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Demand for Education Investment in A Model with Uncertainty

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Demand for Education Investment in A Model with Uncertainty[†]

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Abstract

Many reports of the relevant literature describe studies of education investment. As undertaken by Glomm and Ravikumar (1992), we also consider public and private school education in the study described herein. However, different from the related literature, our model setting incorporates private tutoring and uncertainty about the productivity of human capital accumulation. Based on Bearse, Glomm and Patterson (2005), our paper presents consideration of the uncertainty about education results and presents examination of how demand for education investment is determined. Results show that, because of uncertainty about education results, demand for education investment is less than in a case of no uncertainty.

Keywords: education, uncertainty

JEL Classifications: I22, H52

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1. Introduction

Many studies have examined education investment. Glomm and Ravikumar (1992) presented the fundamental paper on this topic after conducting a study with a human capital accumulation model that incorporated education investment. By that study, Glomm and Ravikumar (1992) show that private education, where parents pay education costs for their children, is associated with a higher human capital growth rate and greater inequality than in a case with public education. Glomm and Ravikumar (1992) assume a model with a Cobb–Douglas human capital accumulation function and a logarithm utility function. Glomm and Ravikumar (1992), Glomm and Kaganovich (2003), Cardak (2004) and others assume a logarithmic utility function. By virtue of these assumptions, the result can be derived easily. As one might expect, some studies have sought avoidance of the assumptions of a Cobb–Douglas human capital accumulation function and a logarithm utility function. Glomm (1997), Glomm and Ravikumar (2003), and Bearse, Glomm and Patterson (2005) assume a Constant Relative Risk Averse (CRRA) utility function. Moreover, Bearse, Glomm and Patterson (2005) assume a Constant Elasticity of Substitution (CES) human capital accumulation function.

The aim of our study is to set a model with uncertainty for education investment and to examine how demand for education investment is determined. Some papers incorporate uncertainty about education. Oshio and Yasuoka (2009), Andersson and Konrad (2002), and Brodaty, and Gary-Bobo and Prieto (2014), based on the expected income that can be gained from education investment, incorporate uncertainty of education and household consideration of demand for education investment. One can consider a utility function by which an increase in the income level, but not an increase in the human capital accumulation level, raises the utility level. For such a function, a redistributive policy with a lump-sum transfer, as considered by Boadway, Marceau and Marchand (1992), reduces demand for education investment.

Although some studies examine how education uncertainty affects demand for education investment, no model exists for uncertainty of education in a model incorporating school education and private tutoring. Based on work by Bearse, Glomm and Patterson (2005), who consider both school education and private tutoring, we examine how education uncertainty affects demand for additional levels of education. Compared with the no-uncertainty model, uncertainty reduces demand for private tutoring in the public school education model. In the case of private school education, demand for private school education decrease.

The remainder of this paper is presented with an explanation of the basic model in Section 2, and with an examination of how education uncertainty affects education investment. Section 3 explains school choice. Section 4 concludes our paper.

2. Model

As described herein, we demonstrate how demand for education investment is determined. As education investment, one can consider school education (public school education, private school education) and private tutoring. We consider school education of two types: public school education and private school education.

2.1 Public school education

This subsection presents consideration of the public-school education case. The preference for consumption and education for children can be shown as

$$u_t = \frac{c_t^{1-\gamma}}{1-\gamma} + E_t \frac{h_{t+1}^{1-\gamma}}{1-\gamma}, 0 < \gamma \tag{1}$$

where c_t and h_{t+1} respectively stand for the consumption of the parents and the human capital stock of children. The utility function assumed for this study is not a logarithm utility function but a Constant Relative Risk Averse (CRRA) utility function.

Next, we consider the Constant Elasticity of Substitution (CES) function as the human capital accumulation function. First, we consider the case of public school education. Then, the human capital accumulation function is assumed as shown below.

$$\begin{cases} h_{t+1}^H = (1+x)h_{t+1}^M & & \pi \\ h_{t+1}^M = A\left(\alpha e_t^\rho + (1-\alpha)E_t^{u\rho}\right)^{\frac{1}{\rho}} & \text{with probability} & 1-\pi \\ h_{t+1}^L = (1-x)h_{t+1}^M & & \pi \end{cases} \tag{2}$$

We assume that $\rho < 1$, $0 < \alpha < 1$, $0 < x < 1$, and $0 < \pi < 1$. This setting is similar to that used by Lord and Rangazas (1998). x denotes the ability of education. x is known by the household ex ante. However, the household can not know whether the children have the ability of education, or not. The household considers the type of children with probability. e_t and E_t^u respectively denote private tutoring and public school education.

Public school education is financed by taxation. Parents need not pay for school education. Nevertheless, they must pay for education investment for private tutoring. Therefore, the budget constraint of parents is

$$c_t + e_t = (1-\tau)h_t. \tag{3}$$

In that equation, τ and h_t respectively denote the tax rate for financing public school education and the human capital stock of parents, which denotes the labor income of parents.

The optimal demand for private tutoring e_t to maximize utility (1) subject to constraints (2) and (3) is given to satisfy the following equation.

$$\begin{aligned}(\pi(1+x)^{1-\gamma} + (1-2\pi) + \pi(1-x)^{1-\gamma})A^{1-\gamma}\left(\alpha e_t^\rho + (1-\alpha)E_t^{u\rho}\right)^{\frac{1-\gamma}{\rho}-1}\alpha e_t^{\rho-1} \\ = \left((1-\tau)h_t - e_t\right)^{-\gamma}\end{aligned} \tag{4}$$

With $\frac{1-\gamma}{\rho} < 1$, the left-hand side of (4) decreases with an increase in e_t. The right-hand side of (4) increases with an increase in e_t.[1] Subsequently, one can obtain the unique solution of e_t shown by the following figure.

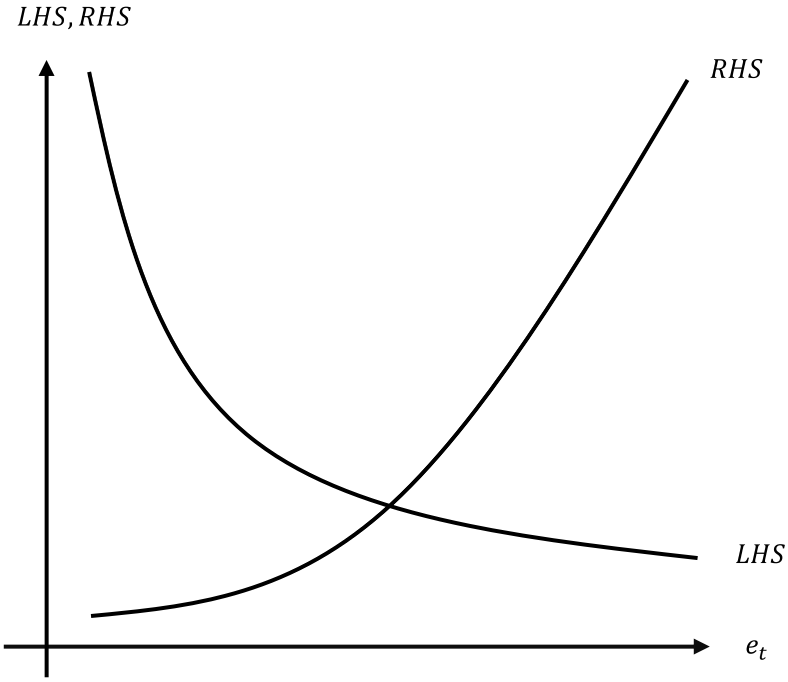

Fig. 1 Optimal demand for private tutoring.

Therein, the LHS and RHS respectively denote the left-hand side of (4) and the right-hand side of (4).

The difference between work presented by Bearse, Glomm and Patterson (2005) and our work is the examination of the uncertainty of education results, which is human capital accumulation. Bearse, Glomm and Patterson (2005) do not consider uncertainty about human capital accumulation. By contrast, we address uncertainty of the human capital accumulation and particularly examine how education investment is changed by uncertainty.

First, we examine the effects of π. With $\pi = 0$, no uncertainty about human capital accumulation exists. We consider the case of $\pi > 0$. Then we calculate $\frac{dLHS}{d\pi}$ of the left-hand side

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[1] We calculate $\frac{dLHS}{de_t}$ of the left-hand side of (4); thereby, we obtain $(\pi(1+x)^{1-\gamma} + (1-2\pi) + \pi(1-x)^{1-\gamma})A^{1-\gamma}\alpha\left((\rho-1)\left(\alpha e_t^{\rho} + (1-\alpha)E_t^{u\rho}\right)^{\frac{1-\gamma}{\rho}-1} e_t^{\rho-2} + \left(\frac{1-\gamma}{\rho}-1\right)\left(\alpha e_t^{\rho} + (1-\alpha)E_t^{u\rho}\right)^{\frac{1-\gamma}{\rho}-2} e_t^{\rho-1}\right)$. Actually, $(\rho-1)$ in the first term of the bracket is always negative. Also, $\left(\frac{1-\gamma}{\rho}-1\right)$ in the second term of the bracket is negative if $\frac{1-\gamma}{\rho} < 1$.

of (4). If the following inequality holds, then the slope of LHS shifts down and the demand for private tutoring e_t decreases. Also, the households increase consumption c_t.

$$(1+x)^{1-\gamma} + (1-x)^{1-\gamma} < 2. \qquad (5)$$

If one considers the logarithm utility function to set $\gamma = 1$, then the change of π does not affect private tutoring e_t.

Second, we examine the effects of x. An increase in x leads to an increase in the variance of the human capital accumulation. We calculate $\frac{dLHS}{dx}$ of the left-hand side of (4). If the following inequality holds, then the LHS slope shifts down; also, demand for private tutoring e_t decreases. The households increase consumption c_t.

$$\pi(1-\gamma)((1+x)^{-\gamma} + (1-x)^{-\gamma}) < 0. \qquad (6)$$

The sign of the left-hand side of (6) is determined by parameter γ. As long as $\gamma > 1$, the inequality (6) holds and the LHS curve shifts down.

Consequently, we can establish the following proposition.

Proposition 1

In public school education, with $(1+x)^{1-\gamma} + (1-x)^{1-\gamma} < 2$, an increase in π reduces private tutoring e_t. With $\gamma > 1$, an increase in x reduces private tutoring e_t.

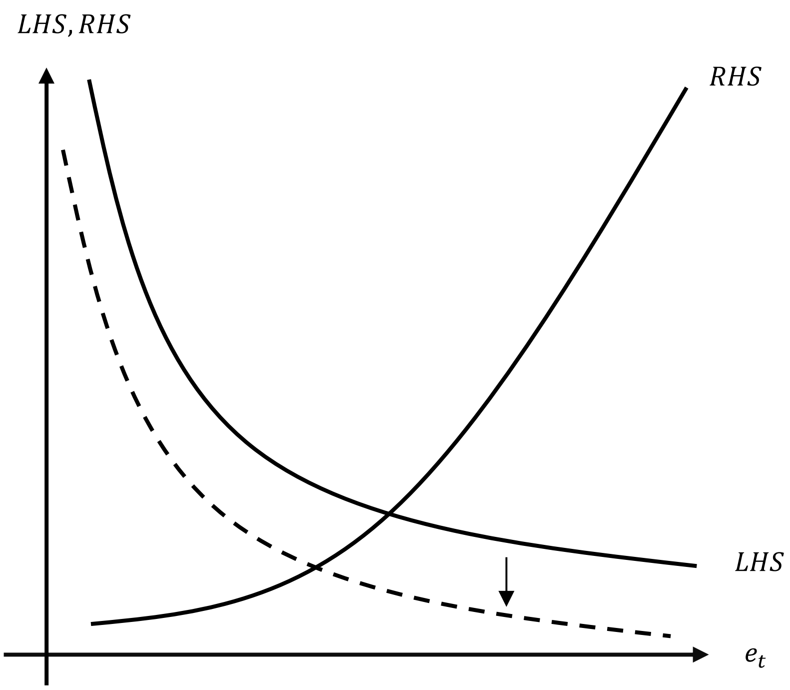

Fig. 2 Demand for private tutoring and uncertainty.

If one considers the logarithm utility function to set $\gamma = 1$, then the uncertainty does not affect private tutoring. The logarithm utility function is assumed in many related papers because of the tractability for the model. The results produced by our study show attention for the model setting if uncertainty is considered.

2.2 Private school education

This subsection presents consideration of optimal household allocation in the case of private school education. In the case of private school education, parents pay the fee for private school education. Human capital accumulation is assumed as[2]

$$\begin{cases} h_{t+1}^{H} = (1+x)h_{t+1}^{M} \\ h_{t+1}^{M} = AE_t^r \\ h_{t+1}^{L} = (1-x)h_{t+1}^{M} \end{cases} \quad \text{with probability} \quad \begin{matrix} \pi \\ 1-\pi \\ \pi \end{matrix} \tag{7}$$

where E_t^r denotes the education investment for private school education. The budget constraint of the parents is

$$c_t + E_t^r = (1-\tau)h_t. \tag{8}$$

The optimal allocations of private school education maximize utility function (1) subject to constraints (7) and (8). Private tutoring is shown as

$$E_t^r = \frac{(1-\tau)h_t}{1 + X^{-\frac{1}{\gamma}} A^{-\frac{1-\gamma}{\gamma}}}. \tag{9}$$

where X is

$$X = (\pi(1+x)^{1-\gamma} + (1-2\pi) + \pi(1-x)^{1-\gamma}). \tag{10}$$

An increase in X raises demand for the private school education E_t^r. If inequality (5) holds, then the level of $(\pi(1+x)^{1-\gamma} + (1-2\pi) + \pi(1-x)^{1-\gamma})$ decreases and private school education E_t^r decreases.

$$(1+x)^{1-\gamma} + (1-x)^{1-\gamma} < 2. \tag{5}$$

In addition, if the inequality (6) holds, then the level of $(\pi(1+x)^{1-\gamma} + (1-2\pi) + \pi(1-x)^{1-\gamma})$ decreases; then private school education E_t^r decreases.

$$\pi(1-\gamma)((1+x)^{-\gamma} + (1-x)^{-\gamma}) < 0. \tag{6}$$

Therefore, we can establish the following proposition.

Proposition 2

For private school education, with $(1+x)^{1-\gamma} + (1-x)^{1-\gamma} < 2$, an increase in π reduces private school education E_t^r. With $\gamma > 1$, an increase in x reduces private school education E_t^r.

3. School choice

[2] This setting is based on Bearse, Gloom and Patterson (2005).

We consider an income distribution by which the cumulative distributive function is $F(h_t)$, with the income distribution range of $[\underline{h}_t, \overline{h}_t]$. Considering (1)–(4), we can obtain the indirect utility function of public school education v_t^{Pub}. The indirect utility function of private school education v_t^{Pri} is obtained using (1), (8) and (9). As long as $v_t^{Pri} > v_t^{Pub}$, the household chooses a private school. However, if household income h_t is low, then the low-income household chooses public school education because of a lack of means to pay for a school education. Therefore, we can obtain $\tilde{h}_t$ such that $v_t^{Pri} = v_t^{Pub}$. Then, the share of $F(\tilde{h}_t)$ chooses the public school education and the share of $1 - F(\tilde{h}_t)$ chooses private school education.

How does probability π affect school choice? As long as (5) holds, the preference for education investment decreases. Then, the household reduces the payment for education investment. Therefore, to cut education investment, households which choose public school education increase: that is, $\tilde{h}_t$ decreases. Demand for private tutoring is decreased by an increase in π because of the associated decrease in preference for education investment.

Bearse, Glomm and Patterson (2005) derive the political equilibrium with a median voter. Our study can also obtain the political equilibrium with a median voter. However, the results are substantially the same as those of Bearse, Glomm and Patterson (2005). Bearse, Glomm and Patterson (2005) derive the condition to have a political equilibrium with a median voter. This condition is the same as that used for our study.

4. Conclusions

This study sets a model of education investment with uncertainty about education productivity. Concretely, the probability distribution of human capital accumulation ability is considered. Because of uncertainty about ability, parents reduce their demands for education investment compared to the no-uncertainty model. Therefore, education investment can be less than the social optimal level. Then an increase in investment for public school education can be supported. Bearse, Glomm and Patterson (2005) derive the political equilibrium with the median voter theorem. In our study based on Bearse, Glomm and Patterson (2005), we can obtain the political equilibrium with the median voter theorem for the level of public school education, as shown by Bearse, Glomm and Patterson (2005).

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