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Abstract

To investigate the macroeconomic effects of capital account liberalization, we apply a dynamic general equilibrium model with two production sectors. In contrast to the literature on belief-driven sunspot fluctuations caused by production externalities, our model does not assume any production externalities. In our model, agents face financial constraints and production heterogeneity. The financial constraints and agents' production heterogeneity are sources of dynamic inefficiency. Although indeterminacy of equilibrium and belief-driven sunspot fluctuations never occur in the closed economy, dynamic inefficiency combined with a negative foreign asset in the steady state produces indeterminacy in the small open economy if financial constraints are fully relaxed under the condition that the investment goods sector is more labor intensive than the consumption goods sector.

Keywords: Two-sector growth model; small open economy; financial constraints; heterogeneous agents; dynamic inefficiency; indeterminacy.

JEL Classification Numbers: E32; F36; F43; O41

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1 Introduction

One of the central issues in international macroeconomics is a potential risk of capital account liberalization. Until the mid 1990s, most mainstream economists recommended capital account liberalization, believing that it was an essential process of economic development. Not just mainstream economists but also the International Monetary Fund (IMF) advocated for the liberalization of capital movement around globe as one of the IMF's purposes (Prasad and Rajin 2008). Economic history, however, revealed that countries that had opened their domestic financial markets in the mid 1980-1990s experienced a surge of capital inflow and outflow that exacerbated economic fluctuations and often caused financial crises (Stiglitz 2004; Ocampo et al. 2008). Motivated by these historical observations, we present a dynamic general equilibrium model with two production sectors in which opening up the domestic financial market to the international financial market can cause belief-driven fluctuations.

In our two-sector model, any production externalities are not assumed to yield belief-driven fluctuations, in contrast to existing studies including Lahiri (2001), Weder (2001), Nishimura and Shimomura (2002a), and Meng and Velasco (2004) that address belief-driven fluctuations in small open economies with two production sectors. Although the two production sectors produce consumption and investment goods as in the models of these studies, the source of inefficiency lies in financial frictions and the production heterogeneity between agents in our model, and these two factors are essential to the occurrence of belief-driven fluctuations. In each period, each agent receives an idiosyncratic productivity shock and optimally chooses whether to become a producer of intermediate goods borrowing in the financial market (henceforth called a producer or a borrower for short) or a lender depending on the agent's productivity shock. Agents who draw higher productivity borrow in the financial market and become producers, and agents who draw lower productivity become lenders. As such, borrowers and lenders endogenously appear in equilibrium. Whereas lenders acquire only the market interest rate on their savings in the financial market, producers can borrow only up to a certain proportion of their own funds, but they can obtain a return on their investment greater than the market interest rate.

Because the availability of consumption and production resources in each period is limited in a closed economy, the optimal allocation of these resources is uniquely determined in equilibrium, and thus, equilibrium is determinate in a closed economy. However, once the capital account is liberalized and the economy becomes a small open economy, agents can borrow and lend at the world interest rate in both domestic and international financial markets. Therefore, if the world interest rate is sufficiently low, agents can borrow from abroad (subject to the extent of financial constraints) at a low cost to produce intermediate goods if they draw high productivity. This means that a small open economy does not face a resource constraint, and thereby indeterminacy of equilibrium and belief-driven fluctuations can occur in a small open economy. Moreover, belief-driven fluctuations are more likely to occur when financial market imperfections are sufficiently resolved. In other words, when financial constraints are fully softened, opening up the domestic financial market to the international market may destabilize the economy. This theoretical implication is consistent with warnings discussed in Stiglitz (2004) and Ocampo et al. (2008) regarding a potential risk of capital account liberalization.

For over two decades, indeterminacy of equilibrium and belief-driven sunspot fluctuations in dynamic general equilibrium models have been studied by many researchers.[1] Extrinsic random variables, called sunspots, do not directly affect economic fundamentals but can impact agents' expectations (Shell, 1977; Azariadis, 1981; Cass and Shell, 1983). An equilibrium is called a sunspot equilibrium if the resource allocation in equilibrium is subject to the realization of a sunspot variable. Belief-driven sunspot fluctuations can occur when indeterminacy of equilibrium arises with extrinsic uncertainty randomizing multiple equilibria. In this case, a sunspot equilibrium can be constructed as a rational expectations equilibrium (Chiappori and Guesnerie, 1991; Benhabib and Farmer, 1999).

[1]For the theoretical work in this literature, see, for instance, Benhabib and Farmer (1994, 1996), Borldrin and Rustichini (1994), Benhabib and Nishimura (1998), Benhabib et al. (2000), Nishimura and Venditti (2004, 2007), and Dufourt, Nishimura and Venditti (2015).

Although many studies in the literature on indeterminacy of equilibrium in dynamic general equilibrium models have investigated the local dynamics in a closed economy, some earlier works also have considered a small open economy. Lahiri (2001) studies an endogenous growth model in a small open economy and obtains indeterminate growth paths in equilibrium. In his model, the economy faces the perfect international financial market and the accumulation of human capital characterizes the increasing returns to scale for the production technology. Weder (2001) and Meng and Velasco (2004) investigate a small open economy, which is open to the perfect international financial market, where two production sectors with production externalities operate and derive indeterminacy of equilibrium. In Weder's model, the production technologies exhibit increasing returns to scale, whereas in Meng and Velasco's model, the production technologies exhibit constant returns to scale. Nishimura and Shimomura (2002a) also study a small open economy with two production sectors in which there is no international financial market but there is an international consumption goods market. In their model, the production technologies exhibit constant returns to scale, and they assume endogenous subjective discount rates.[2] In the abovementioned models, production externalities are assumed, without which indeterminacy cannot occur, and it is more likely to occur when the period utility approaches a linear function. Our study departs from the literature in three crucial respects while employing linear period utility. First, any production externalities are not assumed, second, heterogeneity in productivity between agents is assumed, and third, agents in the economy face financial constraints. The financial constraint and agents' production heterogeneity are a source of dynamic inefficiency, which never arises in the models of Lahiri (2001), Weder (2001), and Meng and Velasco (2004) because the subjective discount rate is equal to the world interest rate in their models. Dynamic inefficiency combined with a negative foreign asset in the steady state can

[2]Nishimura and Shimomura (2002b) and Hu and Mino (2013) investigate a two-country model with two production sectors in which production externalities are present and show that indeterminacy can arise under the moderate parameter conditions. Nishimura and Shimomura (2006) and Bond, Iwasa and Nishimura (2011) also derive indeterminacy of equilibrium in a two-country model with two production sectors. In their models, international lending and borrowing are not allowed, but there are no production externalities.

produce indeterminacy of equilibrium in the small open economy. Schmitt-Grohé and Uribe (2021) also investigate an infinite-horizon small open economy with two sectors and derives a sunspot equilibrium. In contrast with our model, they consider an endowment economy with tradable and nontradable goods. In their model, multiple equilibria occur because not only nontradable goods but also tradable goods serve as collateral and accordingly borrowing constraints become softer as the economy becomes more indebted. This is a different mechanism from ours for indeterminacy equilibrium to occur.

The remainder of this paper proceeds as follows. In the next section, a basic model is presented, which can be applied to both closed and small open economies. In Section 3, market clearing conditions are discussed, and in Section 4, we construct aggregate variables in equilibrium. In Section 5, we derive the local dynamics in a closed economy as a benchmark, and in Section 6, the local dynamics in a small open economy is investigated. In Section 7, concluding remarks are presented.

2 Model

An economy consists of an infinitely lived representative firm and infinitely lived agents, whose population is equal to 1, and continues in discrete time from time 0 to $+\infty$. The representative firm produces both consumption and investment goods. The infinitely lived agents have potential investment opportunities to produce (country-specific) intermediate goods from the investment goods. They receive uninsured idiosyncratic productivity shocks in each period.

In both cases of closed and small open economies, consumption goods are the numeraire throughout the current analysis, and the financial trade of borrowing and lending is denominated in units of consumption goods. In particular, in the case of a small open economy, all the financial trades in the international financial market are performed using only consumption goods. In other words, consumption goods are tradable, and investment and intermediate goods are nontradable.

2.1 Agents

2.1.1 Timing of events

At the beginning of period t, agents have not yet faced the idiosyncratic productivity shocks. Each agent earns a wage income by supplying one unit of labor inelastically. She also acquires a return on her savings at that time. The market for consumption goods in period t is opened at the beginning of the period and closed before the idiosyncratic productivity shock is realized. Therefore, an agent must make a decision about consumption and saving at the beginning of period t without knowing her productivity in the production of intermediate goods.

At the end of period t, the idiosyncratic productivity shock in period $t+1$ is realized. Each agent can utilize two saving methods: one is lending her savings in the financial market and the other is initiating the production of intermediate goods. An agent optimally chooses one of the two saving methods knowing her productivity. Lending one unit of savings in the financial market in period t yields a claim to r_{t+1} units of consumption goods in period $t+1$, where r_{t+1} is the gross interest rate, whereas purchasing one unit of investment goods at price p_t creates Φ_t units of intermediate goods, which are sold at price q_{t+1} to the production sector in period $t+1$. Although agents can borrow in the financial market when they produce intermediate goods, they face a financial constraint and can borrow up to a certain proportion of their own funds.

Productivity Φ_t is a random variable defined on a probability space $(\Omega, \mathcal{B}, P)$, where $\Omega := [0, 1]$ is a sample space, $\mathcal{B}$ is a Borel σ-field on Ω, and P is the probability measure. Φ_t is a function of $\omega_t \in \Omega$. No one can insure against low productivity, which means that there is no insurance market for the productivity shocks. Ω^t is a Cartesian product of t copies of Ω. Denote the history of ω_t by $\omega^{t-1} = \{\omega_0, \omega_1, ..., \omega_{t-1}\}$. Then, ω^{t-1} is an element of Ω^t. Φ_0, Φ_1,..., are independent and identically distributed across both agents and periods (the i.i.d.

assumption), and thus, $\Phi_t(\omega_t)$ is an idiosyncratic productivity shock. Because the measure of the agent population is equal to one and because of the i.i.d. assumption, an individual who experiences the history $\omega^{t-1} = \{\omega_0, \omega_1, ..., \omega_{t-1}\}$ can be denoted by ω^{t-1}. The support of Φ_t is $[d, \eta]$ where $d, \eta \in \mathbb{R}_+ \cup \{+\infty\}$. The cumulative distribution function of Φ_t is given by $G(\Phi) := P(\{\omega_t \in \Omega | \Phi_t(\omega_t) \leq \Phi\})$, which is time-invariant and continuously differentiable on the support, where $\{\omega_t \in \Omega | \Phi_t(\omega_t) \leq \Phi\} \in \mathcal{B}$.

2.1.2 Utility maximization

An agent solves the following maximization problem for her lifetime utility:

$$\max E \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} c_\tau(\omega^{\tau-1}) \middle| \, \omega^{t-1} \right]$$

subject to

$$c_\tau(\omega^{\tau-1}) + s_\tau(\omega^{\tau-1}) = q_\tau \Phi_{\tau-1}(\omega_{\tau-1}) x_{\tau-1}(\omega^{\tau-1}) + r_\tau b_{\tau-1}(\omega^{\tau-1}) + w_\tau \tag{1}$$

$$b_\tau(\omega^\tau) \geq -\theta s_\tau(\omega^{\tau-1}) \tag{2}$$

$$x_\tau(\omega^\tau) \geq 0 \qquad , \tag{3}$$

for $\tau \geq t$, where $\beta \in (0, 1)$ is the subjective discount factor, $c_\tau(\omega^{\tau-1})$ is consumption, w_τ is a wage income, and $E[.|\omega^{t-1}]$ is an expectation operator given the history ω^{t-1}. In Eq. (1), $s_\tau(\omega^{\tau-1}) := b_\tau(\omega^\tau) + p_\tau x_\tau(\omega^\tau)$ is the agent's savings in period τ, where $b_\tau(\omega^\tau)$ is lending if $b_\tau(\omega^\tau) > 0$ and borrowing if $b_\tau(\omega^\tau) < 0$, $x_\tau(\omega^\tau)$ is investment goods used for the production of intermediate goods, and p_τ is the price of investment goods. One can call p_τ the real exchange rate when the economy is a small open economy.

For the production of intermediate goods, a linear technology with respect to investment goods is assumed as $\Phi_{\tau-1}(\omega_{\tau-1}) x_{\tau-1}(\omega^{\tau-1})$, which is intermediate goods used as input for the two final production sectors in period τ. As previously discussed, Eq. (1) implies that when the agent makes a decision in period t on consumption and saving, she does not know her productivity $\Phi_t(\omega_t)$. However, note from the expression for $s_t(\omega^{t-1}) = b_t(\omega^t) + p_t x_t(\omega^t)$ that she knows $\Phi_t(\omega_t)$ when she makes a portfolio decision about investment, lending, and/or borrowing in period t. While Eq. (1) is the flow budget constraint that is effective for $\tau \geq 1$, the flow budget constraint in period 0 is given by $c_0 + s_0 = q_0 K_0 + w_0$ where K_0 is the initial endowment of intermediate goods that is common across agents.

Inequality (2) is the financial constraint that the agent faces in period τ.[3] An agent can borrow in the financial market only up to a limited proportion of her savings, which are her own funds. The extent of financial constraints is given by $\theta \in (0, \infty)$. The smaller θ, the more severe is the financial constraint. Inequality (2) can be rewritten as $b_\tau(\omega^\tau) \geq -\mu p_\tau x_\tau(\omega^\tau)$ where $\mu = \theta/(1+\theta) \in (0,1)$. Because this constraint is more convenient than inequality (2), we use it henceforth. As μ goes to 1, the financial market approaches perfection, and as μ goes to zero, agents are unable to borrow in the financial market. The purchase of investment goods should be nonnegative, and thus, inequality (3) is imposed.

2.1.3 Optimal portfolio within a period

Letting $\phi_t := p_t r_{t+1}/q_{t+1}$, one notes that knowing the productivity, Φ_t, agents who draw $\Phi_t > \phi_t$ optimally borrow up to the limit of the financial constraint and purchase investment goods for the production of intermediate goods, whereas agents who draw $\Phi_t \leq \phi_t$ lend all their savings in the financial market to acquire interest, r_{t+1}.[4] Hence, ϕ_t is the cutoff for the productivity shocks that divides agents into borrowers (producers) and lenders in period t. An agent's optimal portfolio program is given by

$$x_t(\omega^t) = \begin{cases} 0 & \text{if } \Phi_t(\omega_t) \leq \phi_t \\ \frac{s_t(\omega^{t-1})}{p_t(1-\mu)} & \text{if } \Phi_t(\omega_t) > \phi_t, \end{cases} \tag{4}$$

[3]This type of financial constraint is employed by many researchers such as Aghion et al. (1999), Aghion and Banerjee (2005), and Aghion et al. (2005).

[4]The derivation of an optimal portfolio allocation of savings follows Kunieda and Shibata (2016). Although agents who draw $\Phi_t = \phi_t$ are indifferent between initiating the production project and lending in the financial market, it is assumed that they lend their savings in the financial market.

and

$$b_t(\omega^t) = \begin{cases} s_t(\omega^{t-1}) & \text{if } \Phi_t(\omega_t) \le \phi_t \\ -\frac{\mu}{1-\mu} s_t(\omega^{t-1}) & \text{if } \Phi_t(\omega_t) > \phi_t. \end{cases} \tag{5}$$

2.1.4 Euler equation

The portfolio program given by Eqs. (4) and (5) rewrites the flow budget constraint (1) as

$$s_\tau(\omega^{\tau-1}) + c_\tau(\omega^{\tau-1}) = R_\tau(\omega_{\tau-1}) s_{\tau-1}(\omega^{\tau-2}) + w_\tau, \tag{6}$$

where $R_\tau(\omega_{\tau-1}) := \max\{r_\tau, (q_\tau \Phi_{\tau-1}(\omega_{\tau-1})/p_{\tau-1} - r_\tau \mu)/(1-\mu)\}$. The maximization of the agent's lifetime utility subject to (6) yields the following Euler equation:

$$1 = \beta E_t \left[R_{t+1}(\omega_t) \right], \tag{7}$$

where $E_t\left[R_{t+1}(\omega_t)\right] := E\left[R_{t+1}(\omega_t) | \omega^{t-1}\right]$, which is the return in period $t+1$ expected at the beginning of period t with history ω^{t-1} given. Eq. (7) implies that the individual expected return is constant and given by $R := E_t\left[R_{t+1}(\omega_t)\right] = 1/\beta$. Note that R is greater than 1 and is not necessarily equal to the gross interest, r_{t+1}. We impose the no-Ponzi condition such that

$$\lim_{\tau\to\infty} E_t[s_{t+\tau}(\omega^{t+\tau-1})]/R^\tau \ge 0. \tag{8}$$

Remark 1. *The no-Ponzi condition (8) holds with equality in equilibrium such that*

$$\lim_{\tau\to\infty} E_t[s_{t+\tau}(\omega^{t+\tau-1})]/R^\tau = 0. \tag{9}$$

Therefore, applying the Euler equation (7) to Eq. (9), we have the transversality condition such that

$$\lim_{\tau\to\infty} \beta^\tau E_t[s_{t+\tau}(\omega^{t+\tau-1})] = 0. \tag{10}$$

Eq. (9) in Remark 1 is proven as follows. The flow budget constraint in period $t+\tau$ is

$$s_{t+\tau}(\omega^{t+\tau-1}) + c_{t+\tau}(\omega^{t+\tau-1}) = R_{t+\tau}(\omega_{t+\tau-1})s_{t+\tau-1}(\omega^{t+\tau-2}) + w_{t+\tau}. \tag{11}$$

Taking expectations for both sides of Eq. (11) and arranging the resulting equation, we have

$$\frac{E_{t+\tau-1}\left[s_{t+\tau}(\omega^{t+\tau-1})\right]}{R} + \frac{E_{t+\tau-1}\left[c_{t+\tau}(\omega^{t+\tau-1})\right]}{R} = s_{t+\tau-1}(\omega^{t+\tau-2}) + \frac{w_{t+\tau}}{R}. \tag{12}$$

The flow budget constraint in period $t+\tau-1$ is

$$s_{t+\tau-1}(\omega^{t+\tau-2}) + c_{t+\tau-1}(\omega^{t+\tau-2}) = R_{t+\tau-1}(\omega_{t+\tau-2})s_{t+\tau-2}(\omega^{t+\tau-3}) + w_{t+\tau-1}. \tag{13}$$

From Eqs. (12) and (13), it follows that

$$\begin{aligned} &\frac{E_{t+\tau-1}\left[s_{t+\tau}(\omega^{t+\tau-1})\right]}{R} + c_{t+\tau-1}(\omega^{t+\tau-2}) + \frac{E_{t+\tau-1}\left[c_{t+\tau}(\omega^{t+\tau-1})\right]}{R} \\ &= R_{t+\tau-1}(\omega_{t+\tau-2})s_{t+\tau-2}(\omega^{t+\tau-3}) + w_{t+\tau-1} + \frac{w_{t+\tau}}{R}. \end{aligned} \tag{14}$$

Again, taking the expectation for both sides of Eq. (14) with the law of iterated expectations and arranging the resulting equation, we have

$$\begin{aligned} &\frac{E_{t+\tau-2}\left[s_{t+\tau}(\omega^{t+\tau-1})\right]}{R^2} + \frac{E_{t+\tau-2}\left[c_{t+\tau-1}(\omega^{t+\tau-2})\right]}{R} + \frac{E_{t+\tau-2}\left[c_{t+\tau}(\omega^{t+\tau-1})\right]}{R^2} \\ &= s_{t+\tau-2}(\omega^{t+\tau-3}) + \frac{w_{t+\tau-1}}{R} + \frac{w_{t+\tau}}{R^2}. \end{aligned} \tag{15}$$

By iterating these operations, we obtain the following equation:

$$\frac{E_t\left[s_{t+\tau}(\omega^{t+\tau-1})\right]}{R^\tau} + E_t\left[\sum_{s=1}^{\tau} \frac{c_{t+s}(\omega^{t+s-1})}{R^s}\right] = s_t(\omega^{t-1}) + \sum_{s=1}^{\tau} \frac{w_{t+s}}{R^s}. \tag{16}$$

It follows from Eq. (16) and the no-Ponzi condition (8) that

$$E_t\left[\sum_{s=1}^{\infty}\frac{c_{t+s}(\omega^{t+s-1})}{R^s}\right]\le s_t(\omega^{t-1})+\sum_{s=1}^{\infty}\frac{w_{t+s}}{R^s}. \tag{17}$$

This is the lifetime budget constraint of an agent in period t with history ω^{t-1}. The lifetime budget constraint (17) holds with equality because of the local nonsatiation of the period utility, which means that the no-Ponzi condition (8) also holds with equality.

As such, the necessary and sufficient optimality conditions for the lifetime utility maximization problem are given by the Euler equation (7) and the transversality condition Eq. (10).

2.2 Production

The representative firm produces both investment and consumption goods from labor and intermediate goods with Cobb-Douglas technologies: $F^1(l_t^1,k_t^1)=A(l_t^1)^{\alpha^1}(k_t^1)^{1-\alpha^1}$ for investment goods and $F^2(l_t^2,k_t^2)=B(l_t^2)^{\alpha^2}(k_t^2)^{1-\alpha^2}$ for consumption goods, where $\alpha^i\in(0,1)$ for $i=1,2$ and $\alpha^1\neq\alpha^2$. A and B are the productivity parameters. In the production functions, l_t^i and k_t^i are labor and intermediate goods. Intermediate goods entirely depreciate in one period.

The firm solves the following profit maximization problem:

$$\max_{l_t^1,l_t^2,k_t^1,k_t^2}\Pi_t:=p_tF^1(l_t^1,k_t^1)+F^2(l_t^2,k_t^2)-q_tk_t-w_tl_t, \tag{18}$$

where $k_t=k_t^1+k_t^2$ is the total intermediate goods in the economy in period t. The total labor supply is given by $l_t^1+l_t^2=l_t$, which is equal to the population of agents, i.e., $l_t=1$. The first-order conditions are given by

$$p_tA\alpha^1\left(\frac{k_t^1}{l_t^1}\right)^{1-\alpha^1}=B\alpha^2\left(\frac{k_t^2}{l_t^2}\right)^{1-\alpha^2}=w_t. \tag{19}$$

and

$$p_t A(1-\alpha^1)\left(\frac{k_t^1}{l_t^1}\right)^{-\alpha^1} = B(1-\alpha^2)\left(\frac{k_t^2}{l_t^2}\right)^{-\alpha^2} = q_t. \tag{20}$$

Eqs. (19) and (20) yield

$$k_t^1 = \tfrac{(1-\alpha^1)w_t}{\alpha^1 q_t} l_t^1 \quad \text{and} \quad k_t^2 = \tfrac{(1-\alpha^2)w_t}{\alpha^2 q_t} l_t^2. \tag{21}$$

Eqs. (19) and (20) also yield

$$w_t = \Psi p_t^{\frac{1-\alpha^2}{\alpha^1-\alpha^2}} =: w(p_t) \quad \text{and} \quad q_t = \Lambda p_t^{\frac{-\alpha^2}{\alpha^1-\alpha^2}} =: q(p_t), \tag{22}$$

where Ψ and Λ are defined as $\Psi := [(A(\alpha^1)^{\alpha^1}(1-\alpha^1)^{1-\alpha^1})^{1-\alpha^2}(B(\alpha^2)^{\alpha^2}(1-\alpha^2)^{1-\alpha^2})^{\alpha^1-1}]^{1/(\alpha^1-\alpha^2)}$ and $\Lambda := [(A(\alpha^1)^{\alpha^1}(1-\alpha^1)^{1-\alpha^1})^{\alpha^2}(B(\alpha^2)^{\alpha^2}(1-\alpha^2)^{1-\alpha^2})^{-\alpha^1}]^{1/(\alpha^2-\alpha^1)}$.

Applying Eq. (21), $k_t^1 + k_t^2 = k_t$, and $l_t^1 + l_t^2 = 1$, we rewrite the production functions as follows:

$$p_t F^1(l_t^1, k_t^1) = -\frac{\alpha^2 q(p_t)k_t - (1-\alpha^2)w(p_t)}{\alpha^1 - \alpha^2} \tag{23}$$

and

$$F^2(l_t^2, k_t^2) = \frac{\alpha^1 q(p_t)k_t - (1-\alpha^1)w(p_t)}{\alpha^1 - \alpha^2}. \tag{24}$$

Assumption 1. $\alpha^1 < 2\alpha^2$

Assumption 1 implies that even if the investment goods sector is more labor-intensive than the consumption goods sector, it is not so to a greater extent. Takahashi, Mashiyama, and Sakagami (2012) provide the capital intensity ratios of the consumption goods sector to the investment goods sector in the postwar Japanese and main OECD countries (Japan, Canada, France, US, and Germany) and the upper limit of them is less than 2.0. From Eq. (21), the corresponding capital intensity ratio in our model is given by $[\alpha^1(1-\alpha^2)]/[\alpha^2(1-\alpha^1)]$. One can note that $[\alpha^1(1-\alpha^2)]/[\alpha^2(1-\alpha^1)] < 2$ is equivalent to $\alpha^1 < 2\alpha^2 - \alpha^1\alpha^2$ and Assumption 1 is reasonable.

3 Market clearing conditions and net foreign assets

We can study the equilibrium conditions for all markets without considering the economy's openness. In this section, we obtain the market clearing conditions for investment and intermediate goods, which are common across closed and small open economies, and then derive the net foreign assets as a function of the national savings. The difference between closed and small open economies is in that in a closed economy, the financial market clears within the domestic economy and the net foreign assets held by the economy equal zero, whereas in a small open economy, the financial market does not clear within the domestic economy, and instead, the interest rate is exogenously given, being equal to the constant world rate, and net foreign assets are positive or negative.

3.1 Investment goods market

Eq. (4) shows that the investment goods are purchased by agents who draw higher productivity such that $\Phi_t(\omega_t) > \phi_t$. Therefore, the investment goods market clearing condition is given by

$$\int_{\Omega^t \times (\Omega \setminus E_t)} x_t(\omega^t) dP^{t+1}(\omega^t) = F^1(k_t^1, l_t^1). \tag{25}$$

where $E_t = \{\omega_t \in \Omega | \Phi_t(\omega_t) \leq \phi_t\}$.

3.2 Intermediate goods market

Intermediate goods are demanded by the representative firm and supplied by agents with $\Phi_t(\omega_t) > \phi_t$. In Lemma 1 below, we derive the intermediate goods market clearing condition.

Lemma 1. *The intermediate goods market clearing condition is given by*

$$k_{t+1} = k_{t+1}^1 + k_{t+1}^2 = \frac{H(\phi_t)}{p_t(1-\mu)} \int_{\Omega^t} s_t(\omega^{t-1}) dP^t(\omega^{t-1}), \tag{26}$$

where $H(\phi_t) := \int_{\phi_t}^{\eta} \Phi_t(\omega_t) dG(\Phi)$.

Proof. See the Appendix.

3.3 Net foreign assets

Net foreign assets are given by $B_t := \int_{\Omega^{t+1}} b_t(\omega^t) dP^{t+1}(\omega^t)$. One notes that $B_t = 0$ for all $t \geq 0$ in a closed economy. B_t is rewritten in terms of the national savings, $\int_{\Omega^t} s_t(\omega^{t-1}) dP^t(\omega^{t-1})$, as in Lemma 2 below.

Lemma 2. *The net foreign assets in period t are computed as*

$$B_t = \frac{G(\phi_t) - \mu}{1 - \mu} \int_{\Omega^t} s_t(\omega^{t-1}) dP^t(\omega^{t-1}). \tag{27}$$

Proof. See the Appendix.

From Eq. (1), we obtain the law of motion of the net foreign assets as in Lemma 3.

Lemma 3. *The law of motion of the net foreign assets when the financial market is open to the world market is given by*

$$B_t = r_t B_{t-1} + F^2(k_t^2, l_t^2) - C_t, \tag{28}$$

where $C_t := \int_{\Omega^t} c_t(\omega^{t-1}) dP^t(\omega^{t-1})$ is the total consumption.

Proof. See the Appendix.

4 Aggregation

The aggregation of the flow budget constraint (6) across all agents leads to the law of motion of the national savings, $\int_{\Omega^t} s_t(\omega^{t-1}) dP^t(\omega^{t-1})$, as in Lemma 4 below.

Lemma 4. *The law of motion of the national savings,* $\int_{\Omega^t} s_t(\omega^{t-1})dP^t(\omega^{t-1})$*, is given by*

$$\int_{\Omega^t} s_t(\omega^{t-1})dP^t(\omega^{t-1}) = r_t \left(\frac{G(\phi_{t-1}) - \mu}{1-\mu}\right) \int_{\Omega^{t-1}} s_{t-1}(\omega^{t-2})dP^{t-1}(\omega^{t-2})$$
$$+ p_t F^1(l_t^1, k_t^1) + F^2(l_t^2, k_t^2) - C_t. \tag{29}$$

Proof. See the Appendix.

From (27)-(29), it is straightforward to show that the national savings, $\int_{\Omega^t} s_t(\omega^{t-1})dP^t(\omega^{t-1})$, can be written by the value of the total investment goods as follows:

$$\int_{\Omega^t} s_t(\omega^{t-1})dP^t(\omega^{t-1}) = \frac{1-\mu}{1-G(\phi_t)} p_t F^1(l_t^1, k_t^1). \tag{30}$$

From (23), (26), and (30), we obtain a dynamic equation with respect to intermediate goods as follows:

$$\begin{aligned} k_{t+1} &= \frac{H(\phi_t)}{1-G(\phi_t)} F^1(l_t^1, k_t^1) \\ &= \frac{H(\phi_t)q(p_t)}{(1-G(\phi_t))(\alpha^1 - \alpha^2)p_t} \left(-\alpha^2 k_t + (1-\alpha^2)\frac{w(p_t)}{q(p_t)}\right). \end{aligned} \tag{31}$$

The use of $\phi_t = p_t r_{t+1}/q_{t+1}$ computes the expected return, $E[R_{t+1}(\omega_t)|\omega^{t-1}]$, as in Lemma 5 below.

Lemma 5.

$$E[R_{t+1}(\omega_t)|\omega^{t-1}] = \frac{r_{t+1}}{\phi_t}\left[\frac{\phi_t(G(\phi_t) - \mu) + H(\phi_t)}{1-\mu}\right]. \tag{32}$$

Proof. See the Appendix.

Remark 2. *Given the interest rate,* r_{t+1}*, and the extent of financial constraints,* μ*, the expected return,* $E[R_{t+1}(\omega_t)|\omega^{t-1}]$*, decreases as the cutoff,* ϕ_t*, increases.*

Proof. Define $\Gamma(\phi) := G(\phi) - \mu + H(\phi)/\phi$. From Lemma 5, $sign(\partial E[R_{t+1}(\omega_t)|\omega^{t-1}]/\partial\phi_t) = sign(\partial\Gamma(\phi)/\partial\phi_t)$. Because $\partial\Gamma(\phi)/\partial\phi_t = -H(\phi_t)/(\phi_t)^2 < 0$, the claim holds. □

Remark 2 implies that other things being equal, as the number of producers (borrowers) decreases, the expected return from saving decreases. This is because as ϕ_t increases, the chance for an agent to be a producer who acquires a greater return than lenders becomes thin. Inserting (32) into the Euler equation (7) yields

$$\frac{r_{t+1}}{\phi_t}\left[\frac{\phi_t(G(\phi_t)-\mu)+H(\phi_t)}{1-\mu}\right]=\frac{1}{\beta}. \tag{33}$$

5 Closed economy

5.1 Equilibrium

Sequences of prices, $\{w_t, q_t, p_t, r_{t+1}\}$ for all $t \geq 0$ and allocation, $\{k_t, k_t^1, k_t^2, l_t, l_t^1, l_t^2\}$ and $\{c_t(\omega^{t-1}), s_t(\omega^{t-1}), x_t(\omega^t), b_t(\omega^t)\}$ for all $t \geq 0$, ω^t, and ω^{t-1} form a competitive equilibrium in the closed economy, so that (i) for each ω^t and ω^{t-1}, consumers maximize their lifetime utility from period t onward, (ii) the representative firm maximizes its profits in each period, and (iii) the consumption and investment goods markets, the intermediate goods market, and the labor market all clear, and additionally, the financial market clears within the economy.[5]

As discussed in the previous section, all equations derived up to Section 4 are applicable to the closed economy by adding the domestic financial market clearing condition. Because the domestic financial market clears within the closed economy, the net foreign assets, B_t, in (27) are consistently zero for all $t \geq 0$, and accordingly, we obtain a time-invariant cutoff, $\phi_t = \phi^*$, in equilibrium as in the following proposition.

Proposition 1. *The cutoff, ϕ^*, in equilibrium in the closed economy is given by*

$$G(\phi^*) = \mu. \tag{34}$$

Proof. In the closed economy, the net foreign assets are equal to zero. Therefore, letting

[5] At period 0, r_0 does not appear because the initial endowment of intermediate goods, K_0, is commonly distributed across agents. To be accurate, c_0 is not subject to any history and ω^{-1} is empty.

$B_t = 0$ in Eq. (27) yields $G(\phi^*) = \mu$. $\square$

Substituting $\phi_t = \phi^*$ with $G(\phi^*) = \mu$ and $r_{t+1} = \phi^* q_{t+1}/p_t$ into Eqs. (31) and (33) yields

$$k_{t+1} = \frac{H(\phi^*)q(p_t)}{(1-\mu)(\alpha^1 - \alpha^2)p_t} \left(-\alpha^2 k_t + (1-\alpha^2)\frac{w(p_t)}{q(p_t)} \right) \tag{35}$$

and

$$\left(\frac{H(\phi^*)}{1-\mu} \right) \frac{q(p_{t+1})}{p_t} = \frac{1}{\beta}, \tag{36}$$

respectively. Eqs. (35) and (36) form a dynamical system in equilibrium in the closed economy.

5.2 Steady state

From Eqs. (22) and (36), the investment goods price, p^*, in the steady state is obtained as follows:

$$p^* = \left(\frac{1-\mu}{\Lambda \beta H(\phi^*)} \right)^{\frac{\alpha^2 - \alpha^1}{\alpha^1}}. \tag{37}$$

Moreover, from Eqs. (22), (35), and (36), the capital stock, k^*, in the steady state is obtained as follows:

$$k^* = \frac{\Psi(1-\alpha^2)}{\Lambda^{\frac{\alpha^1 - 1}{\alpha^1}} [(1-\beta)\alpha^2 + \beta\alpha^1]} \left(\frac{\beta H(\phi^*)}{1-\mu} \right)^{\frac{1}{\alpha^1}}. \tag{38}$$

To confirm that the current closed economy produces both intermediate and consumption goods in the steady state, we obtain Lemma 6 below.

Lemma 6. *In the current closed economy, it holds that*

$$\frac{(1-\alpha^i)w(p^*)}{\alpha^i q(p^*)} < k^* < \frac{(1-\alpha^j)w(p^*)}{\alpha^j q(p^*)}, \tag{39}$$

where $\alpha^i > \alpha^j$ *for* (i,j)=$(1,2)$ *or* $(2,1)$.

Proof. See the Appendix.

One can note from Eqs. (23), (24), and Lemma 6 that both investment and consumption goods are produced in the steady state. By continuity, in the neighborhood of the steady state, the economy imperfectly specializes in production.

5.3 Local dynamics

The dynamical system with respect to k_t and p_t formed by Eqs. (35) and (36) is rewritten as follows:

$$\begin{cases} k_{t+1} = J^c(k_t, p_t) \\ p_{t+1} = \left(\frac{1-\mu}{\Lambda\beta H(\phi^*)}\right)^{\frac{\alpha^2-\alpha^1}{\alpha^2}} p_t^{\frac{\alpha^2-\alpha^1}{\alpha^2}}, \end{cases} \tag{40}$$

where

$$J^c(k_t, p_t) = \frac{\alpha^2 H(\phi^*)\Lambda p_t^{\frac{\alpha^1}{\alpha^2-\alpha^1}}}{(1-\mu)(\alpha^2-\alpha^1)} k_t - \frac{(1-\alpha^2)H(\phi^*)\Psi p_t^{\frac{\alpha^1-1}{\alpha^2-\alpha^1}}}{(1-\mu)(\alpha^2-\alpha^1)}.$$

The linearization of the dynamical system (40) around the steady state yields

$$\begin{pmatrix} k_{t+1} - k^* \\ p_{t+1} - p^* \end{pmatrix} = \begin{pmatrix} \frac{\alpha^2}{\beta(\alpha^2-\alpha^1)} & J_p^c(k^*, p^*) \\ 0 & \frac{\alpha^2-\alpha^1}{\alpha^2} \end{pmatrix} \begin{pmatrix} k_t - k^* \\ p_t - p^* \end{pmatrix}, \tag{41}$$

where $J_p^c(k,p) := \partial J^c(k,p)/\partial p$. We easily obtain the eigenvalues, λ_1^c and λ_2^c, from the dynamical system (41) as follows:

$$\lambda_1^c = \frac{\alpha^2}{\beta(\alpha^2-\alpha^1)} \quad \text{and} \quad \lambda_2^c = \frac{\alpha^2-\alpha^1}{\alpha^2}.$$

Theorem 1. *Suppose that the economy is closed. Then, under Assumption 1, the following hold.*

(i) Suppose that the consumption goods sector is more labor-intensive than the investment goods sector such that $\alpha^1 < \alpha^2$. Then, the steady state, $\{k^*, p^*\}$, is a saddle point.

(ii) Suppose that the investment goods sector is more labor-intensive than the consumption goods sector such that $\alpha^2 < \alpha^1 < 2\alpha^2$. Then, the steady state, $\{k^*, p^*\}$, is a saddle

point.

Proof. See the Appendix.

6 Small open economy

6.1 Equilibrium

A competitive equilibrium in the small open economy is defined similarly to that in the closed economy except that the interest rate, r_{t+1}, is exogenously given by the constant world rate, r, for all $t \geq 0$ and the financial market does not clear within the economy. Then, sequences of prices, $\{w_t, q_t, p_t\}$ for all $t \geq 0$, allocation, $\{k_t, k_t^1, k_t^2, l_t, l_t^1, l_t^2\}$ and $\{c_t(\omega^{t-1}), s_t(\omega^{t-1}), x_t(\omega^t), b_t(\omega^t)\}$ for all $t \geq 0$, ω^t, and ω^{t-1}, and the net foreign assets, $\{B_t\}$, for all $t \geq 0$ form a competitive equilibrium in the small open economy.

In the case of the small open economy, a time-invariant cutoff, ϕ^{**}, is obtained from (33) as in the following proposition.

Proposition 2. *The cutoff, ϕ^{**}, in equilibrium in the small open economy is given by*

$$\frac{r}{\phi^{**}}\left[\frac{\phi^{**}(G(\phi^{**})-\mu)+H(\phi^{**})}{1-\mu}\right]=\frac{1}{\beta}. \tag{42}$$

Proof. In the small open economy, the economy faces the constant world interest rate. Therefore, substituting $r_{t+1} = r$ and $\phi_t = \phi^{**}$ into (33) yields (42). □

To derive explicit solutions for $G(\phi^{**})$ and ϕ^{**}, we assume that Φ follows a Pareto distribution in what follows as formally stated in Assumption 2.

Assumption 2. Φ *follows a Pareto distribution, i.e., $G(\Phi) = 1 - (d/\Phi)^{\xi}$, where $d > 0$ and $\xi > 1$.*

It is straightforward to compute $H(\Phi)$ under Assumption 2 as $H(\Phi) = [\xi/(\xi-1)]\Phi(1-G(\Phi))$. Note that the mean of Φ with the Pareto distribution is given by $H(d) = [\xi/(\xi-1)]d$, which decreases with ξ and increases with d. To guarantee the existence of ϕ^{**}, a parameter condition is imposed as in Assumption 3.

Assumption 3. $\frac{(\xi-1)(1-\mu)}{(\xi-1)(1-\mu)+1} \leq \beta r < 1$.

Under Assumptions 2 and 3, $G(\phi^{**})$ and thus ϕ^{**} are uniquely derived from (42) as follows:

$$G(\phi^{**}) = 1 - (\xi - 1)(1-\mu)\left(\frac{1}{\beta r} - 1\right) \tag{43}$$

and

$$\phi^{**} = \frac{d}{\left[(\xi-1)(1-\mu)\left(\frac{1}{\beta r}-1\right)\right]^{\frac{1}{\xi}}}. \tag{44}$$

Substituting $\phi_t = \phi^{**}$ into (31) yields

$$k_{t+1} = \frac{H(\phi^{**})q(p_t)}{(1-G(\phi^{**}))(\alpha^1-\alpha^2)p_t}\left(-\alpha^2 k_t + (1-\alpha^2)\frac{w(p_t)}{q(p_t)}\right). \tag{45}$$

From the definition of the cutoff, $\phi_t = p_t r/q(p_{t+1})$, the dynamic equation of the price of intermediate goods becomes

$$q(p_{t+1}) = \frac{r}{\phi^{**}}p_t. \tag{46}$$

Eqs. (45) and (46) form a dynamical system in equilibrium in the small open economy.

Under Assumption 2, it follows from Proposition 1 that $\phi^* = d/(1-\mu)^{1/\xi}$ in the closed economy. Note from Proposition 1 and (27) that if $\phi^{**} > \phi^*$, the net foreign assets are positive and if $\phi^{**} < \phi^*$, the net foreign assets are negative. Proposition 3 below provides parameter conditions for the small open economy to be a lender and a borrower in the international financial market.

Proposition 3. *Suppose that Assumptions 2 and 3 hold. Then, if* $(\xi-1)(1/(\beta r)-1) < 1$, *the current small open economy is a lender and if* $(\xi-1)(1/(\beta r)-1) > 1$, *it is a borrower in the international financial market.*

Proof. The claim follows from the fact that $\phi^{**} > (<)\phi^* \iff (\xi-1)(1/(\beta r)-1) < (>)1$.

□

From Proposition 3, it is noted that as ξ becomes greater (smaller) with the mean of the productivity shocks decreasing (increasing), it is more likely that the small open economy becomes a borrower (a lender) in the international financial market. As the mean of the productivity shocks decreases (increases), the expected return to saving, which is the left-hand side of (42), is downward (upward) pressured. In the current small open economy, however, the expected return is equal to the inverse of the subjective discount factor, which is the right-hand side of (42). Therefore, the decreased (increased) cutoff offsets the downward (upward) pressure that ξ produces on the expected return.[6]

We also note from Proposition 3 that as the world interest rate, r, becomes smaller (greater), it is more likely that the small open economy becomes a borrower (a lender) in the international financial market. This is a normal result in international finance. In terms of our model, as the world interest rate becomes smaller (greater), the expected return from saving is downward (upward) pressured as seen in the left-hand side of (42). Again, in the small open economy, the expected return is equal to the inverse of the subjective discount factor, and by decreasing (increasing) the cutoff, the economy offsets the downward (upward) pressure on the expected return.

6.2 Steady State

We consider the case in which both investment and consumption goods are produced in the small open economy. To guarantee this, a parameter condition is imposed as in the following assumption.

Assumption 4. $\frac{1-\alpha^i}{\alpha^i} < \frac{1-\alpha^2}{\alpha^2}\left(1 - \frac{(\alpha^1-\alpha^2)(\xi-1)}{\alpha^2 r\xi + (\alpha^1-\alpha^2)(\xi-1)}\right) < \frac{1-\alpha^j}{\alpha^j}$ *where* $\alpha^i > \alpha^j$ *for* $(i,j)=(1,2)$ *or* $(2,1)$.

Under Assumption 2, from Eqs. (22), (44), and (46), the investment goods price, p^{**}, in the

[6] Note from Remark 2 that other things being equal, the expected return decreases (increases) as the cutoff increases (decreases).

steady state in the small open economy is derived as follows:

$$p^{**} = \left[\frac{r}{\Lambda d}\left((\xi-1)(1-\mu)\left(\frac{1}{\beta r}-1\right)\right)^{\frac{1}{\xi}}\right]^{\frac{\alpha^2-\alpha^1}{\alpha^1}}. \tag{47}$$

From the first inequality of Assumption 4, it follows that $\alpha^2 r\xi + (\alpha^1-\alpha^2)(\xi-1) > 0$. Then, from Eqs. (22), (45), and (46), the capital stock, k^{**}, in the steady state is obtained as follows:

$$k^{**} = \frac{r\xi(1-\alpha^2)}{\alpha^2 r\xi + (\alpha^1-\alpha^2)(\xi-1)}\left(\frac{\Psi}{\Lambda}\right)\left(\frac{\Lambda\phi^{**}}{r}\right)^{\frac{1}{\alpha^1}}. \tag{48}$$

Furthermore, from Eqs. (27), (30), and (31), we have the net foreign asset, B^{**}, in the steady state as follows:

$$B^{**} = \frac{G(\phi^{**})-\mu}{H(\phi^{**})}p^{**}k^{**}. \tag{49}$$

Note from Eqs. (43) and (49) that

$$\mathrm{sign}(B^{**}) = \mathrm{sign}(G(\phi^{**})-\mu) = \mathrm{sign}\left(1-\frac{\xi-1}{\beta r\xi}\right), \tag{50}$$

which will be used in the later investigation.

Under Assumptions 2-4, Lemma 7 below confirms that both investment and consumption goods are produced in the steady state in the current small open economy as in the case of the closed economy.

Lemma 7. *Under Assumptions 2-4, it holds that*

$$\frac{(1-\alpha^i)w(p^{**})}{\alpha^i q(p^{**})} < k^{**} < \frac{(1-\alpha^j)w(p^{**})}{\alpha^j q(p^{**})} \tag{51}$$

in the current small open economy, where $\alpha^i > \alpha^j$ *for* (i,j)=$(1,2)$ *or* $(2,1)$.

Proof. See the Appendix.

By continuity, in the neighborhood of the steady state, the small open economy imperfectly specializes in production under Assumptions 2-4.

6.3 Local dynamics

Eqs. (45) and (46) are the dynamical system with respect to k_t and p_t in the small open economy, which is rewritten as

$$\begin{cases} k_{t+1} = J^s(k_t, p_t) \\ p_{t+1} = \left(\frac{r}{\Lambda\phi^{**}}\right)^{\frac{\alpha^2-\alpha^1}{\alpha^2}} p_t^{\frac{\alpha^2-\alpha^1}{\alpha^2}}, \end{cases} \tag{52}$$

where

$$J^s(k_t, p_t) = \frac{\alpha^2 H(\phi^{**}) q(p_t)}{(1-G(\phi^{**}))(\alpha^2-\alpha^1)p_t} k_t - \frac{(1-\alpha^2)H(\phi^{**})\Psi p_t^{\frac{\alpha^1-1}{\alpha^2-\alpha^1}}}{(1-G(\phi^{**}))(\alpha^2-\alpha^1)}.$$

In the steady state, it holds that $\phi^{**} = p^{**}r/q(p^{**})$. By using this equation and $H(\phi^{**}) = [\xi/(\xi-1)]\phi^{**}(1-G(\phi^{**}))$, we linearize the dynamical system (52) around the steady state as follows:

$$\begin{pmatrix} k_{t+1} - k^{**} \\ p_{t+1} - p^{**} \end{pmatrix} = \begin{pmatrix} \frac{r\xi\alpha^2}{(\xi-1)(\alpha^2-\alpha^1)} & J_p^s(k^{**}, p^{**}) \\ 0 & \frac{\alpha^2-\alpha^1}{\alpha^2} \end{pmatrix} \begin{pmatrix} k_t - k^{**} \\ p_t - p^{**} \end{pmatrix}, \tag{53}$$

where $J_p^s(k,p) := \partial J^s(k,p)/\partial p$. The eigenvalues, λ_1^s and λ_2^s, from the dynamical system (53) are obtained as follows:

$$\lambda_1^s = \frac{r\xi\alpha^2}{(\xi-1)(\alpha^2-\alpha^1)} \quad \text{and} \quad \lambda_2^s = \frac{\alpha^2-\alpha^1}{\alpha^2}.$$

Compared with the case of the closed economy, one of the eigenvalues in the small open economy is the same as that in the closed economy. In contrast, another eigenvalue is affected by the world interest rate, r, and the parameter of the productivity distribution, ξ, whereas the corresponding eigenvalue in the closed economy is affected by the subjective discount factor.

Theorem 2. *Suppose that the economy is a small open economy. Then, under Assumptions 1-4, the following hold.*

(i) Suppose that the consumption goods sector is more labor-intensive than the intermediate goods sector such that $\alpha^1 < \alpha^2$. *Then, the steady state,* $\{k^{**}, p^{**}\}$, *is a saddle point.*

(ii) Suppose that the investment goods sector is more labor-intensive than the consumption goods sector such that $\alpha^2 < \alpha^1 < 2\alpha^2$.

(a) If $\alpha^2/(\alpha^1 - \alpha^2) > \frac{\xi-1}{r\xi}$, *the steady state,* $\{k^{**}, p^{**}\}$, *is a saddle point.*

(b) If $\alpha^2/(\alpha^1 - \alpha^2) < \frac{\xi-1}{r\xi}$, *the steady state,* $\{k^{**}, p^{**}\}$, *is totally stable.*

Proof. See the Appendix.

In the dynamical systems given by Eq. (41) in the closed economy and Eq. (53) in the small open economy, k_t is a state variable that is predetermined in period t and p_t is a jump variable. Therefore, in Theorems 1 and 2, if the steady state is a saddle point, equilibrium around the steady state is uniquely determined. This is because when the steady state is a saddle point, for any given initial intermediate goods, k_0, only the initial price of investment goods determines a transitional path for $\{k_t, p_t\}$ in equilibrium. In contrast, in the case of (ii)-(b) of Theorem 2, the steady state is totally stable. In this case, there exists a continuum of the initial values of k_0 and p_0, each of which is an initial point of an equilibrium sequence of $\{k_t, p_t\}$ that converges to the steady state. In other words, equilibrium is indeterminate. When equilibrium is indeterminate, extrinsic uncertainty can cause belief-driven sunspot fluctuations. The case of (ii)-(b) in Theorem 2 is comparable to the case in which the consumption goods sector is more capital-intensive than the capital goods sector in the standard two-sector growth model. Takahashi, Mashiyama, and Sakagami (2012) provide empirical evidence showing that the consumption goods sector has been more capital-intensive than the capital goods sector in the postwar Japanese and main OECD countries.

In Remark 3 below, we investigate the characteristics of the steady state in the case of (ii)-(b) of Theorem 2.

Remark 3. *Regarding the steady state in the case of (ii)-(b) of Theorem 2, the following hold.*

- *The world interest rate, r, is less than* 1.
- *It holds that* $B^{**} < 0$.

Proof. For the first part, the condition of (ii)-(b) of Theorem 2 implies that $r < (\alpha^1/\alpha^2 - 1)(1 - 1/\xi)$. Because $\alpha^1 < 2\alpha^2$, the last inequality is followed by $r < (1 - 1/\xi) < 1$. For the second part, it suffices to show that $(\xi - 1)/(\beta r \xi) > 1$ because of Eq. (50). Again, the condition of (ii)-(b) implies that $(\xi - 1)/(\beta r \xi) > \alpha^2/(\alpha^1 - \alpha^2) > \alpha^2/(2\alpha^2 - \alpha^2) = 1$. □

6.4 Financial constraints, dynamic inefficiency, and indeterminacy

In our model, if the domestic financial market is closed to the international financial market, the market interest rate increases as financial constraints are relaxed. As the financial market approaches perfection, the interest rate acquired by lenders becomes equal to a return obtained by the highest-productivity producers with their arbitrage opportunity vanishing. In this case, indeterminacy of equilibrium does not occur. In a small open economy, however, indeterminacy can occur even though the domestic financial market approaches perfection. Although this outcome is somewhat surprising, one should note that the market interest rate remains constant in a small open economy even though financial constraints are fully relaxed. Then, producers can still take an arbitrage opportunity by borrowing in the financial market and producing intermediate goods. Although producers purchase more investment goods to produce intermediate goods to acquire a higher return, they do not have to reduce their consumption from the optimal plan by borrowing consumption goods in the financial market.

Intuitively, consider an equilibrium path in the small open economy with a certain price of investment goods p_t given intermediate goods k_t in period t. Let us examine whether another path with a price p'_t greater than p_t can be an equilibrium. If $p'_t > p_t$, whereas the cost of purchasing one unit of investment goods in period t increases, producers can borrow consumption goods in the international financial market to keep their optimal plan when financial constraints are soft. If the investment goods sector is more labor-intensive than the consumption sector (i.e., the consumption sector is more intermediate goods-intensive than the investment goods sector), the economy can save more intermediate goods in period t than when this is not the case. Moreover, as the price of investment goods becomes high, the representative firm increases the production of investment goods using saved intermediate goods in period t. Additionally, as the price of investment goods becomes high, the cutoff ϕ_t increases and low-productivity agents are ruled out of the production of intermediate goods used in period $t+1$. As a result, the aggregate productivity of intermediate goods production becomes high and the economy produces more intermediate goods. Then, consumption goods in period $t+1$ are sufficiently produced to repay in the international financial market because the consumption goods sector is more intermediate goods-intensive. One can provide a similar explanation for the case in which $p'_t < p_t$ by considering the inverse allocation of goods. As such, the path with the price of investment goods that deviates from the original equilibrium path is also an equilibrium when imperfections in the domestic financial market are fully resolved, and indeterminacy of equilibrium can arise in the small open economy.

Thus far, we have understood that it is important for indeterminacy of equilibrium to occur that the consumption sector is more intermediate goods-intensive than the investment goods sector, and thus, it cannot occur in the case of (i) of Theorem 2 but can in the case of (ii). Furthermore, one should note that it can occur only in the case of (ii)-(b) of Theorem 2 because of dynamic inefficiency and the presence of the negative net foreign asset. Whereas each agent's no-Ponzi condition holds as investigated in Section 2.1 because the individual expected return R is greater than 1, the world interest r is less than 1 in the case of (ii)-(b) as investigated in Remark 3. Because the gross growth rate in the steady state is 1 and the gross interest is less than 1, the steady state in the case of (ii)-(b) is dynamically inefficient.

The financial constraint and agents' production heterogeneity, which cause the deviation between R and r, are a source of dynamic inefficiency. By using Eq. (28) iteratively, we have an intertemporal feasibility constraint of the economy from period t onward as follows:

$$\sum_{s=1}^{\infty} \frac{C_{t+s}}{r^s} = B_t + \sum_{s=1}^{\infty} \frac{\tilde{F}^2(k_t, p_t)}{r^s} - \lim_{\tau \to \infty} \frac{B_{t+\tau}}{r^\tau}, \tag{54}$$

where $\tilde{F}^2(k_t, p_t) := F^2(l_t^2, k_t^2)$ (see Eq. (24)).

Remark 4. *In equilibrium under the case of (ii)-(b) of Theorem 2, it holds that* $\sum_{s=1}^{\infty} C_{t+s}/r^s = \infty$ *and* $\sum_{s=1}^{\infty} \tilde{F}^2(k_t, p_t)/r^s = \infty$ *(because* $\lim_{s\to\infty} C^{**}/r^s = \infty$ *and* $\lim_{s\to\infty} \tilde{F}^2(k^{**}, p^{**})/r^s = \infty$*). Therefore,* $-\lim_{\tau\to\infty} B_{t+\tau}/r^\tau = \infty$ *is not inconsistent with the intertemporal feasibility constraint (54).*

Remark 4 is a typical situation for dynamic inefficiency. In this case, whereas the individual no-Ponzi condition (8) holds with equality as stated in Remark 1, the Ponzi debt in the economy (i.e., negative net foreign assets) can be present in the steady state and infinitely many equilibrium paths of B_t can exist, each of which is consistent with Eq. (54). Dynamic inefficiency never occurs in the models in the literature, such as Lahiri (2001), Weder (2001), and Meng and Velasco (2004), because they assume that the world interest rate is equal to the subjective discount rate (i.e., $r = 1/\beta$ from the perspective of our model).

Now, we more concretely investigate the parameter conditions for indeterminacy to occur. From the first inequality of Assumption 4 and the condition in (ii)-(b) in Theorem 2, the following inequalities are obtained.[7]

$$\frac{(\xi-1)(1-\alpha^1)}{\xi} < r < \frac{(\xi-1)(\alpha^1-\alpha^2)}{\xi\alpha^2}. \tag{55}$$

[7]Given the value of ξ, there exists r that satisfies inequality (55) if and only if $1-\alpha^1 < (\alpha^1-\alpha^2)/\alpha^2 \iff \alpha^1 > 2\alpha^2/(1+\alpha^2)$.

Furthermore, from Assumption 3, it holds that

$$\frac{(\xi-1)(1-\mu)}{\beta[(\xi-1)(1-\mu)+1]} \le r < \frac{1}{\beta}. \tag{56}$$

The world interest rate r must satisfy both inequalities (55) and (56) for equilibrium to be indeterminate. Because the right-hand side of (55) is less than one and the right-hand side of (56) is greater than one, the right-hand side of (55) should be strictly grater than the left-hand side of (56) for the existence of such a world interest rate, which is stated as a parameter condition in Theorem 3.

Theorem 3. *Suppose that Assumptions 1-4 hold. Then, the world interest rate r for which the steady state, $\{k^{**}, p^{**}\}$, is totally stable exists if and only if*

$$\mu > \frac{\xi\alpha^2 - \beta\xi(\alpha^1-\alpha^2)}{\xi\alpha^2 - \beta(\xi-1)(\alpha^1-\alpha^2)} =: \bar{\mu} \in (0,1). \tag{57}$$

Proof. It is straightforward to show that the right-hand side of inequality (55) is strictly grater than the left-hand side of inequality (56) if and only if inequality (57) holds. □

As noted from inequality (57), the severe financial constraint does not induce indeterminacy. The lower limit of financial constraints for indeterminacy, which is given by $\bar{\mu}$, depends on the productivity distribution. Fig 1 provides a numerical example for $\bar{\mu}$ by varying ξ with other parameter values remaining fixed as $\alpha_1 = 0.70$, $\alpha_2 = 0.50$, and $\beta = 0.98$.[8] Note from Fig 1 that $\bar{\mu}$ takes a value from approximately 0.61 to 0.94 when ξ increases from 1 to 10. As ξ increases, the mean of productivity shocks decreases. As a result, the chance to acquire the higher return from arbitrage by borrowing and producing intermediate goods decreases. To mitigate this negative effect on the return, the volume of borrowing should be increased for indeterminacy to occur by relaxing the financial constraints.

In summary, our findings are as follows. Indeterminacy of equilibrium can arise in the

[8]Under this parameter setting, the labor share obtained is consistently equal to 0.667, which is similar to that used in real business cycle theory. Furthermore, under this setting, it holds that $\alpha^1 > 2\alpha^2/(1+\alpha^2)$ (see footnote 7). Additionally, $\beta = 0.98$ is a reasonable setting, again relative to real business cycle theory.

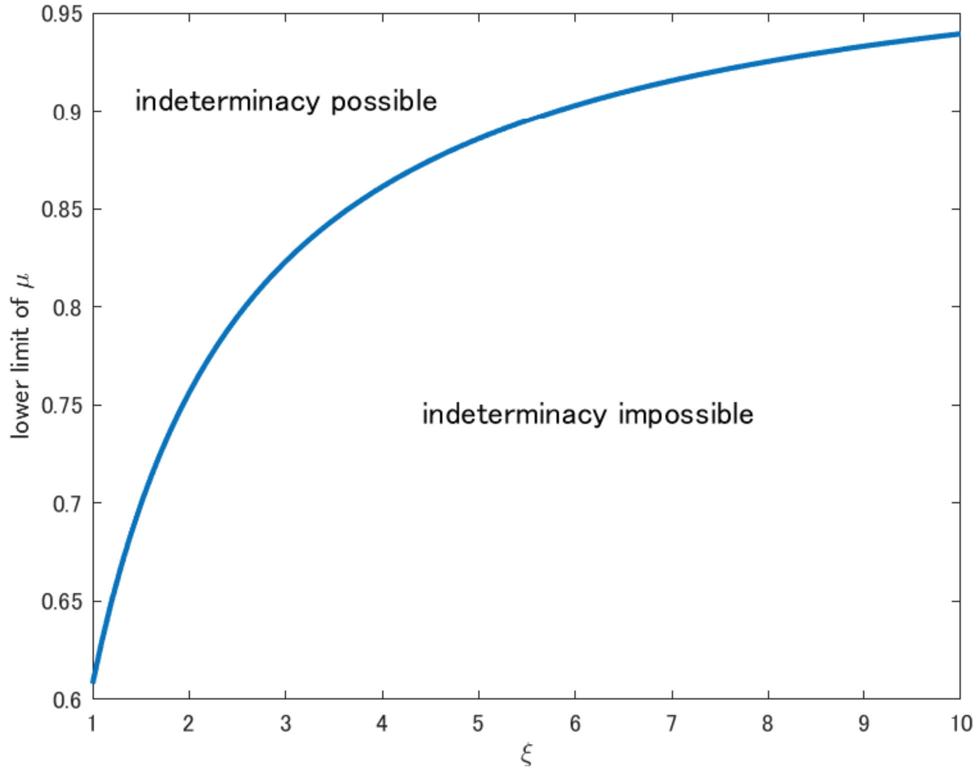

Fig 1. The lower limit of financial constraints for indeterminacy

small open economy only if financial constraints are relaxed under the condition that the investment goods sector is labor intensive relative to the consumption goods sector. Even though financial market imperfections are sufficiently resolved with μ approaching 1, indeterminacy can still occur in the small open economy. In other words, when financial constraints are fully relaxed, opening the domestic financial market to the international market can destabilize the economy. This outcome is somewhat surprising because without any imperfections, multiple equilibria typically cannot occur.

7 Concluding remarks

Our dynamic general equilibrium model demonstrates that indeterminacy of equilibrium and belief-driven sunspot fluctuations are more likely to occur in the small open economy. More concretely, under the condition that the investment goods sector is more labor intensive than the consumption goods sector, indeterminacy can occur in the small open economy only when financial constraints are fully relaxed.

As a worldwide trend in the relaxation of regulations in the domestic and international financial markets, many countries liberalized and opened their domestic markets to the international financial market in the 1980-1990s. As documented by Stiglitz (2004) and Ocampo et al. (2008), witnessing the process of capital account liberalization, many researchers warned of risks that are inherently present in the deregulation process such as volatile capital inflow and outflow that amplify economic fluctuations and often cause financial crises. Although the risk inherent in capital account liberalization is empirically acknowledged by Prasad et al. (2005), the effects that capital account liberalization has on economies have not been fully investigated theoretically thus far. Our paper must be one of the theoretical grounds and references for the policy debate on the effects of the capital account liberalization.

Appendix

Proof of Lemma 1

Because intermediate goods are supplied by agents who draw higher productivity such that $\Phi_t(\omega_t) > \phi_t$ as shown in Eq. (4), the intermediate goods market clearing condition is computed as follows:

$$\begin{aligned} k_{t+1}^1 + k_{t+1}^2 &= \int_{\Omega^t \times (\Omega \setminus E_t)} \Phi_t(\omega_t) x_t(\omega^t) dP^{t+1}(\omega^t) \\ &= \int_{\Omega \setminus E_t} \int_{\Omega^t} \Phi_t(\omega_t) \frac{s_t(\omega^{t-1})}{p_t(1-\mu)} dP^t(\omega^{t-1}) dP(\omega_t) \\ &= \int_{\phi_t}^{\eta} \frac{\Phi_t(\omega_t)}{p_t(1-\mu)} dG(\Phi) \times \int_{\Omega^t} s_t(\omega^{t-1}) dP^t(\omega^{t-1}) \\ &= \frac{H(\phi_t)}{p_t(1-\mu)} \int_{\Omega^t} s_t(\omega^{t-1}) dP^t(\omega^{t-1}). \end{aligned}$$

The third equality follows from the i.i.d. assumption. $\square$

Proof of Lemma 2

It follows from (5) and $B_t = \int_{\Omega^{t+1}} b_t(\omega^t) dP^{t+1}(\omega^t)$ that

$$\begin{aligned} B_t &= \int_{\Omega} \int_{\Omega^t} b_t(\omega^t) dP^t(\omega^{t-1}) dP(\omega_t) \\ &= \int_{E_t} \int_{\Omega^t} s_t(\omega^{t-1}) dP^t(\omega^{t-1}) dP(\omega_t) - \frac{\mu}{1-\mu} \int_{\Omega \setminus E_t} \int_{\Omega^t} s_t(\omega^{t-1}) dP^t(\omega^{t-1}) dP(\omega_t) \end{aligned} \quad \text{(A.1)}$$

where $E_t = \{\omega_t \in \Omega | \Phi_t(\omega_t) \leq \phi_t\}$. Because ω_t and ω^{t-1} are independent, Eq. (A.1) can be rewritten as

$$\begin{aligned} B_t &= \int_{\Omega^t} s_t(\omega^{t-1}) dP^t(\omega^{t-1}) \int_0^{\phi_t} dG(\Phi) - \frac{\mu}{1-\mu} \int_{\Omega^t} s_t(\omega^{t-1}) dP(\omega^{t-1}) \int_{\phi_t}^{\eta} dG(\Phi) \\ &= \frac{G(\phi_t) - \mu}{1-\mu} \int_{\Omega^t} s_t(\omega^{t-1}) dP^t(\omega^{t-1}). \quad \square \end{aligned}$$

Proof of Lemma 3

By aggregating Eq. (1) across all agents, we have

$$\begin{aligned} &\int_{\Omega^{t+1}} c_t(\omega^{t-1}) dP^{t+1}(\omega^t) + p_t \int_{\Omega^{t+1}} x_t(\omega^t) dP^{t+1}(\omega^t) + \int_{\Omega^{t+1}} b_t(\omega^t) dP^{t+1}(\omega^t) \\ &= q_t \int_{\Omega^{t+1}} \Phi_{t-1}(\omega_{t-1}) x_{t-1}(\omega^{t-1}) dP^{t+1}(\omega^t) + r_t \int_{\Omega^{t+1}} b_{t-1}(\omega^{t-1}) dP^{t+1}(\omega^t) + \int_{\Omega^{t+1}} w_t dP^{t+1}(\omega^t). \end{aligned} \quad \text{(B.1)}$$

Note that

$$\int_{\Omega^{t+1}} c_t(\omega^{t-1}) dP^{t+1}(\omega^t) = \int_{\Omega} \int_{\Omega^t} c_t(\omega^{t-1}) dP^t(\omega^{t-1}) dP(\omega_t) = C_t, \quad \text{(B.2)}$$

$$\int_{\Omega^{t+1}} x_t(\omega^t) dP^{t+1}(\omega^t) = \int_{\Omega^t \times (\Omega \setminus E_t)} x_t(\omega^t) dP^{t+1}(\omega^t) = F^1(l_t^1, k_t^1) \quad \because \text{Eq. (25)}, \quad \text{(B.3)}$$

$$\int_{\Omega^{t+1}} \Phi_{t-1}(\omega_{t-1})x_{t-1}(\omega^{t-1})dP^{t+1}(\omega^t) = \int_{\Omega}\int_{\Omega^t} \Phi_{t-1}(\omega_{t-1})x_{t-1}(\omega^{t-1})dP^t(\omega^{t-1})dP(\omega_t) = k_t, \tag{B.4}$$

$$\int_{\Omega^{t+1}} b_{t-1}(\omega^{t-1})dP^{t+1}(\omega^t) = \int_{\Omega}\int_{\Omega^t} b_{t-1}(\omega^{t-1})dP^t(\omega^{t-1})dP(\omega_t) = B_{t-1}, \tag{B.5}$$

and

$$\int_{\Omega^{t+1}} w_t dP^{t+1}(\omega^t) = w_t. \tag{B.6}$$

Because $q_t k_t + w_t = p_t F^1(l_t^1, k_t^1) + F^2(l_t^2, k_t^2)$, Eqs. (B.2)-(B.6) rewrites (B.1) as follows:

$$B_t = r_t B_{t-1} + F^2(l_t^2, k_t^2) - C_t. \quad \square$$

Proof of Lemma 4

By aggregating the flow budget constraint (6) across all agents, we obtain

$$\begin{aligned}
&\int_{\Omega^t} s_t(\omega^{t-1})dP^t(\omega^{t-1}) \\
&= \int_{\Omega^t} \left[R_t(\omega_{t-1})s_{t-1}(\omega^{t-2}) + w_t - c_t(\omega^{t-1})\right] dP^t(\omega^{t-1}) \\
&= \int_{\Omega^{t-1}\times E_{t-1}} r_t s_{t-1}(\omega^{t-2})dP^t(\omega^{t-1}) \\
&+ \int_{\Omega^{t-1}\times(\Omega\setminus E_{t-1})} \frac{q_t\Phi_{t-1}(\omega_{t-1})/p_{t-1} - r_t\mu}{1-\mu} s_{t-1}(\omega^{t-2})dP^t(\omega^{t-1}) + w_t - C_t \\
&= r_t\left[\int_{E_{t-1}}\int_{\Omega^{t-1}} s_{t-1}(\omega^{t-2})dP^{t-1}(\omega^{t-2})dP(\omega_{t-1}) - \frac{\mu}{1-\mu}\int_{\Omega\setminus E_{t-1}}\int_{\Omega^{t-1}} s_{t-1}(\omega^{t-2})dP^{t-1}(\omega^{t-2})dP(\omega_{t-1})\right] \\
&+ \frac{q_t}{(1-\mu)p_{t-1}}\int_{\Omega\setminus E_{t-1}}\int_{\Omega^{t-1}} \Phi_{t-1}(\omega_{t-1})s_{t-1}(\omega^{t-2})dP^{t-1}(\omega^{t-2})dP(\omega_{t-1}) + w_t - C_t \\
&= r_t\left[G(\phi_{t-1})\int_{\Omega^{t-1}} s_{t-1}(\omega^{t-2})dP^{t-1}(\omega^{t-2}) - \frac{\mu(1-G(\phi_{t-1}))}{1-\mu}\int_{\Omega^{t-1}} s_{t-1}(\omega^{t-2})dP^{t-1}(\omega^{t-2})\right] \\
&+ \frac{q_t H(\phi_{t-1})}{(1-\mu)p_{t-1}}\int_{\Omega^{t-1}} s_{t-1}(\omega^{t-2})dP^{t-1}(\omega^{t-2}) + w_t - C_t \\
&= r_t\left(\frac{G(\phi_{t-1})-\mu}{1-\mu}\right)\int_{\Omega^{t-1}} s_{t-1}(\omega^{t-2})dP^{t-1}(\omega^{t-2}) + q_t k_t + w_t - C_t \quad \because \text{Lemma 1} \\
&= r_t\left(\frac{G(\phi_{t-1})-\mu}{1-\mu}\right)\int_{\Omega^{t-1}} s_{t-1}(\omega^{t-2})dP^{t-1}(\omega^{t-2}) + p_t F^1(l_t^1, k_t^1) + F^2(l_t^2, k_t^2) - C_t.
\end{aligned}$$

The fourth equality follows from the i.i.d. assumption and the last equality follows from Eqs. (23) and (24). $\square$

Proof of Lemma 5

Because $\phi_t = p_t r_{t+1}/q(p_{t+1})$, it follows that

$$\begin{aligned} E[R_{t+1}(\omega_t)|\omega^{t-1}] &= E\left[\max\left\{r_{t+1}, \frac{q_{t+1}\Phi_t(\omega_t)/p_t - r_{t+1}\mu}{1-\mu}\right\}\middle|\omega^{t-1}\right] \\ &= \frac{q_{t+1}}{p_t}E\left[\max\left\{\phi_t, \frac{\Phi_t(\omega_t) - \phi_t\mu}{1-\mu}\right\}\middle|\omega^{t-1}\right] \\ &= \frac{q_{t+1}}{p_t}\left[\int_0^{\phi_t}\phi_t dG(\Phi) + \int_{\phi_t}^{\eta}\frac{\Phi_t(\omega_t) - \phi_t\mu}{1-\mu}dG(\Phi)\right] \\ &= \frac{r_{t+1}}{\phi_t}\left[\frac{\phi_t(G(\phi_t) - \mu) + H(\phi_t)}{1-\mu}\right]. \quad \square \end{aligned}$$

Proof of Lemma 6

From Eqs. (22), (37), and (38), one notes that inequality (39) is equivalent to

$$\frac{1-\alpha^i}{\alpha^i} < \frac{1-\alpha^2}{(1-\beta)\alpha^2 + \beta\alpha^1} < \frac{1-\alpha^j}{\alpha^j}. \tag{C.1}$$

Therefore, we prove inequality (C.1).

Case 1: $\alpha^1 < \alpha^2$

Because $\alpha^1 < \alpha^2$, it follows that

$$\frac{1-\alpha^2}{(1-\beta)\alpha^2 + \beta\alpha^1} > \frac{1-\alpha^2}{(1-\beta)\alpha^2 + \beta\alpha^2} = \frac{1-\alpha^2}{\alpha^2}. \tag{C.2}$$

Similarly, it follows that

$$\frac{1-\alpha^1}{\alpha^1} > \frac{1-\alpha^2}{(1-\beta)\alpha^1 + \beta\alpha^1} > \frac{1-\alpha^2}{(1-\beta)\alpha^2 + \beta\alpha^1}. \tag{C.3}$$

From inequalities (C.2) and (C.3), we obtain (C.1).

Case 2: $\alpha^2 < \alpha^1$

Because $\alpha^2 < \alpha^1$, it follows that

$$\frac{1-\alpha^2}{(1-\beta)\alpha^2+\beta\alpha^1} > \frac{1-\alpha^2}{(1-\beta)\alpha^1+\beta\alpha^1} = \frac{1-\alpha^1}{\alpha^1}. \tag{C.4}$$

Similarly, it follows that

$$\frac{1-\alpha^2}{\alpha^2} = \frac{1-\alpha^2}{(1-\beta)\alpha^2+\beta\alpha^2} > \frac{1-\alpha^2}{(1-\beta)\alpha^2+\beta\alpha^1}. \tag{C.5}$$

From inequalities (C.4) and (C.5), we obtain (C.1). □

Proof of Theorem 1

(i) Because $\alpha^2 > \alpha^1$, it holds that $0 < \lambda_2^c < 1$ and $\lambda_1^c > 1$. (ii) Because $\alpha^1 > \alpha^2$ and $(\alpha^1 - \alpha^2)/\alpha^2 < 1$, it holds that $-1 < \lambda_2^c < 0$. Because $\alpha^2/(\alpha^1-\alpha^2) > 1$, it holds that $\lambda_1^c < -1$. □

Proof of Lemma 7

From Assumption 4, we have

$$\frac{1-\alpha^i}{\alpha^i} < \frac{r\xi(1-\alpha^2)}{\alpha^2 r\xi + (\alpha^1-\alpha^2)(\xi-1)} < \frac{1-\alpha^j}{\alpha^j}, \tag{D.1}$$

where $\alpha^i > \alpha^j$ for (i,j)=$(1,2)$ or $(2,1)$. It follows from inequality (D.1) that

$$\begin{aligned}\frac{1-\alpha^i}{\alpha^i}\left(\frac{\Psi}{\Lambda}\right)\left(\frac{\Lambda\phi^{**}}{r}\right)^{\frac{1}{\alpha^1}} &< \frac{r\xi(1-\alpha^2)}{\alpha^2 r\xi + (\alpha^1-\alpha^2)(\xi-1)}\left(\frac{\Psi}{\Lambda}\right)\left(\frac{\Lambda\phi^{**}}{r}\right)^{\frac{1}{\alpha^1}} \\ &< \frac{1-\alpha^j}{\alpha^j}\left(\frac{\Psi}{\Lambda}\right)\left(\frac{\Lambda\phi^{**}}{r}\right)^{\frac{1}{\alpha^1}},\end{aligned}$$

or equivalently,

$$\frac{(1-\alpha^i)w(p^{**})}{\alpha^i q(p^{**})} < k^{**} < \frac{(1-\alpha^j)w(p^{**})}{\alpha^j q(p^{**})},$$

for (i,j)=$(1,2)$ or $(2,1)$. $\square$

Proof of Theorem 2

(i) From the first inequality of Assumption 4, it follows that $\alpha^2 r\xi + (\alpha^1 - \alpha^2)(\xi - 1) > 0$, from which we obtain $\lambda_1^s = \frac{r\xi\alpha^2}{(\xi-1)(\alpha^2-\alpha^1)} > 1$. Because $\alpha^2 > \alpha^1$, it follows that $\lambda_2^s = (\alpha^2 - \alpha^1)/\alpha^2 \in (0,1)$. (ii)-(a) Because $\alpha^2 < \alpha^1 < 2\alpha^2$, we obtain $\lambda_2^s \in (-1,0)$. Because $\alpha^1 > \alpha^2$ and $\alpha^2/(\alpha^1-\alpha^2) > (\xi-1)/(r\xi)$, it follows that $\lambda_1^s = r\xi\alpha^2/[(\xi-1)(\alpha^2-\alpha^1)] < -1$. (ii)-(b) As in the case of (ii)-(a), it holds that $\lambda_2^s = (\alpha^2-\alpha^1)/\alpha^2 \in (-1,0)$. Because $\alpha^2/(\alpha^1-\alpha^2) > (\xi-1)/(r\xi)$, it follows that $\lambda_1^s = r\xi\alpha^2/[(\xi-1)(\alpha^2-\alpha^1)] \in (-1,0)$. $\square$

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