinkaikaku to Shusshouritsu (in Japanese)), Quarterly Social Security Research (Kikan Shakaihoshou Kenkyu (in Japanese), 36(4), 535-546.

Ono T. (2010) Growth and unemployment in an OLG economy with public pensions, Journal of Population Economics 23, 737–767.

van Groezen B. & Leers T. & Meijdam L. (2003). Social security and endogenous fertility: pensions and child allowances as Siamese twins, Journal of Public Economics, 87(2), 233-251.

van Groezen B. & Meijdam L. (2008). Growing old and staying young: population policy in an ageing closed economy, Journal of Population Economics, 21(3), 573-588.

Yasuoka M. & Miyake A. (2010). Change in the transition of the fertility rate. Economics Letters, 106(2), 78-80.

Yasuoka M. (2014) Child-care policies and pension in an endogenous fertility model, Discussion Paper Series 114, School of Economics, Kwansei Gakuin University, Jan. 2014.

Yasuoka M. (2021) Endogenous Fertility and Social Security, Economic Analysis, 202, 201-219.