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School Choice and Private Tutoring

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Abstract

Many related studies reported in the literature examine how public school education and private school education affect human capital accumulation. Public school education is financed by taxation but private school education is financed by households. Our paper sets a human capital accumulation model with school education and private tutoring and examines how private tutoring affects human capital accumulation. As shown by our paper, because of private tutoring, inequality of human capital accumulation exists in the case of public school education. Moreover, our paper shows that if public school education is complementary with private tutoring, then low income households are unable to choose public school education because of private tutoring costs. Therefore, public school education is not a redistribution policy for low income households.

Keywords: Human capital accumulation, Private tutoring, School education
JEL Classifications: I22, H52

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1. Introduction

The aim of our study is setting a model with both school education investment and private tutoring and then examining how households choose their own level of education investment. Some papers report studies of education investment with school education. Cardak (2004) and Glomm (1997) consider public education and private education while examining school choice. In their model with income inequality, higher-income households select private school education and lower-income households select public school education because no payment is necessary for the latter school education. Public school education is financed by taxation and is equally provided. Therefore, public school education reduces the inequality of human capital accumulation. By contrast, Glomm and Ravikumar (2003) show that public school education expands the inequality of human capital accumulation if certain conditions hold.

Gamlath and Lahiri (2018) set a model by which human capital accumulation is determined not only by the input of public education but also by private education. They examine how the equilibrium is determined. Gamlath and Lahiri (2018) do not consider school choice or income inequality. Bearse, Glomm, and Patterson (2005) consider private tutoring in addition to school education and examine how school choice and political equilibrium with median voting are determined. The human capital of children is therefore determined by parents in these theoretical papers. Empirically, Houtenville and Conway (2008) show that the education effect of the parents is important.

Based on Gamlath and Lahiti (2018), our study sets the dynamics model with school choice and examines how school choice and human capital accumulation are determined. The results are presented hereinafter. First, because of private tutoring, inequality of human capital accumulation exists in the case of public school education. Second, our study shows that, if public school education is complementary with private tutoring, then low income households are unable to choose public school education because of the private tutoring cost. Then, public school education is not a redistribution policy for the low income households. Private school education is demanded for low income households and high income households. Middle income households choose public school education. Therefore, if the government considers public school education as a redistribution policy to reduce the inequality of human capital accumulation, then public school education an inappropriate redistribution policy as long as public school education is complementary to private tutoring.

The remainder of our paper is presented as described hereinafter. Section 2 sets the model with education choice. Section 3 derives the equilibrium of the model. Section 4 concludes our manuscript.

2. The model

In this model, a household chooses public school education and private school education. In addition, the household decides the level of private tutoring.

2.1 Public school education

Human capital accumulation is assumed as

$$h_{i,t+1} = e_{i,t}^{\gamma} h_{i,t}^{\delta}, 0 < \gamma, 0 < \delta, \tag{1}$$

where

$$e_{i,t} = (e_t^G)^{\alpha}\left(e_{i,t}^s + abe_t^G\right)^{1-\alpha}, 0 < a, 0 < \alpha < 1. \tag{2}$$

The function of education investment is assumed by Gamlath and Lahiri (2018). The education attainment level is achieved using both private tutoring $e_{i,t}^s$ and public school education $e_{i,t}^G$. Also, i and t respectively denote the indexes of the household and the time. The case of $0 < b$ shows the substitutive relation between private tutoring and school education. Otherwise, that is the case of $b < 0$, the relation between private tutoring and school education is complementary. Finally, $h_{i,t}$ and $h_{i,t+1}$ respectively denote the human capital stock of parents and of children of the i th household. The human capital of parents is not homogeneous; $h_{i,t}$ is assumed to be distributed uniformly in $\left[\underline{h}_t, \overline{h}_t\right]$.

The utility function of parents $U_{i,t}$ is assumed as

$$U_{i,t} = \beta lnc_{i,t} + (1-\beta) lnh_{i,t+1}, 0 < \beta < 1. \tag{3}$$

This setting is standard form in the field of human capital accumulation. Therein, $c_{i,t}$ represents consumption.

The budget constraint of parents is

$$(1-\tau)h_{i,t} = c_{i,t} + e_{i,t}^s. \tag{4}$$

τ denotes the tax rate for public school education. Next, we consider the optimal household allocations to maximize utility (3) subject to constraints (1), (2) and (4). The optimal household allocations are derived as

$$c_{i,t} = \frac{\beta\left((1-\tau)h_{i,t} + abe_t^G\right)}{\beta + (1-\alpha)(1-\beta)\gamma} \text{ and} \tag{5}$$

$$e_{i,t}^s = \frac{(1-\alpha)(1-\beta)\gamma}{\beta + (1-\alpha)(1-\beta)\gamma}(1-\tau)h_{i,t} - ab\left(1 - \frac{(1-\alpha)(1-\beta)\gamma}{\beta + (1-\alpha)(1-\beta)\gamma}\right)e_t^G. \tag{6}$$

Moreover, (6) should be non-negative. Therefore, if

$$b < \frac{(1-\alpha)(1-\beta)\gamma(1-\tau)h_{i,t}}{a\beta e_t^G}, \tag{7}$$

then the households select no private tutoring:

$$e_{i,t}^s = 0. \tag{8}$$

As shown by (7), when human capital h_t is smaller, the inequality holds: the low income households select no private tutoring.

The education function of the case of $e_{i,t}^{s} > 0$ is shown as

$$e_{i,t} = (e_t^G)^{\alpha}\left(\frac{(1-\alpha)(1-\beta)\gamma\left((1-\tau)h_{i,t} + abe_t^G\right)}{\beta + (1-\alpha)(1-\beta)\gamma}\right)^{1-\alpha}. \quad (9)$$

Human capital accumulation can be presented as

$$h_{i,t+1} = (e_t^G)^{\alpha\gamma}\left(\frac{(1-\alpha)(1-\beta)\gamma\left((1-\tau)h_{i,t} + abe_t^G\right)}{\beta + (1-\alpha)(1-\beta)\gamma}\right)^{(1-\alpha)\gamma} h_{i,t}^{\delta}. \quad (10)$$

In the case of $e_t^s = 0$, the education function and the human capital accumulation are

$$e_{i,t} = (ab)^{1-\alpha}e_t^G. \quad (11)$$

$$h_{i,t+1} = (e_t^G)^{\gamma}(ab)^{(1-\alpha)\gamma}h_{i,t}^{\delta}. \quad (12)$$

2.2 Private school education

In the case of private school education, the households pay for school education. The education function is assumed as

$$e_{i,t} = \left(e_{i,t}^P\right)^{\alpha}\left(e_{i,t}^s + abe_{i,t}^P\right)^{1-\alpha}, 0 < a, 0 < \alpha < 1. \quad (13)$$

In that equation, $e_{i,t}^P$ denotes the investment for private school education. The budget constraint of the case of private school education is

$$(1-\tau)h_{i,t} = c_{i,t} + e_{i,t}^s + e_{i,t}^P. \quad (14)$$

The optimal allocations to maximize the household utility function (3) subject to the constraint (1), (13) and (14) are derived as

$$c_{i,t} = \frac{\beta(1-\tau)h_{i,t}}{\beta + (1-\beta)\gamma}, \quad (15)$$

$$e_{i,t}^P = \frac{\alpha\gamma(1-\beta)}{1-ab}\frac{(1-\tau)h_{i,t}}{\beta + (1-\beta)\gamma}, \text{and} \quad (16)$$

$$e_{i,t}^s = \frac{\gamma(1-\beta)(1-\alpha-ab)}{1-ab}\frac{(1-\tau)h_{i,t}}{\beta + (1-\beta)\gamma}. \quad (17)$$

To be a positive value for $e_{i,t}^P$ and $e_{i,t}^s$, $1-ab > 0$ must hold. In the case of $1-ab < 0$, we derive the following allocations of

$$c_{i,t} = \frac{\beta(1-\tau)h_{i,t}}{\beta + (1-\beta)\gamma} \text{ and} \quad (18)$$

$$e_{i,t}^P = \frac{\gamma(1-\beta)}{1-ab}\frac{(1-\tau)h_{i,t}}{\beta + (1-\beta)\gamma}. \quad (19)$$

$$e_{i,t}^s = 0. \quad (20)$$

3. Equilibrium

This section presents derivation of the equilibrium. First, the government budget constraint of public school education is addressed. Considering the balanced budget constraint, the budget constraint is

$$\tau \int_{\underline{h}}^{\overline{h}} f(h_{i,t}) h_{i,t} dh_{i,t} = e_t^G F(h_t^*). \tag{21}$$

Therein, $f(h_{i,t})$ denotes the density function; $F(h_t^*)$ denotes cumulative distribution function, which shows the share of households which select public school education. In the case of the uniform distribution of $h_{i,t}$, e_t^G is derived as presented below:[1]

$$e_t^G = \frac{\tau}{2} \frac{\left(\overline{h}_t - \underline{h}_t\right)^2}{h_t^*}. \tag{22}$$

We assume $\gamma + \delta = 1$, representing constant returns to scale. Then, human capital accumulation (the growth rate of human capital accumulation of i th household) in the case of public education is shown as

$$\frac{h_{i,t+1}}{h_{i,t}} = \left(\frac{e_t^G}{h_{i,t}}\right)^{\alpha\gamma} \left(\frac{(1-\alpha)(1-\beta)\gamma}{\beta + (1-\alpha)(1-\beta)\gamma} \left((1-\tau) + ab\frac{e_t^G}{h_{i,t}}\right)\right)^{(1-\alpha)\gamma}. \tag{23}$$

In the case of (8)

$$\frac{h_{i,t+1}}{h_{i,t}} = (ab)^{(1-\alpha)\gamma} \frac{e_t^G}{h_{i,t}}. \tag{24}$$

An increase in tax rate τ has two effects. One is the positive effect on human capital accumulation because of an increase in public school education investment. The other is the negative effect on private tutoring because the household disposable income decreases. If the household selects no private tutoring, then this effect does not influence a household which selects no private tutoring.

Considering (3), (5), and (23), the indirect utility function of public school education with $e_t^S > 0$, $V_{i,t}^{Pub,p}$ is

$$V_{i,t}^{Pub,p} = \beta ln \frac{\beta\left((1-\tau) + ab\frac{e_t^G}{h_{i,t}}\right)}{\beta + (1-\alpha)(1-\beta)\gamma}$$

$$+(1-\beta) ln \left(\frac{e_t^G}{h_{i,t}}\right)^{\alpha\gamma} \left(\frac{(1-\alpha)(1-\beta)\gamma}{\beta + (1-\alpha)(1-\beta)\gamma} \left((1-\tau) + ab\frac{e_t^G}{h_{i,t}}\right)\right)^{(1-\alpha)\gamma} \tag{25}$$

$$+ln h_{i,t}.$$

As shown by (7), if human capital stock $h_{i,t}$ is small, then the household does not pay for private tutoring. Defining $\hat{h}_t$ such that the following equation holds, we have

[1] In the case of uniform distribution, the density function is $f(h_t) = \frac{1}{\overline{h}_t - \underline{h}_t}$.

$$b = \frac{\frac{(1-\alpha)(1-\beta)\gamma}{\beta+(1-\alpha)(1-\beta)\gamma}(1-\tau)h_{i,t}}{a\left(1-\frac{(1-\alpha)(1-\beta)\gamma}{\beta+(1-\alpha)(1-\beta)\gamma}\right)e_t^G}. \quad (26)$$

Then, the share of household $\frac{\hat{h}_t-\underline{h}_t}{\overline{h}_t-\underline{h}_t}$ chooses public school education without private tutoring.

Second, we derive the indirect utility function of the case of private school education. Considering (11), (14), and (15), human capital accumulation with private tutoring is derived as

$$\frac{h_{i,t+1}}{h_{i,t}} = (1-\tau)^\gamma \left(\frac{\alpha\gamma(1-\beta)}{1-ab}\frac{1}{\beta+(1-\beta)\gamma}\right)^\alpha \left(\frac{(1-\alpha)(1-\beta)\gamma}{\beta+(1-\beta)\gamma}\right)^{1-\alpha}. \quad (27)$$

Without private tutoring, human capital accumulation is derived as

$$\frac{h_{i,t+1}}{h_{i,t}} = \left(\frac{(ab)^{1-\alpha}\gamma(1-\beta)(1-\tau)}{\beta+(1-\beta)\gamma}\right)^\gamma. \quad (27)$$

Considering (3), (15), and (27), the indirect utility function of the case of private school education with private tutoring, $V_{i,t}^{Pri,p}$ is derived as shown below:

$$V_{i,t}^{Pri,p} = \beta ln\frac{\beta}{\beta+(1-\beta)\gamma}$$
$$+(1-\beta)\gamma ln\left(\frac{\alpha\gamma(1-\beta)}{1-ab}\frac{1}{\beta+(1-\beta)\gamma}\right)^\alpha \left(\frac{(1-\alpha)(1-\beta)\gamma}{\beta+(1-\beta)\gamma}\right)^{1-\alpha} \quad (29)$$
$$+(\beta+(1-\beta)\gamma)ln(1-\tau)+lnh_{i,t}.$$

Considering (3), (18)–(20), the indirect utility function of the case of private school education without private tutoring, $V_{i,t}^{Pri,n}$ is derived as

$$V_{i,t}^{Pri,n} = \beta ln\beta(1-\tau)+(1-\beta)\gamma ln\left((ab)^{1-\alpha}\frac{\gamma(1-\beta)(1-\tau)}{\beta+\gamma(1-\beta)}\right)+lnh_{i,t}. \quad (30)$$

We examine the equilibrium in the two cases $b \geq 0$ and $b < 0$, which respectively represent the substitution and complementary cases.

3.1 Case of $b \geq 0$

With $1-ab > 0$, the households which choose private school education pay for private tutoring: no household chooses private school education without private tutoring. However, with $1-ab < 0$, the households which choose private school education do not pay for private tutoring: there is no household which chooses private school education with private tutoring.

3.1.1 Case of $1-ab > 0$

The case of $1-ab > 0$ shows that substitution between school education and private tutoring is slight. In this case, there exist households of three types.

Type I. Households that choose public school education without private tutoring
Type II. Households that choose public school education with private tutoring
Type III. Households that choose private school education with private tutoring

In the case of $b \geq 0$ and $1 - ab > 0$, substitution between school education and private tutoring is slight. Consequently, the demand for private tutoring in households which choose private school education is positive.

Compare (25) with (29), if the following inequality holds, the households choose public school education with private tutoring. Otherwise, the households choose private school education with private tutoring.

$$\begin{aligned}
&\beta ln \frac{\beta\left((1-\tau)+ab\frac{e_t^G}{h_{i,t}}\right)}{\beta+(1-\alpha)(1-\beta)\gamma} \\
&+(1-\beta)ln\left(\frac{e_t^G}{h_{i,t}}\right)^{\alpha\gamma}\left(\frac{(1-\alpha)(1-\beta)\gamma}{\beta+(1-\alpha)(1-\beta)\gamma}\left((1-\tau)+ab\frac{e_t^G}{h_{i,t}}\right)\right)^{(1-\alpha)\gamma} \\
&> \beta ln \frac{\beta}{\beta+(1-\alpha)(1-\beta)\gamma} \\
&\quad (1-\beta)\gamma ln\left(\frac{\alpha\gamma(1-\beta)}{1-ab}\frac{1}{\beta+(1-\beta)\gamma}\right)^{\alpha}\left(\frac{(1-\alpha)(1-\beta)\gamma}{\beta+(1-\beta)\gamma}\right)^{1-\alpha} \\
&+(\beta+(1-\beta)\gamma)ln(1-\tau)
\end{aligned} \tag{31}$$

The left-hand-side of (31) decreases with increased human capital stock $h_{i,t}$. The right-hand-side of (31) is constant over time, irrespective of human capital accumulation. Therefore, we can obtain the human capital stock h_t^* such that $V_{i,t}^{Pri,p} = V_{i,t}^{Pub,p}$ holds. With $h_t^* > h_{i,t}$, households choose public school education. Otherwise, households choose private school education, as shown in Fig. 1. The share of $\frac{\widehat{h}_t - \underline{h}_t}{\overline{h}_t - \underline{h}_t}$ denotes those households which choose public school education without private tutoring (Type I). The share of $\frac{h_t^* - \widehat{h}_t}{\overline{h}_t - \underline{h}_t}$ denotes those households which choose the public school education with the private tutoring (Type II). Also, the share of $\frac{\overline{h}_t - h_t^*}{\overline{h}_t - \underline{h}_t}$ denotes those households which choose private school education without private tutoring (Type III). Then, the following proposition can be established.

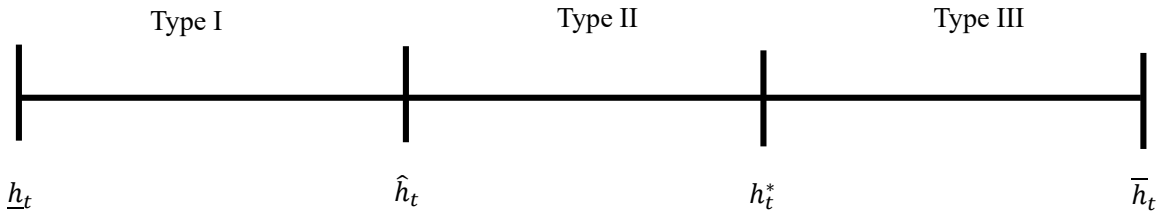

Fig. 1 Shares of households.

Proposition 1

We assume the case of $b \geq 0$ and $1 - ab > 0$. There exist households of three types: the share of households $\frac{\hat{h}_t - \underline{h}_t}{\overline{h}_t - \underline{h}_t}$ choosing public school education without private tutoring, the share of households $\frac{h_t^* - \hat{h}_t}{\overline{h}_t - \underline{h}_t}$ choosing public school education with private tutoring, and the share of $\frac{\overline{h}_t - h_t^*}{\overline{h}_t - \underline{h}_t}$ choosing private school education with private tutoring.

3.1.2 Case of $1 - ab < 0$

The case of $1 - ab < 0$ shows that the substitution between school education and the private tutoring is large. In the case of $b \geq 0$ and $1 - ab < 0$, there exist households of three types:

Type I. Households that choose public school education without private tutoring
Type II. Households that choose public school education with private tutoring
Type IV. Households that choose private school education without private tutoring

Comparison of (25) with (29) demonstrates that, if the following inequality holds, then households choose public school education with private tutoring. Otherwise, households choose private school education with private tutoring.

$$\beta ln \frac{\beta\left((1-\tau) + ab\frac{e_t^G}{h_{i,t}}\right)}{\beta + (1-\alpha)(1-\beta)\gamma} + (1-\beta) ln\left(\frac{e_t^G}{h_{i,t}}\right)^{\alpha\gamma}\left(\frac{(1-\alpha)(1-\beta)\gamma}{\beta + (1-\alpha)(1-\beta)\gamma}\left((1-\tau) + ab\frac{e_t^G}{h_{i,t}}\right)\right)^{(1-\alpha)\gamma} \tag{32}$$

$$> \beta ln\beta(1-\tau) + (1-\beta)\gamma ln\left((ab)^{1-\alpha}\frac{\gamma(1-\beta)(1-\tau)}{\beta+\gamma(1-\beta)}\right)$$

The left-hand-side of (32) decreases with an increase in human capital stock $h_{i,t}$. The right-hand-side of (32) is constant over time, irrespective of human capital accumulation. Therefore, one can obtain the human capital stock h_t^* such that $V_{i,t}^{Pri,p} = V_{i,t}^{Pub,n}$ holds. With $h_t^* > h_{i,t}$, the households choose public school education. Otherwise, households choose private school education. Then, the share of $\frac{\hat{h}_t - \underline{h}_t}{\overline{h}_t - \underline{h}_t}$ denotes those households which choose public school education without private tutoring (Type I). The share of $\frac{h_t^* - \hat{h}_t}{\overline{h}_t - \underline{h}_t}$ denotes those households which choose public school education with private tutoring (Type II). Also, the share of $\frac{\overline{h}_t - h_t^*}{\overline{h}_t - \underline{h}_t}$ denotes those households which choose private school education without private tutoring (Type IV) as shown in Fig. 2. Then, the following proposition can be established.

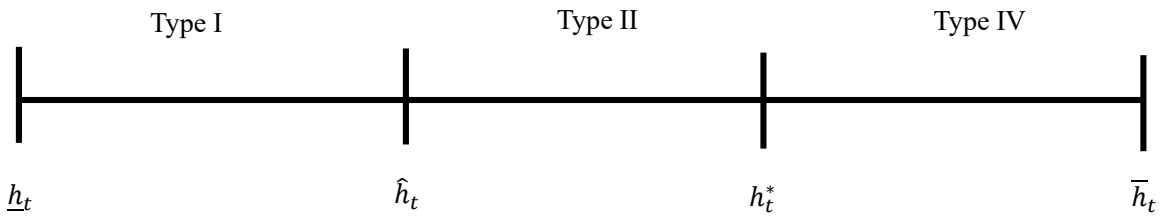

Fig. 2 Shares of households.

Proposition 2

We assume the case of $b \geq 0$ and $1 - ab < 0$. There exist households of three types: households $\frac{h_t^* - \hat{h}_t}{\overline{h}_t - \underline{h}_t}$ choosing public school education without private tutoring, households $\frac{h_t^* - \hat{h}_t}{\overline{h}_t - \underline{h}_t}$ choosing public school education with private tutoring, and households $\frac{\overline{h}_t - h_t^*}{\overline{h}_t - \underline{h}_t}$ choosing private school education without private tutoring.

In the case of $b \geq 0$ and $1 - ab < 0$, substitution between school education and private tutoring is large. Then, the household reduces demand for private tutoring.

3. 2 Case of $b < 0$

If $b < 0$, then the households pay for private tutoring: $e_{i,t}^s > 0$. Then, no household chooses public

school education without private tutoring. There are households of two types: one for public school education with private tutoring and the other for private school education with private tutoring. That is, households of two types exist, as shown by the following.

Type II. Households that choose public school education with private tutoring
Type III. Households that choose private school education with private tutoring

Therefore, as shown by (6), the demand for the private tutoring must be positive. If the household is of low income, that is, of low human capital stock $h_{i,t}$, then the household is unable to choose public school education because of the private tutoring cost. Then, the share of $\frac{\hat{h}_t-\underline{h}_t}{\overline{h}_t-\underline{h}_t}$ households chooses private school education with private tutoring. Low-income households reduce private school education to decrease private tutoring.

From comparison of (25) with (29), where inequality (31) holds, the households choose public school education with private tutoring. Otherwise, the households choose private school education with private tutoring. The shares of households are given respectively as $\frac{h_t^*-\hat{h}_t}{\overline{h}_t-\underline{h}_t}$ (Type II) and $\frac{\overline{h}_t-h_t^*}{\overline{h}_t-\underline{h}_t}$ (Type III), as depicted in Fig. 3. Then, the following proposition can be established.

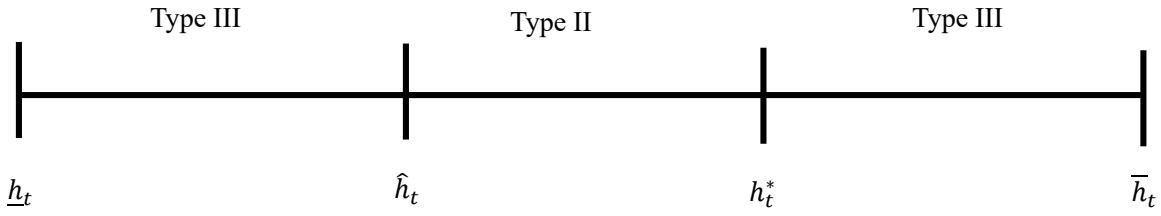

Fig. 3 Shares of households.

Proposition 3

We assume the case of $b < 0$. There exist households of two types: households $\frac{h_t^*-\hat{h}_t}{\overline{h}_t-\underline{h}_t}$ choosing the public school education with private tutoring, and households $\frac{\overline{h}_t-h_t^*}{\overline{h}_t-\underline{h}_t}$ and $\frac{\hat{h}_t-\underline{h}_t}{\overline{h}_t-\underline{h}_t}$ choosing private school education without private tutoring.

In the case of $b < 0$, the low-income households choose private school education to reduce private

tutoring. Public school education is regarded as a redistribution policy for low income households. However, as long as public school education is complementary with private tutoring, public school education is not chosen by low income households. Then, public school education is not a redistribution policy for low income households.

We consider how substitution parameter b affects human capital accumulation. As shown by (23), (24), and (27), an increase in b raises human capital accumulation because an increase in b raises the marginal productivity of school education. However, with $b < 0$, that is, the complementary case, human capital accumulation is reduced compared with the substitution case because of $b < 0$.

4. Conclusions

Our paper sets the human capital accumulation model with school education and private tutoring and examines how private tutoring affects human capital accumulation. As shown by the findings from our study, inequality of human capital accumulation exists in the case of public school education because of private tutoring. Moreover, our paper shows that if public school education is complementary with private tutoring, then low income households are unable to choose public school education because of the costs of private tutoring. Then, public school education is not a redistribution policy for low income households. Private school education is demanded for low income households and high income households. Middle income households choose public school education. Therefore, if the government considers public school education as a redistributive policy to reduce the inequality of human capital accumulation, then public school education is an inappropriate redistribution policy as long as public school education is complementary with private tutoring. This case does not occur in the case of substitution between public school education and private tutoring.

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