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The Provision of High-powered Incentives under Multitasking

Kohei Daido

(Kwansei Gakuin University)

Takeshi Murooka

(Osaka University)

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KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho

Nishinomiya 662-8501, Japan

The Provision of High-powered Incentives under Multitasking*

Kohei Daido† Takeshi Murooka‡

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Abstract

We study multitasking problems where an agent engages in both a contractible task and a non-contractible task, which are substitutes. The agent has private information on the value of the non-contractible task, and there are followers who can also contribute to this task. We highlight a new mechanism by incorporating leading-by-example (Hermalin, 1998) in a multitasking model. To prevent excessive effort by the agent with low value on the non-contractible task, the principal provides high-powered incentives for the contractible task. We discuss its organizational implications to pay for performance, incentives to help colleagues, and prevention of overwork.

JEL Codes: D82, D86, J33, M52.

Keywords: Multitasking, Signaling, Leadership, Pay for Performance, Help, Overwork

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†School of Economics, Kwansei Gakuin University, 1-155 Ichiban-cho Uegahara Nishinomiya, Hyogo, JAPAN 662-8501 (Email: daido@kwansei.ac.jp).

‡Osaka School of International Public Policy, Osaka University, Machikaneyama 1-31, Toyonaka, Osaka, JAPAN 560-0043 (Email: murooka@osipp.osaka-u.ac.jp).

1 Introduction

After the seminal work by Holmstrom and Milgrom (1991), multitasking problems have been one of the most important topics in contract theory and organizational economics, as well as many other fields.[1] In a standard multitasking model, an agent is responsible for multiple tasks, some of which are not easy to monitor and reward (called "non-contractible tasks"). One of the main results is that when the tasks are substitutes or effort costs are complements, a principal prefers to provide a *lower-powered* incentive for a task which outcome is verifiable (called a "contractible task") compared to a case in which the agent works only on the contractible task. However, even when some tasks are non-contractible, the effort spent on the task can be observed by other agents (e.g., helping colleagues or working while on vacation). In addition, the value of such a non-contractible task is often uncertain, and some agents know more about it than other agents (e.g., senior employees know more about the importance of non-contractible work than junior employees do, or a salesperson who directly meets customers knows more about their demand than administrative officers).

This study builds a multitasking model in which an agent's effort can signal the non-contractible task's value to others. We highlight a new rationale that, for an agent to work harder on a non-contractible task, a principal may prefer to provide *higher-powered incentives* to a contractible task. Because the signaling game between agents (leading-by-example by Hermalin, 1998) is embedded in the multitasking model, the principal can indirectly control the incentives for agents to work on the non-contractible task by designing a payment scheme for the contractible task. We derive the condition in which the principal offers higher-powered incentives to a contractible task, in contrast to the result of the standard multitasking model (with no signaling). The results have implications to pay for performance, incentives to help other agents, and the prevention of overwork in multitasking situations.

In this model, the agent engages in two substitute tasks simultaneously. The principal can verify the performance of one task, while she cannot write any contract with regard to the other task. A key element in our model is that the agent has private information on the value of the non-contractible task (either high or low), and there is a follower (e.g., another agent, third party,

[1] For example, 2016 Scientific Background on the Prize in Economic Sciences in Memory of Alfred Nobel features three extensions of the basic moral-hazard model as highly influential and important: the multitasking model, incentives in teams, and career concerns. `https://www.nobelprize.org/prizes/economic-sciences/2016/press-release/` (accessed October 19, 2022).

or principal) who can also contribute to the non-verifiable task after observing the agent's action. In this setting, the agent and follower play a signaling game concerning the value of the non-contractible task. The signaling role of the agent's effort can be interpreted as "leading-by-example" studied by Hermalin (1998). The signaling effect has significant implications on the optimal incentive contracts the principal offers: the optimal contract may specify a high-powered incentive to the contractible task to suppress the agent's excessive effort on the non-contractible task. Intuitively, by providing a sufficiently high bonus to the contractible task, the agent does not work hard on the non-contractible task if its value is low. Hence, under the high-powered incentive to the contractible task, working hard on the non-contractible task becomes a credible signal with regard to the value of the non-contractible task. If this effect is sufficiently important for the principal, she provides a high-powered incentive to the contractible task (with a low fixed wage).[2] The difference between the optimal contract in our model and the one in the standard multitasking model can be substantial: The bonus upon the success of the contractible task may exceed the highest possible revenue from the contractible task.

These results have implications for managerial incentives in organizations. For example, suppose that a salesperson engages in both the main sales activity and other informal activities (e.g., participating in events in the community of a sales territory). Unlike the main sales activity, the informal activities do not directly contribute to sales performance, but they might affect the company's overall performance. Only the salesperson knows how each informal activity contributes to overall performance. Even if an informal activity is worthwhile, the company's support is necessary to succeed. Hence, if the salesperson knows that the informal activity contributes to overall performance, the company should support it. On the other hand, if the salesperson indulges in the informal activity just for fun (e.g., to host parties), the company should not support it. To alleviate inefficiencies from information asymmetry, it may be optimal for the company to provide a high bonus for the main sales activity (with low fixed payment), which suppresses excessive effort in the informal activity by the salesperson who knows that the informal activity is not worthwhile.

Related literature. This paper relates mainly to two literatures: multitasking and leadership.[3] In the literature on multitasking initiated by Holmstrom and Milgrom (1991), our study is mainly

[2] We use female pronouns to refer to a principal and male pronouns to refer to an agent.
[3] We will introduce other related studies in section 5 which provides some possible applications.

related to those in which some agents have private information. Bénabou and Tirole (2016) investigate the impact of labor-market competition on contracts in a Hotelling model, in which both multitasking and screening problems matter. They show that competition for unobservable talents causes high-powered incentives for the task easily measured, which leads to distorted decisions and efficiency losses. Although they mainly focus on private information about the quality of a contractible task, they also highlight private information about the quality of a non-contractible task (called "ethical motivation") as an alternative setting. However, in this setting, they focus on screening and show that the first-best outcome could be achieved under perfect labor-market competition. Kosfeld and von Siemens (2011) study multitasking problems with worker heterogeneity regarding social preferences. They show that firms employing "conditionally cooperative" workers moght make strictly positive profits even in a competitive labor market due to adverse selection of the workers and discuss its implications for different corporate cultures. While these studies focus on adverse selection and labor market competition, we shed light on the signaling role in the standard multitasking setting. We show that the principal may offer a high-powered incentive with a lower fixed wage relative to the symmetric information situation because the high-powered incentive improves the credibility of the signal for the value of the non-contractible task. By this mechanism, the bonus on the contractible task may exceed the highest possible revenue from the contractible task.[4]

Our signaling effect is closely related to the literature on the economics of leadership. Specifically, Hermalin (1998) and the following literature on leading-by-example investigate how an action taken by a leader agent can credibly convey the agent's private information about the value of a task to the following agents. Hermalin (1998) studies a team-production problem and shows that the sequence of moves may improve team performance and enhance efficiency because the leader's action serves as a signal of the value of the task.[5] While Hermalin (1998) assumes that an agent's action fully conveys the true value, Komai et al. (2007) extend Hermalin (1998) to state that the leader's action partially informs the value. They show that efficiency can be improved under partial

[4] Sinclair-Desgagné (1999) shows how selective auditing can lead to high-powered incentives under multitasking. Our model features a different mechanism and also provides additional insights, e.g., when the bonus upon the success on the contractible task exceeds the highest benefit of the contractible task.

[5] Kobayashi and Suehiro (2005) and Huck and Rey-Biel (2006) study how leadership can arise endogenously in leading-by-example models. Costa (2014) provides historical evidence on leading-by-example. Vesterlund (2003)and Andreoni (2006) investigate dynamic public-good provision problems with a focus on leading-by-example. For the review on the economics of leadership, see Hermalin (2012).

information. Komai and Stegeman (2010) analyze the above team-production model by incorporating coordination problems, and highlight a new trade-off between shirking and coordination. Komai and Stegeman (2010) also study the problem of choosing an optimal leader by assuming heterogeneous agents. Zhou (2016) studies leadership in a hierarchical structure and optimal organizational structure endogenously. To the best of our knowledge, however, previous studies on leading-by-example have not investigated the implications of performance-based payment scheme. We analyze optimal performance-based payment schemes in the presence of leading-by-example by focusing on multitasking situations, highlighting the new mechanism and implications discussed above.

Organization of the paper. This remainder of this article is organized as follows. Section 2 sets up our model. Section 3 illustrates our result with a binary action space. Section 4 analyzes the optimal contract and discusses its implications with providing an example. Section 5 covers applications of our model to CEO payments, incentives to help other agents, and overwork. Section 6 concludes. All proofs are in Appendix.

2 Setup

There are three parties: a principal (P), an agent (A), and a follower (F).[6] When the principal hires the agent, the agent can make non-verifiable efforts on two tasks ($n = 1, 2$). The outcome of task 1 is verifiable and solely depends on the agent's effort. Following the standard multitasking framework, we focus on a situation in which the principal cannot write any contract with regard to the other task (task 2). The outcome of task 2 depends both on the agent's effort and on the decision made by the follower after observing the agent's effort in task 2.

At the beginning, the principal offers the agent a contract (w, b), where w is an unconditional fixed payment and b is a bonus upon success in task 1. If the agent rejects the contract, then he receives the outside option $\bar{u} \in \mathbb{R}$. If the agent accepts the contract, he exerts effort $e_n \in \mathbb{R}_+$ for $n = 1, 2$. Let $e = (e_1, e_2)$ denote the agent's effort profile.[7] Task 1 succeeds with probability $q(e_1)$ and fails with probability $1 - q(e_1)$ where $q(e_1) \in [0, 1)$ for all $e_1 \in \mathbb{R}_+$. $q(\cdot)$ is strictly increasing,

[6] Throughout the analysis, we assume that there are three different parties. However, the follower may not necessary be a third party. For example, the principal can also serve the role of the follower.

[7] Section 3 analyzes the illustrative case in which the agent's effort choice is binary.

twice continuously differentiable, and $\lim_{e_1 \to 0} q'(e_1) = +\infty$. When task 1 succeeds, the principal receives $V > 0$; otherwise, she receives 0 from task 1.

The agent has private information on the value of task 2, denoted by $\theta \in \{\underline{\theta}, \overline{\theta}\}$ with $0 < \underline{\theta} < \overline{\theta}$ and $\Delta_\theta \equiv \overline{\theta} - \underline{\theta}$. There is a common prior belief that $\theta = \overline{\theta}$ with probability $\mu \in (0,1)$. Denote by $E(\theta) = \mu\overline{\theta} + (1-\mu)\underline{\theta}$ and $\Delta_\theta = \overline{\theta} - \underline{\theta}$. When the agent takes up the contract, he privately observes the true value of θ. Because the follower cannot observe θ, the follower infers the value of θ by observing the agent's effort for task 2. The decision of the follower is denoted by $d \in \mathbb{R}_+$.

The payoff from task 2 for each player i ($i = P, A, F$) is $(x_i e_2 + y_i d)\theta$, where $x_i, y_i \geq 0$ and $x_A, y_A > 0$. The agent's effort cost for (e_1, e_2) is $c_A(e_1, e_2)$, where $c_A(e_1, e_2)$ is strictly increasing, twice continuously differentiable, strictly convex in (e_1, e_2), and $\frac{\partial^2 c_A(e_1,e_2)}{\partial e_1 \partial e_2} > 0$. The follower's effort cost is $c_F(d)$, where $c_F(0) = c'_F(0) = 0$, $c_F(\cdot)$ is strictly increasing, strictly convex, twice continuously differentiable, and $\lim_{d\to\infty} c'_F(d) = \infty$. We assume that for any fixed $d \geq 0$ and $b > 0$, the agent's optimal effort choice (e_1, e_2) is unique, interior, and satisfies the second-order condition.[8]

The timing is summarized as follows:

1. The principal offers a contract (w, b) to the agent.
2. The agent decides whether or not to accept the contract.
3. The agent receives a signal about θ.
4. The agent chooses $e = (e_1, e_2)$.
5. The follower observes e_2 and then makes a decision d.
6. The outputs are realized and payments are executed.

Note that the agent's action depends on θ and the follower's action depends on e_2. Let $e^\theta = (e_1^\theta, e_2^\theta)$ be the action profile for the agent with type θ. Let I_{e_2} be the information set in which the follower who observed e_2 makes a decision. Let $\mu_{e_2} = Prob\{\theta = \overline{\theta}|e_2\}$ be the follower's posterior belief

[8] Specifically, we assume $q''(e_1)b - \frac{\partial^2 c_A(e_1,e_2)}{\partial e_1^2} < 0$ and $-\left(q''(e_1)b - \frac{\partial^2 c_A(e_1,e_2)}{\partial e_1^2}\right)\frac{\partial^2 c_A(e_1,e_2)}{\partial e_2^2} - \left(\frac{\partial^2 c_A(e_1,e_2)}{\partial e_1 \partial e_2}\right)^2 > 0$ for the second-order condition. A sufficient condition for the unique and interior effort choice is that $x_A > 0$, $q''(\cdot) \leq 0$, and $\frac{\partial c_A(e_1,e_2)}{\partial e_1}|_{e_1=0} = \frac{\partial c_A(e_1,e_2)}{\partial e_2}|_{e_2=0} = 0$.

on θ after observing e_2.[9] Let $d(\mu_{e_2})$ be the decision of the follower who holds the posterior belief μ_{e_2}. For continuation games played between the agent and follower, we adopt perfect-Bayesian equilibria (PBE).

3 Illustration with Binary Effort

This section illustrates a simple case in which each party's effort choice is binary: $e \in \{(1,0),(0,1)\}$, $d \in \{0,1\}$, task 1 succeeds with probability $q(1) = q \in (0,1)$ and $q(0) = 0$, the agent's effort cost is $c_A = 0$, and the follower's effort cost is $c_F \cdot d$ with $c_F > 0$.[10] To focus on the most interesting case, we make the following assumptions throughout this section:

Assumption 1. (i) $qV > \overline{u}$, (ii) $y_F E(\theta) < c_F < y_F \overline{\theta}$, (iii) $x_A \overline{\theta} < (x_A + y_A)\underline{\theta}$.

Assumption 1 (i) states that working on task 1 yields a positive social surplus. Assumption 1 (ii) implies that the follower does not prefer to work on task 2 if there is no information. Assumption 1 (iii) is to focus on our most interesting case in which the agent's payoff on task 2 crucially depends on the follower's decision. In the rest of this section, we analyze the case where the principal sets the lowest bonus by which her intended equilibrium is played.[11] For continuation games played between the agent and the follower, we focus on a pure-strategy PBE whenever it exists.

We first illustrate optimal contracts in the benchmark case where there is no asymmetric information; all parties observe θ when the agent observes it. Let (w^*, b^*) denote the optimal contract for this benchmark case. In this case, the follower chooses $d = 1$ if and only if $\theta = \overline{\theta}$ regardless of the agent's effort. Also, the type-θ agent chooses $(1,0)$ if and only if $qb \geq x_A\theta$. Given the above actions, the optimal bonus level is either $b^* = \frac{x_A\overline{\theta}}{q}$ (when the principal prefers both types of agent to work on task 1), $b^* = \frac{x_A\underline{\theta}}{q}$ (when the principal prefers only the type-$\underline{\theta}$ agent to work on task 1), or $b^* = 0$ (when the principal prefers both types of agent to work on task 2). Hence, the highest possible bonus level in this benchmark case is $b^* = \frac{x_A\overline{\theta}}{q}$.

[9] For simplicity, we assume that the follower observes e_2 but not e_1. All our results remain qualitatively the same if the follower can observe both e_1 and e_2.

[10] In this setup, none of the following results changes when $e = (0,0)$ is included in the agent's effort choice. This is because $e = (0,0)$ never constitutes the optimal contract under $c_A = 0$.

[11] Under binary effort and linear utility, multiple optimal contracts exist under certain parameters. As we show in section 4, the optimal contract becomes unique under continuous effort. Alternatively, if we incorporate the agent's risk aversion into the binary model, the optimal contract will be uniquely pinned down to the specified one.

In the following, we demonstrate that the principal may set a bonus strictly higher than $\frac{x_A\overline{\theta}}{q}$ when θ is the private information of the agent. After observing the principal's offer (w, b), there are three pure-strategy PBE played by the agent and follower, which depend on the values of b.[12] First, there exists a separating equilibrium in which the type-$\overline{\theta}$ agent works on task 2 and the type-$\underline{\theta}$ agent works on task 1 if $\frac{(x_A+y_A)\underline{\theta}}{q} \leq b \leq \frac{(x_A+y_A)\overline{\theta}}{q}$. In this equilibrium, the follower works on task 2 if and only if the agent exerts effort on task 2. Second, a pooling equilibrium exists in which both types of the agent work on task 1 if $b \geq \frac{x_A\overline{\theta}}{q}$.[13] Third, there is a pooling equilibrium in which both types of the agent work on task 2 if $b \leq \frac{x_A\underline{\theta}}{q}$. In both pooling equilibria, the follower never works on task 2 on the equilibrium path by Assumption 1 (ii).

We now derive the conditions under which the principal offers higher-powered incentives than the benchmark cases. Let (w^{**}, b^{**}) denote the optimal contract in the case of asymmetric information. By Assumption 1 (i), the principal prefers the agent to accept a contract. Under the optimal contract, the agent's individual rationality constraint is binding. There are three possible cases. First, given that both types of the agent work on task 1, the optimal contract is $(w^{**}, b^{**}) = \left(\overline{u} - x_A\overline{\theta}, \frac{x_A\overline{\theta}}{q}\right)$ and the principal's payoff is $\pi_{pool-1} = qV - \overline{u}$. Second, given that both types of the agent work on task 2, the optimal contract is $(w^{**}, b^{**}) = (\overline{u} - x_A E(\theta), 0)$ and the principal's payoff is $\pi_{pool-2} = (x_P + x_A)E(\theta) - \overline{u}$. Third, given that the type-$\overline{\theta}$ agent works on task 2 and the type-$\underline{\theta}$ agent works on task 1, the optimal contract is $(w^{**}, b^{**}) = \left(\overline{u} - (x_A + y_A)E(\theta), \frac{(x_A+y_A)\underline{\theta}}{q}\right)$ and the principal's payoff is $\pi_{sep} = \mu(x_P + x_A + y_P + y_A)\overline{\theta} + (1-\mu)qV - \overline{u}$.

We derive the following proposition by comparing among the principal's payoffs in these three cases.

Proposition 1. Suppose Assumption 1 holds. Then, the optimal contract is $(w^{**}, b^{**}) = \left(\overline{u} - (x_A + y_A)E(\theta), \frac{(x_A+y_A)\underline{\theta}}{q}\right)$ if and only if the following condition holds:

$$(x_P + x_A)\underline{\theta} - \frac{\mu}{1-\mu}(y_P + y_A)\overline{\theta} < qV < (x_P + x_A + y_P + y_A)\overline{\theta}. \quad (1)$$

[12] For some values of b, a pure-strategy PBE does not exist; hence a mixed-strategy PBE is played. In the proof of Proposition 1, we show that such mixed-strategy PBE is never implemented in the optimal contract. Furthermore, in the Supplementary Material, we derive all the PBE (including mixed strategies) and show that the optimal contract stated in Proposition 1 remains the same.

[13] If we impose the Intuitive Criterion (Cho and Kreps, 1987), then a pooling equilibrium in which both types of the agent working on task 1 fails to survive for $\frac{(x_A+y_A)\underline{\theta}}{q} < b < \frac{(x_A+y_A)\overline{\theta}}{q}$. However, note that the lowest bonus in this pooling equilibrium, which survives the Intuitive Criterion, is still $b = \frac{x_A\overline{\theta}}{q}$ by Assumption 1 (iii). A full characterization by imposing the Intuitive Criterion is presented in the Supplementary Material.

Proposition 1 characterizes the condition in which the optimal contract is supported by the separating equilibrium. From Assumption 1 (iii), $b^{**} = \frac{(x_A+y_A)\theta}{q}$ is strictly higher than $\frac{x_A\overline{\theta}}{q}$; the principal offers a strictly higher bonus level than the benchmark case. Intuitively, by providing a sufficiently high bonus to task 1 so that the agent never works on task 2 if the value of task 2 is low, the principal can credibly infer the value of the task 2. The following comparative statics holds. First, Condition (1) is more likely to be satisfied as $\overline{\theta}$ is higher. Second, Condition (1) is more likely to be satisfied as the benefit from the follower's support for task 2 (i.e., y_P or y_A) increases. Third, Condition (1) is more likely to be satisfied as the likelihood of $\overline{\theta}$ (i.e., μ) increases.

The signaling game between the agent and the follower can be viewed as a model of "leadership" by Hermalin (1998) and the following literature. In our model, the principal facing a multitasking problem attempts to take advantage of the agent's leadership. The type-$\overline{\theta}$ agent increases his effort level, which is a credible signal to make the follower support task 2. Our multitasking model — which embeds a signaling game — shows that the principal needs to provide a high-powered incentive for task 1 to prevent the type-$\underline{\theta}$ agent from pretending to be the type-$\overline{\theta}$ agent. As a result, the principal can save the fixed wage payment and increase her expected payoff. We examine this mechanism in detail in the next continuous-effort model.

4 Analysis

This section analyzes the model with continuous effort choices and derives its implications. Section 4.1 presents a benchmark case with symmetric information. Section 4.2 characterizes the PBE played between the agent and follower. Section 4.3 analyzes the optimal contract and provide an example.

4.1 Benchmark Result

In this subsection, we analyze a benchmark case with symmetric information. Suppose that all parties observe θ when the agent observes it.

We first characterize the actions of the follower and agent. Suppose that contract (w, b) is accepted. The follower maximizes $y_F d\theta - c_F(d)$ and hence chooses d^* such that $y_F\theta = c_F'(d^*)$. For consistency of notation with the analysis in the main model with private information, let $d^*(1)$ denote the follower's action when $\theta = \overline{\theta}$ and $d^*(0)$ denote the follower's action when $\theta = \underline{\theta}$.

Anticipating the follower's action, an agent with type θ maximizes $q(e_1)b + [x_A e_2 + y_A d^*]\theta - c_A(e_1, e_2)$. In this benchmark case, the agent with type θ chooses $(e_1^{\theta *}, e_2^{\theta *})$ such that

$$q'(e_1^{\theta *})b - \frac{\partial c_A(e_1^{\theta *}, e_2^{\theta *})}{\partial e_1} = 0, \tag{2}$$

$$x_A\theta - \frac{\partial c_A(e_1^{\theta *}, e_2^{\theta *})}{\partial e_2} = 0. \tag{3}$$

In the proof of Proposition 2, we show that $\frac{\partial e_1^{\theta *}}{\partial b} > 0$ and $\frac{\partial e_2^{\theta *}}{\partial b} < 0$.

Next, we derive the optimal contract. Let (w^*, b^*) denote the optimal contract for this benchmark case. In the optimal contract, the agent's individual rationality constraint binds (otherwise, the principal can reduce w^*):

$$w = \bar{u} - \mu[q(e_1^{\overline{\theta}*})b + (x_A e_2^{\overline{\theta}*} + y_A d(1))\overline{\theta} - c_A(e_1^{\overline{\theta}*}, e_2^{\overline{\theta}*})] - (1-\mu)[q(e_1^{\underline{\theta}*})b + (x_A e_2^{\underline{\theta}*} + y_A d(0))\underline{\theta} - c_A(e_1^{\underline{\theta}*}, e_2^{\underline{\theta}*})].$$

Then, the principal's profits are

$$\begin{aligned}\Pi =& \mu \left\{ q(e_1^{\overline{\theta}*})(V - b) + [x_P e_2^{\overline{\theta}*} + y_P d(1)]\overline{\theta} \right\} + (1-\mu) \left\{ q(e_1^{\underline{\theta}*})(V - b) + [x_P e_2^{\underline{\theta}*} + y_P d(0)]\underline{\theta} \right\} - w \\ =& \mu \left\{ q(e_1^{\overline{\theta}*})V + [(x_P + x_A)e_2^{\overline{\theta}*} + (y_P + y_A)d(1)]\overline{\theta} - c_A(e_1^{\overline{\theta}*}, e_2^{\overline{\theta}*}) \right\} \\ &+ (1-\mu) \left\{ q(e_1^{\underline{\theta}*})V + [(x_P + x_A)e_2^{\underline{\theta}*} + (y_P + y_A)d(0)]\underline{\theta} - c_A(e_1^{\underline{\theta}*}, e_2^{\underline{\theta}*}) \right\} - \bar{u},\end{aligned} \tag{4}$$

Proposition 2 provides the optimal level of bonus in the benchmark case:

Proposition 2 (Symmetric Information)**.** Suppose all parties observe θ. Then, the optimal contract (w^*, b^*) specifies

$$b^* = V + \frac{x_P \overbrace{\left[\mu \frac{\partial e_2^{\overline{\theta}*}}{\partial b}\overline{\theta} + (1-\mu)\frac{\partial e_2^{\underline{\theta}*}}{\partial b}\underline{\theta} \right]}^{<0}}{\underbrace{\mu \frac{\partial e_1^{\overline{\theta}*}}{\partial b} q'(e_1^{\overline{\theta}*}) + (1-\mu)\frac{\partial e_1^{\underline{\theta}*}}{\partial b} q'(e_1^{\underline{\theta}*})}_{>0}}. \tag{5}$$

In particular, $b^* \leq V$ holds strict inequality when $x_P > 0$.

The intuition behind Proposition 2 is akin to Holmstrom and Milgrom (1991): when effort costs are complements, the principal sets a low incentive for a contractible task.[14]

[14] As another benchmark case, if there is no benefit from the follower's decision (i.e., $y_i = 0$ for $i = P, A$), then the optimal contract is the same as that specified in Proposition 2.

4.2 Perfect-Bayesian Equilibria played by the Agent and the Follower

We now analyze the main model where θ is the agent's private information. This subsection analyzes the continuation games played by the agent and follower after observing the principal's offer (w, b). We adopt perfect-Bayesian equilibria that satisfy the Intuitive Criterion (Cho and Kreps, 1987).

Follower's Best Responses Given the follower's belief $\mu_{e_2} \in [0,1]$, the follower maximizes $[\mu_{e_2}\overline{\theta} + (1-\mu_{e_2})\underline{\theta}]y_F d - c_F(d)$. Hence, the follower chooses $d(\mu_{e_2})$ such that $[\mu_{e_2}\overline{\theta} + (1-\mu_{e_2})\underline{\theta}]y_F = c_F'(d(\mu_{e_2}))$. By the assumption of $c_F(\cdot)$, $d(\mu_{e_2})$ is unique and strictly increasing in μ_{e_2}.

Perfect-Bayesian equilibria Given μ_{e_2} and b, an agent with type θ maximizes $q(e_1)b + [x_A e_2 + y_A d(\mu_{e_2})]\theta - c_A(e_1, e_2)$. Let $e_i^{\theta**}$ denote the equilibrium action of the type-θ agent in task i. In what follows, we derive a separating equilibrium by restricting the off-path belief that the follower always thinks that a deviating agent must be the low type: $\mu_{e_2} = 0$ for any $e_2 \neq e_2^{\overline{\theta}**}$.

Then, the type-$\underline{\theta}$ agent's action profile must satisfy the following conditions:

$$q'(e_1^{\underline{\theta}**})b - \frac{\partial c_A(e_1^{\underline{\theta}**}, e_2^{\underline{\theta}**})}{\partial e_1} = 0, \tag{6}$$

$$x_A\underline{\theta} - \frac{\partial c_A(e_1^{\underline{\theta}**}, e_2^{\underline{\theta}**})}{\partial e_2} = 0, \tag{7}$$

$$q(e_1^{\underline{\theta}**})b + [x_A e_2^{\underline{\theta}**} + y_A d(0)]\underline{\theta} - c_A(e_1^{\underline{\theta}**}, e_2^{\underline{\theta}**}) \geq \max_{e_1} \left\{ q(e_1)b + [x_A e_2^{\overline{\theta}**} + y_A d(1)]\underline{\theta} - c_A(e_1, e_2^{\overline{\theta}**}) \right\}. \tag{8}$$

(6) and (7) are conditions in which the type-$\underline{\theta}$ agent takes his best responses for $e_2 \neq e_2^{\overline{\theta}**}$. (8) means that the type-$\underline{\theta}$ agent does not mimic the type-$\overline{\theta}$ agent by choosing $e_2^{\overline{\theta}**}$.

The type-$\overline{\theta}$ agent's action profile must satisfy the following conditions:

$$q'(e_1^{\overline{\theta}**})b - \frac{\partial c_A(e_1^{\overline{\theta}**}, e_2^{\overline{\theta}**})}{\partial e_1} = 0, \tag{9}$$

$$q(e_1^{\overline{\theta}**})b + [x_A e_2^{\overline{\theta}**} + y_A d(1)]\overline{\theta} - c_A(e_1^{\overline{\theta}**}, e_2^{\overline{\theta}**}) \geq \max_{e_1, e_2} \left\{ q(e_1)b + [x_A e_2 + y_A d(0)]\overline{\theta} - c_A(e_1, e_2) \right\}. \tag{10}$$

(9) implies that the type-$\overline{\theta}$ agent optimally chooses e_1 given $e_2 = e_2^{\overline{\theta}**}$. (10) represents the condition in which the type-$\overline{\theta}$ agent does not have the incentive to choose $e_2 \neq e_2^{\overline{\theta}**}$. By the assumption of the second-order condition, the set of separating equilibria given $d(\mu_{e_2})$ is characterized as a tuple of actions that satisfies (6)-(10).

Note that $e^{\overline{\theta}*}$, which is characterized by (2) and (3) in the benchmark case, maximizes the type-$\overline{\theta}$ agent's payoff when $\mu_{e_2} = 1$, i.e.,

$$e^{\overline{\theta}*} = \text{argmax}_{(e_1,e_2)} \left\{ q(e_1)b + [x_A e_2 + y_A d(1)]\overline{\theta} - c_A(e_1, e_2) \right\}.$$

Given $e^{\underline{\theta}**}$ and $e^{\overline{\theta}**}$, let $\tilde{e}_2$ denote the effort level of task 2 where $\tilde{e}_2$ is the highest value among which (8) binds. Let $\tilde{e}_1$ denote the associated effort level of task 1.

In the proof of Proposition 3, we show that if the type-$\overline{\theta}$ agent's action profile in a separating equilibrium survives the Intuitive Criterion, then it is either $e^{\overline{\theta}**} = e^{\overline{\theta}*}$ or $e^{\overline{\theta}**} = \tilde{e}_2$ with $\tilde{e}_2 > e_2^{\overline{\theta}*}$. In either case, the type-$\underline{\theta}$ agent's equilibrium action profile is the same as in the benchmark case: $e^{\underline{\theta}**} = e^{\underline{\theta}*}$. Note that each action profile of the type-$\overline{\theta}$ agent is uniquely determined, implying that the separating equilibrium that survives the Intuitive Criterion is unique. We also show that all pooling equilibria (i.e., both types of the agent choose the same task-2 effort level) fail to survive the Intuitive Criterion. Proposition 3 summarizes the results.

Proposition 3. In continuation games played by the agent and follower, a perfect-Bayesian equilibrium that survives the Intuitive Criterion is unique. It is a separating equilibrium: different types of the agent choose different levels of e_2.

4.3 Optimal Contract

We now characterize the optimal contract given the above equilibrium actions. Note that the equilibrium effort levels of the type-$\underline{\theta}$ agent are $e^{\underline{\theta}**} = e^{\underline{\theta}*}$, and the equilibrium effort level of task 2 of the type-$\overline{\theta}$ agent is given by $e_2^{\overline{\theta}**} = \max\{e_2^{\overline{\theta}*}, \tilde{e}_2\}$.

In the optimal contract, the individual rationality constraint binds; otherwise, the principal can reduce w. Hence, the principal sets:

$$\begin{aligned} w = \bar{u} &- \mu[q(e_1^{\overline{\theta}**})b + (x_A e_2^{\overline{\theta}**} + y_A d(1))\overline{\theta} - c_A(e_1^{\overline{\theta}**}, e_2^{\overline{\theta}**})] \\ &-(1-\mu)[q(e_1^{\underline{\theta}**})b + (x_A e_2^{\underline{\theta}**} + y_A d(0))\underline{\theta} - c_A(e_1^{\underline{\theta}**}, e_2^{\underline{\theta}**})]. \end{aligned}$$

The principal's profits are

$$\begin{aligned} \Pi =& \mu \left\{ q(e_1^{\overline{\theta}**})(V-b) + [x_P e_2^{\overline{\theta}**} + y_P d(1)]\overline{\theta} \right\} + (1-\mu) \left\{ q(e_1^{\underline{\theta}**})(V-b) + [x_P e_2^{\underline{\theta}**} + y_P d(0)]\underline{\theta} \right\} - w \\ =& \mu \left\{ q(e_1^{\overline{\theta}**})V + [(x_P + x_A)e_2^{\overline{\theta}**} + (y_P + y_A)d(1)]\overline{\theta} - c_A(e_1^{\overline{\theta}**}, e_2^{\overline{\theta}**}) \right\} \\ &+ (1-\mu) \left\{ q(e_1^{\underline{\theta}**})V + [(x_P + x_A)e_2^{\underline{\theta}**} + (y_P + y_A)d(0)]\underline{\theta} - c_A(e_1^{\underline{\theta}**}, e_2^{\underline{\theta}**}) \right\} - \bar{u}, \end{aligned} \quad (11)$$

where $e_2^{\overline{\theta}**} = \max\{e_2^*, \tilde{e}_2\}$. By solving this with respect to b in each case of $e_2^{\overline{\theta}**}$, Proposition 4 provides the optimal level of bonus in each case:

Proposition 4. (i) Consider the case where $e_2^{\overline{\theta}*} \geq \tilde{e}_2$. Then, the optimal contract (w^{**}, b^{**}) is given by (5). In particular, $b^{**} \leq V$ holds strict inequality if $x_P > 0$.

(ii) Consider the case where $e_2^{\overline{\theta}*} < \tilde{e}_2$. Then, the optimal contract (w^{**}, b^{**}) specifies

$$b^{**} = V + \frac{\overbrace{\mu \frac{\partial \tilde{e}_2}{\partial b}}^{<0} \left[x_A \overline{\theta} - \frac{\partial c_A(\tilde{e}_1, \tilde{e}_2)}{\partial e_2} \right] + x_P \overbrace{\left[\mu \frac{\partial \tilde{e}_2}{\partial b} \overline{\theta} + (1-\mu) \frac{\partial e_2^{\underline{\theta}**}}{\partial b} \underline{\theta} \right]}^{<0}}{\underbrace{\mu \frac{\partial \tilde{e}_1}{\partial b} q'(\tilde{e}_1) + (1-\mu) \frac{\partial e_1^{\underline{\theta}**}}{\partial b} q'(e_1^{\underline{\theta}**})}_{>0}}. \tag{12}$$

Specifically, when $x_P = 0$, $b^{**} > V$ if and only if $x_A \overline{\theta} < \frac{\partial c_A(\tilde{e}_1, \tilde{e}_2)}{\partial e_2}$.

Proposition 4 (i) covers the case where the signaling effect is not a relevant concern. In this case, the optimal contract is derived using the same logic as in the benchmark case under symmetric information. The intuition behind Proposition 4 (ii) is as follows. In this case, the type-$\underline{\theta}$ agent exerts excessive effort on task 2 because of a signaling motive. Then, setting a higher bonus on task 1 reduces the type-$\underline{\theta}$ agent's effort level on task 2, which improves efficiency under excessive signaling. By doing so, the principal can lower the fixed wage and increase expected profits.

Note that if $x_A \overline{\theta} \geq \frac{\partial c_A(\tilde{e}_1, \tilde{e}_2)}{\partial e_2}$ (i.e., the marginal return on task 2 for the type-$\overline{\theta}$ agent is non-negative), the optimal contract specifies $b^{**} \leq V$. When $x_P = 0$ so that $b^{**} = V$ in the benchmark case, the inverse is also true: if $x_A \overline{\theta} < \frac{\partial c_A(\tilde{e}_1, \tilde{e}_2)}{\partial e_2}$, then the optimal contract specifies $b^{**} > V$. When $x_P > 0$ so that $b^{**} < V$ in the benchmark case, another effect arises: the principal directly receives payoffs from task 2, which puts downward pressure on the optimal bonus level. Still, $b^{**} > V$ is optimal if this effect is outweighed by the effect of mitigating the excessive signaling by the type-$\overline{\theta}$ agent.

Example We describe an example for Proposition 4 in which $q(e_1) = e_1$ (< 1), $c_F(d) = \frac{d^2}{2}$, $c_A(e) = \frac{(e_1)^2}{2} + \frac{(e_2)^2}{2} + s e_1 e_2$ where $s \in (0, 1)$, $x_A = x_F = y_A = y_F = y_P = 1$, and $x_P \geq 0$.

We first analyze the benchmark case under symmetric information. Because the follower also observes θ, $d^* = \theta$ for $\theta \in \{\underline{\theta}, \overline{\theta}\}$. The type-$\theta$ agent maximizes his expected payoff $w + e_1^{\theta} b + (e_2^{\theta} +$

$d^*)\theta - c_A(e^\theta)$ given (w, b); hence, the optimal action profile is $(e_1^{\theta*}, e_2^{\theta*}) = \left(\frac{b-s\theta}{1-s^2}, \frac{\theta-sb}{1-s^2}\right)$. Given the above and binding individual rationality constraint, the principal solves the following problem:

$$\max_b \mu\left[e_1^{\overline{\theta}*}V + \{(x_P+1)e_2^{\overline{\theta}*} + 2\overline{\theta}\}\overline{\theta} - c_A(e^{\overline{\theta}*})\right] + (1-\mu)\left[e_1^{\underline{\theta}*}V + \{(x_P+1)e_2^{\underline{\theta}*} + 2\underline{\theta}\}\underline{\theta} - c_A(e^{\underline{\theta}*})\right] - \bar{u}.$$

By solving it, $b^* = V - sx_PE(\theta)$.

Next, we analyze our main case. Let $e_i^{\theta**}$ denote the equilibrium action of a type-θ agent in task i. For the type-$\underline{\theta}$ agent, $(e_1^{\underline{\theta}**}, e_2^{\underline{\theta}**}) = (e_1^{\underline{\theta}*}, e_2^{\underline{\theta}*}) = \left(\frac{b-s\underline{\theta}}{1-s^2}, \frac{\underline{\theta}-sb}{1-s^2}\right)$. For the type-$\overline{\theta}$ agent, $e_2^{\overline{\theta}**} = \max\{e_2^{\overline{\theta}*}, \tilde{e}_2\}$, where $e_2^{\overline{\theta}*}$ is determined at the type-$\overline{\theta}$ agent's payoff and $\tilde{e}_2$ is determined by holding the type-$\underline{\theta}$ agent's incentive compatibility constraint with equality. When $\max\{e_2^{\overline{\theta}*}, \tilde{e}_2\} = e_2^{\overline{\theta}*}$, the optimal action profile and the optimal contract are the same as those in the benchmark case. When $\max\{e_2^{\overline{\theta}*}, \tilde{e}_2\} = \tilde{e}_2$, $(e_1^{\overline{\theta}**}, e_2^{\overline{\theta}**}) = \left(\frac{b-s\underline{\theta}-sD}{1-s^2}, \frac{\underline{\theta}-sb+D}{1-s^2}\right)$ where $D = \sqrt{2(1-s^2)\underline{\theta}\Delta_\theta}$. The optimal bonus is $b^{**} = V - sx_PE(\theta) + \mu s(D - \Delta_\theta)$ and w^{**} is determined as holding the individual rationality constraint with equality. Consequently, when $D > \Delta_\theta$ or $2(1-s^2)\underline{\theta} > \Delta_\theta$, the optimal bonus for task 1 when the agent has private information on task 2 is higher than when all parties have symmetric information (i.e., $b^{**} > b^*$). This example illustrates how the bonus upon the success of the contractible task may exceed the revenue from contractible task V.

5 Applications

This section presents three applications of our model: pay for performance, incentives to help other agents, and prevention of overwork.

5.1 Pay for Performance

Murphy and Zábojnik (2004) discuss that there are two components for CEOs: general management skills (i.e., the skills transferable across companies) that are priced in a labor market and firm-specific managerial skills (i.e., information about its products, suppliers, and clients) that are unpriced.

In our model, unpriced managerial skills are private information. Investing in such an unpriced skill by closely communicating with suppliers and clients can be beneficial to the company. On the other hand, a CEO might enjoy having parties, wining and dining, or empire-building. As mentioned in Hermalin (2016), J.P. Morgan estimated that US corporate expenditure on entertainment was about \$7.85 billion in 2011.[15] Harford (1999) reports that CEOs may use abundant internal resources for empire-building: making bad acquisitions and destroying shareholder value.

For optimal signaling (to stakeholders, creditors, or employees), our results imply that the price for the general management skills, which is discussed in Murphy and Zábojnik (2004), is even higher in the multitasking setting than in the corresponding single-task setting. We highlight that high payments for general management skills can *improve* the efficiency of unpriced firm-specific management skills, because it alleviates excessive unpriced activities, such as wining and dining or empire-building.

5.2 Help

Our results have implications for an agent's incentives to help other agents (Itoh, 1991; Ishihara, 2017; Tymula, 2017). An agent may allocate his effort to a non-contractible activity that helps other agents (i.e., task 2 in our model). We show that a lower (indirect) incentive to help other agents (i.e., increasing b) may increase the credibility of the signal and incentives to help other agents. In this sense, our results highlight a new rationale of the enhancement of cooperation among agents by providing high-powered incentives to individual performance.

For example, consider a sales department that provides high-powered incentives for individual sales performance. While each agent can work on individual sales, he can also help his colleagues (e.g., by sharing market information among agents), though it crowds out his sales performance. While such helpful activities can be beneficial, an agent may indulge in them even when it does not contribute much to organizational performance. The problem is that it is difficult for the principal to evaluate the extent to which how each helpful activity contributes to organizational performance. Hence, the principal may prefer to indirectly control the agents' informal activities through the individual sales activities, which is objectively easier to measure its performance. We show that when the value of the helpful activity is private information of the agent, the principal may provide high-powered incentives to the contractible activity. This leads to a separation between beneficial and non-beneficial activities.

Although we are unaware of any empirical study that jointly investigates incentives to help and agents' signaling motives, Drago and Garvey (1998) and Danilov et al. (2019) provide evidence of

[15] Hermalin (2016) analyzes how such wining and dining can facilitate inter-firm cooperation, even when firms can use their own incentive systems.

incentives to help other work members. From the perspective of the classical multitasking model, in which tasks are substitutes, an increased incentive would be detrimental as it reduces the effort on the non-contractible task. However, given our signaling model, the reduced effort may be efficiency-improving, as it alleviates excessive signaling in the non-contractible task.

5.3 Overwork

Our mechanism could also have implications for overworking. As an example of overwork, the Ministry of Health, Labour and Welfare in Japan reported that more than half of full-time employees experienced a high (or an extremely high) degree of fatigue (Mizuho Information & Research Institute Inc., 2016). Galinsky et al. (2005) review overwork in the US and argue that one third of all US employees could be viewed as chronically overworked. Importantly, Galinsky et al. (2005) report that being overworked seems to be caused by multitasking (e.g., jumping from task to task) and exerting effort on non-contractable work (e.g., cell-phone calls during non-work times).

In contrast to the conventional wisdom that high-powered incentives lead to excessive effort, our multitasking model implies that the principal may provide high-powered incentives to the contractible task to *mitigate* an agent's excessive effort on the non-contractible task. In this sense, our results offer new insights into bonus cultures and overwork.

6 Concluding Remarks

This study investigates multitasking problems in which some agents have private information on the value of a non-contractible task. We characterize the conditions in which the principal provides high-powered incentives for the contractible task, which contrasts with the finding by Holmstrom and Milgrom (1991) Our results help to understand leadership in organizations, CEO pay for performance, incentives to help other workers, and prevention of overwork.

Beyond the applications discussed, our mechanism can be applied to several important topics. One potential application is signaling in charitable giving (e.g., Vesterlund 2003; Andreoni 2006). Suppose a donor has private information (e.g, quality of charity) and initially contributes. This lead donor's contribution may signal to other potential donors who might also contribute. Designing optimal charity schemes (e.g., tax deduction) in such a situation could be worth considering. Another potential application is the introduction of monitoring technologies, such as Sinclair-Desgagné

(1999) and Gil and Mondria (2011). How our signaling effect interacts with monitoring is left for future research.

Appendix: Proofs

Proof of Proposition 1.

We first study a benchmark case in which all parties observe the signal on the value of task 2. Next, we characterize PBE in the binary model with private information. Then, we derive optimal contracts.

Benchmark: Symmetric Information Because there is no private information, the follower's choice does not depend on the agent's choice of task 2. The follower chooses $d = 1$ when $\theta = \overline{\theta}$ and $d = 0$ when $\theta = \underline{\theta}$ by Assumption 1 (ii).

We first characterize the agent's behavior. Given (w, b) and θ is realized, the agent's payoff for his each choice is $u_\theta(1, 0) = w+qb+y_A d(\theta)\theta$ if he chooses $e = (1, 0)$ and $u_\theta(0, 1) = w+(x_A+y_A d(\theta))\theta$ if he chooses $e = (0, 1)$, where $d(\overline{\theta}) = 1$ and $d(\underline{\theta}) = 0$ which do not depend on the agent's action. Because $u_\theta(1, 0) \geq u_\theta(0, 1)$ if and only if $b \geq \frac{x_A\theta}{q}$, the agent with type θ chooses $e = (1, 0)$ if and only if $b \geq \frac{x_A\theta}{q}$.

We next derive the optimal contract in the benchmark case (w^*, b^*) depending on parameters. In each case, w^* is determined at holding the agent's individual rationality constraint with equality:

$$\mu[w^* + qb^* + y_A d(\overline{\theta})\overline{\theta}] + (1-\mu)[w^* + qb^* + y_A d(\underline{\theta})\underline{\theta}] = w^* + qb^* + \mu y_A\overline{\theta} = \overline{u} \quad \text{if } b^* \geq \frac{x_A\overline{\theta}}{q},$$

$$\mu[w^* + (x_A + y_A d(\overline{\theta}))\overline{\theta}] + (1-\mu)[w^* + qb^* + y_A d(\underline{\theta})\underline{\theta}] = w^* + (1-\mu)qb^* + \mu(x_A + y_A)\overline{\theta} = \overline{u} \quad \text{if } b^* \in \left[\frac{x_A\underline{\theta}}{q}, \frac{x_A\overline{\theta}}{q}\right),$$

$$\mu[w^* + (x_A + y_A d(\overline{\theta}))\overline{\theta}] + (1-\mu)[w + (x_A + y_A d(\underline{\theta}))\underline{\theta}] = w^* + \mu(x_A + y_A)\overline{\theta} + (1-\mu)x_A\underline{\theta} = \overline{u} \quad \text{if } b^* < \frac{x_A\underline{\theta}}{q}.$$

Then, the principal's expected payoffs for each care are as follows:

$$\pi_{b^* \geq \frac{x_A\overline{\theta}}{q}} = q(V - b^*) + \mu y_P\overline{\theta} - w^* = qV + \mu(y_A + y_P)\overline{\theta} - \overline{u},$$

$$\pi_{b^* \in \left[\frac{x_A\underline{\theta}}{q}, \frac{x_A\overline{\theta}}{q}\right)} = (1-\mu)q(V - b^*) + \mu x_P\overline{\theta} + \mu y_P\overline{\theta} - w^* = (1-\mu)qV + \mu(x_A + x_P + y_A + y_P)\overline{\theta} - \overline{u},$$

$$\pi_{b^* < \frac{x_A\underline{\theta}}{q}} = x_P E(\theta) + \mu y_P\overline{\theta} - w^* = \mu(x_A + x_P + y_A + y_P)\overline{\theta} + (1-\mu)(x_A + x_P)\underline{\theta} - \overline{u}.$$

By comparing these payoffs and by the assumption that the principal sets the lowest bonus by which her intended equilibrium is played, the optimal contracts are derived as follows:

$$(w^*, b^*) = \left(\overline{u} - x_A\overline{\theta} - \mu y_A\overline{\theta}, \frac{x_A\overline{\theta}}{q}\right) \quad \text{if } V \geq \frac{(x_A + x_P)\overline{\theta}}{q},$$
$$(w^*, b^*) = \left(\overline{u} - (1-\mu)x_A\underline{\theta} - \mu(x_A + y_A)\overline{\theta}, \frac{x_A\underline{\theta}}{q}\right) \quad \text{if } V \in \left[\frac{(x_A + x_P)\underline{\theta}}{q}, \frac{(x_A + x_P)\overline{\theta}}{q}\right),$$
$$(w^*, b^*) = \left(\overline{u} - (1-\mu)x_A\underline{\theta} - \mu(x_A + y_A)\overline{\theta}, 0\right) \quad \text{if } V < \frac{(x_A + x_P)\underline{\theta}}{q}.$$

Note that in any of the three cases, the principal sets $b^* \leq \frac{x_A\overline{\theta}}{q}$. Hence, the optimal bonus under symmetric information is at most $b^* = \frac{x_A\overline{\theta}}{q}$.

Perfect-Bayesian Equilibria Now we analyze the binary model with private information.

First, we derive separating equilibria. Because of Assumption 1 (ii), the candidate of a separating equilibrium is unique: $e^{\overline{\theta}} = (0,1)$, $e^{\underline{\theta}} = (1,0)$, $d(0) = 0$, and $d(1) = 1$. It is indeed an equilibrium if and only if $u_{\overline{\theta}}(1,0) \leq u_{\overline{\theta}}(0,1)$ and $u_{\underline{\theta}}(0,1) \leq u_{\underline{\theta}}(1,0)$, or equivalently, $\frac{(x_A+y_A)\underline{\theta}}{q} \leq b \leq \frac{(x_A+y_A)\overline{\theta}}{q}$.

Second, we derive pooling equilibria. Note that in any pooling equilibrium, because of Assumption 1 (ii), the follower chooses $d = 0$ on the equilibrium path. We focus on off-path beliefs in which the follower's belief is $\theta = \underline{\theta}$ after observing any deviation. We first consider the pooling equilibrium in which $e^{\overline{\theta}} = e^{\underline{\theta}} = (1,0)$. In this case, $u_\theta(1,0) = w + qb$. This kind of pooling equilibrium exists when $u_\theta(0,1) \leq u_\theta(1,0)$ for all θ, that is, $b \geq \frac{x_A\overline{\theta}}{q}$.[16] We next consider the pooling equilibrium in which $e^{\overline{\theta}} = e^{\underline{\theta}} = (0,1)$. In this case, $u_\theta(0,1) = w + x_A\theta$. This kind of pooling equilibrium exists when $u_\theta(1,0) \leq u_\theta(0,1)$ for all θ, that is, $b \leq \frac{x_A\underline{\theta}}{q}$.

Third, for parameters under a pure-strategy PBE does not exists, i.e., $b \in \left(\frac{x_A\underline{\theta}}{q}, \frac{x_A\overline{\theta}}{q}\right)$, we derive mixed-strategy equilibria.[17] In this case, the type-$\underline{\theta}$ agent randomizes his actions. Suppose that the type-$\underline{\theta}$ agent takes $e^{\underline{\theta}} = (0,1)$ with probability $l \in (0,1)$ and that the follower takes $d(1) = 1$ with probability $r \in (0,1)$ and $d(0) = 0$ with probability one. Then, the type-$\overline{\theta}$ agent takes $e^{\overline{\theta}} = (0,1)$ with probability one. Under the mixed-strategy equilibrium, the follower must choose r^{**} which induces the type-$\underline{\theta}$ to be indifferent: $qb = (1 - r^{**})x_A\underline{\theta} + r^{**}(x_A + y_A)\underline{\theta} \iff r^{**} = \frac{qb - x_A\underline{\theta}}{y_A\underline{\theta}}$. Note that $r^{**} \in (0,1)$ when $b \in (\frac{x_A\underline{\theta}}{q}, \frac{(x_A+y_A)\underline{\theta}}{q})$. Also, the type-$\underline{\theta}$ agent must choose l^{**} which induces

[16] In Supplementary Material, we show that this type of equilibrium fails the Intuitive Criterion when $\frac{(x_A+y_A)\underline{\theta}}{q} < b < \frac{(x_A+y_A)\overline{\theta}}{q}$.

[17] In Supplementary Material, we derive all mixed-strategy PBE. We also show that the optimal contract never induces a mixed-strategy PBE to be played.

the follower to be indifferent: $\frac{\mu}{\mu+(1-\mu)l^{**}} y_F\overline{\theta} + \frac{(1-\mu)l^{**}}{\mu+(1-\mu)l^{**}} y_F\underline{\theta} - c_F = 0 \Longleftrightarrow l^{**} = \frac{\mu}{1-\mu}\frac{\overline{\theta}y_F - c_F}{c_F - \underline{\theta}y_F}$. Note that $l^{**} \in (0,1)$ by Assumption 1 (ii).

Optimal Contracts We now derive the conditions in which (even when the principal sets the lowest bonus by which her intended equilibrium is played) the principal offers higher-powered incentives compared with the benchmark case. Note that the principal prefers the agent to accept a contract and work on some task because $\bar{u} < qV - c_A$ and $x_P, y_P \geq 0$. In the optimal contract, the agent's individual rationality constraint must bind:

$$w^{**} = \bar{u} - \mu[q(e_1^{\overline{\theta}**})b + \{x_A e_2^{\overline{\theta}**} + y_A d(e_2^{\overline{\theta}**})\}\overline{\theta}] - (1-\mu)[q(e_1^{\underline{\theta}**})b + \{x_A e_2^{\underline{\theta}**} + y_A d(e_2^{\underline{\theta}**})\}\underline{\theta}],$$

where $(e_1^{\theta}, e_2^{\theta})$ is the optimal action profile of the agent with $\theta \in \{\underline{\theta}, \overline{\theta}\}$ derived above. Then, the principal's expected payoff is

$$\begin{aligned}\Pi =&\mu\left\{q(e_1^{\overline{\theta}**})(V-b) + [x_P e_2^{\overline{\theta}**} + y_P d(e_2^{\overline{\theta}**})]\overline{\theta}\right\} + (1-\mu)\left\{q(e_1^{\underline{\theta}**})(V-b) + [x_P e_2^{\underline{\theta}**} + y_P d(e_2^{\underline{\theta}**})]\underline{\theta}\right\} - w^{**}\\ =&\mu\left\{q(e_1^{\overline{\theta}**})V + [(x_P + x_A)e_2^{\overline{\theta}**} + (y_P + y_A)d(e_2^{\overline{\theta}**})]\overline{\theta}\right\}\\ &+ (1-\mu)\left\{q(e_1^{\underline{\theta}**})V + [(x_P + x_A)e_2^{\underline{\theta}**} + (y_P + y_A)d(e_2^{\underline{\theta}**})]\underline{\theta}\right\} - \overline{u}.\end{aligned}$$

There are three possible cases under pure-strategy PBE. First, given that both types of the agent work on task 1, the optimal contract with the lowest bonus is $(w^{**}, b^{**}) = (\overline{u} - x_A\overline{\theta}, \frac{x_A\overline{\theta}}{q})$ and the principal's payoff is $\pi_{pool-1} = qV - \overline{u}$.

Second, given that both types of the agent work on task 2, the optimal contract with the lowest bonus is $(w^{**}, b^{**}) = (\overline{u} - x_A E(\theta), 0)$ and the principal's payoff is $\pi_{pool-2} = (x_A + x_p)E(\theta) - \overline{u}$.

Third, given that the type-$\overline{\theta}$ agent works on task 2 and the type-$\underline{\theta}$ agent works on task 1, the optimal contract with the lowest bonus is $(w^{**}, b^{**}) = (\overline{u} - (x_A + x_P)E(\theta), \frac{(x_A+y_A)\underline{\theta}}{q})$ and the principal's payoff is $\pi_{sep} = \mu[(x_P + x_A + y_P + y_A)\overline{\theta}] + (1-\mu)qV - \overline{u}$.

Note that

$$\pi_{sep} > \pi_{pool-1} \Longleftrightarrow (x_P + x_A + y_P + y_A)\overline{\theta} > qV, \tag{13}$$

$$\pi_{sep} > \pi_{pool-2} \Longleftrightarrow \mu[(y_P + y_A)\overline{\theta}] + (1-\mu)[qV - (x_A + x_P)\underline{\theta}] > 0. \tag{14}$$

From (13) and (14), we derive Condition (1) in Proposition 1.

Finally, we show that inducing a mixed strategy is not an optimal contract. Consider the mixed strategy equilibrium in which the type-$\underline{\theta}$ agent takes $e^{\underline{\theta}} = (0,1)$ with probability $l^{**} = \frac{\mu}{1-\mu}\frac{\overline{\theta}y_F - c_F}{c_F - \underline{\theta}y_F}$

and the follower takes $d(1) = 1$ with probability $r^{**} = \frac{qb - x_A\underline{\theta}}{y_A\underline{\theta}}$. In the optimal contract, the principal chooses (w^{**}, b^{**}) which holds the agent's individual rationality constraint with equality: $w^{**} + (1-\mu)(1-l^{**})qb^{**} + (1-r^{**})\{\mu\overline{\theta} + (1-\mu)l^{**}\underline{\theta}\}x_A + r^{**}\{\mu\overline{\theta} + (1-\mu)l^{**}\underline{\theta}\}(x_A + y_A) = \overline{u}$. The principal chooses $b^{**} \in \left[\frac{x_A\underline{\theta}}{q}, \frac{(x_A+y_A)\underline{\theta}}{q}\right]$ by which she maximizes her expected payoff:

$$\begin{aligned}
\Pi =&(1-\mu)(1-l^{**})q(V-b) + (1-r^{**})\{\mu\overline{\theta} + (1-\mu)l^{**}\underline{\theta}\}x_P + r^{**}\{\mu\overline{\theta} + (1-\mu)l^{**}\underline{\theta}\}(x_P + y_P) \\
&- \left[\overline{u} - (1-\mu)(1-l^{**})qb - (1-r^{**})\{\mu\overline{\theta} + (1-\mu)l^{**}\underline{\theta}\}x_A - r^{**}\{\mu\overline{\theta} + (1-\mu)l^{**}\underline{\theta}\}(x_A + y_A)\right] \\
=&(1-\mu)(1-l^{**})qV - \overline{u} + [\mu\overline{\theta} + (1-\mu)l^{**}\underline{\theta}](x_A + x_P) + [\mu\overline{\theta} + (1-\mu)l^{**}\underline{\theta}](y_A + y_P)r^{**} \\
=&(1-\mu)\left(1 - \frac{\mu}{1-\mu}\frac{\overline{\theta}y_F - c_F}{c_F - \underline{\theta}y_F}\right)qV - \overline{u} + \left[\mu\overline{\theta} + (1-\mu)\frac{\mu}{1-\mu}\frac{\overline{\theta}y_F - c_F}{c_F - \underline{\theta}y_F}\underline{\theta}\right](x_A + x_P) \\
&+ \left[\mu\overline{\theta} + (1-\mu)\frac{\mu}{1-\mu}\frac{\overline{\theta}y_F - c_F}{c_F - \underline{\theta}y_F}\underline{\theta}\right](y_A + y_P)\frac{qb - x_A\underline{\theta}}{y_A\underline{\theta}}.
\end{aligned}$$

Note that the payoff function is linear in b and its coefficient is positive because $y_A > 0$. Hence, the principal should choose $b^{**} = \frac{(x_A+y_A)\underline{\theta}}{q}$, which implies $r^* = 1$ and the optimal contract induces a separating equilibrium. □

Proof of Proposition 2.

We first prove $\frac{\partial e_1^{\theta*}}{\partial b} > 0$ and $\frac{\partial e_2^{\theta*}}{\partial b} < 0$. By applying the Implicit Function Theorem to (2) and (3), we obtain:

$$\begin{bmatrix} q''(e_1^{\theta*})b - \frac{\partial^2 c_A(e_1^{\theta*}, e_2^{\theta*})}{\partial e_1^2} & -\frac{\partial^2 c_A(e_1^{\theta*}, e_2^{\theta*})}{\partial e_1 \partial e_2} \\ -\frac{\partial^2 c_A(e_1^{\theta*}, e_2^{\theta*})}{\partial e_1 \partial e_2} & -\frac{\partial^2 c_A(e_1^{\theta*}, e_2^{\theta*})}{\partial e_2^2} \end{bmatrix} \begin{bmatrix} \frac{\partial e_1^{\theta*}}{\partial b} \\ \frac{\partial e_2^{\theta*}}{\partial b} \end{bmatrix} = \begin{bmatrix} -q'(e_1^{\theta*}) \\ 0 \end{bmatrix},$$

Solving this system of equations,

$$\begin{aligned}
\frac{\partial e_1^{\theta*}}{\partial b} &= \frac{q'(e_1^{\theta*})\frac{\partial^2 c_A(e_1^{\theta*}, e_2^{\theta*})}{\partial e_2^2}}{-\left(q''(e_1^{\theta*})b - \frac{\partial^2 c_A(e_1^{\theta*}, e_2^{\theta*})}{\partial e_1^2}\right)\frac{\partial^2 c_A(e_1^{\theta*}, e_2^{\theta*})}{\partial e_2^2} - \left(\frac{\partial^2 c_A(e_1^{\theta*}, e_2^{\theta*})}{\partial e_1 \partial e_2}\right)^2} > 0, \\
\frac{\partial e_2^{\theta*}}{\partial b} &= \frac{-q'(e_1^{\theta*})\frac{\partial^2 c_A(e_1^{\theta*}, e_2^{\theta*})}{\partial e_1 \partial e_2}}{-\left(q''(e_1^{\theta*})b - \frac{\partial^2 c_A(e_1^{\theta*}, e_2^{\theta*})}{\partial e_1^2}\right)\frac{\partial^2 c_A(e_1^{\theta*}, e_2^{\theta*})}{\partial e_2^2} - \left(\frac{\partial^2 c_A(e_1^{\theta*}, e_2^{\theta*})}{\partial e_1 \partial e_2}\right)^2} < 0,
\end{aligned}$$

where the denominator is positive by the assumption.

We next derive the optimal level of the bonus b^*. By taking the derivative of (4) with respect to b, b^* is characterized by:

$$\mu \frac{\partial e_1^{\overline{\theta}*}}{\partial b}\left[q'(e_1^{\overline{\theta}*})V - \frac{\partial c_A(e_1^{\overline{\theta}*}, e_2^{\overline{\theta}*})}{\partial e_1}\right] + \mu \frac{\partial e_2^{\overline{\theta}*}}{\partial b}\left[(x_P + x_A)\overline{\theta} - \frac{\partial c_A(e_1^{\overline{\theta}*}, e_2^{\overline{\theta}*})}{\partial e_2}\right]$$
$$+ (1-\mu)\frac{\partial e_1^{\underline{\theta}*}}{\partial b}\left[q'(e_1^{\underline{\theta}*})V - \frac{\partial c_A(e_1^{\underline{\theta}*}, e_2^{\underline{\theta}*})}{\partial e_1}\right] + (1-\mu)\frac{\partial e_2^{\underline{\theta}*}}{\partial b}\left[(x_P + x_A)\underline{\theta} - \frac{\partial c_A(e_1^{\underline{\theta}*}, e_2^{\underline{\theta}*})}{\partial e_2}\right] = 0.$$

By substituting (2) and (3) into the above equality, we obtain:

$$\mu \frac{\partial e_1^{\overline{\theta}*}}{\partial b} q'(e_1^{\overline{\theta}*})(V - b^*) + \mu \frac{\partial e_2^{\overline{\theta}*}}{\partial b} x_P \overline{\theta} + (1-\mu)\frac{\partial e_1^{\underline{\theta}*}}{\partial b} q'(e_1^{\underline{\theta}*})(V - b^*) + (1-\mu)\frac{\partial e_2^{\underline{\theta}*}}{\partial b} x_P \underline{\theta} = 0.$$

By rearranging this equality, we obtain (5). □

Proof of Proposition 3.

Separating Equilibria We refine the separating equilibria derived in the main text by adopting the Intuitive Criterion. We first show that if $(e_1^{\overline{\theta}**}, e_2^{\overline{\theta}**}) \neq e^{\overline{\theta}*}$, then (8) must hold with equality. Suppose, toward a contradiction, that there is a separating equilibrium with $(e_1^{\overline{\theta}**}, e_2^{\overline{\theta}**}) \neq e^{\overline{\theta}*}$ and (8) holds with strict inequality. Note that $e_2^{\overline{\theta}**} \neq e_2^{\overline{\theta}*}$; otherwise, $e_1^{\overline{\theta}**} = e_1^{\overline{\theta}*}$ and hence $(e_1^{\overline{\theta}**}, e_2^{\overline{\theta}**}) = e^{\overline{\theta}*}$ by the right hand side of (8) and (9). Because (8) holds with strict inequality, there exists $\epsilon > 0$ such that

$$q(e_1^{\underline{\theta}**})b + [x_A e_2^{\underline{\theta}**} + y_A d(0)]\underline{\theta} - c_A(e_1^{\underline{\theta}**}, e_2^{\underline{\theta}**}) > \max_{e_1}\left\{q(e_1)b + [x_A e_2' + y_A d(1)]\underline{\theta} - c_A(e_1, e_2')\right\}$$

for any $e_2' \in (e_2^{\overline{\theta}**} - \epsilon, e_2^{\overline{\theta}**} + \epsilon)$. This implies that any action $e_2' \in (e_2^{\overline{\theta}**} - \epsilon, e_2^{\overline{\theta}**} + \epsilon)$ is equilibrium dominated for the type-$\underline{\theta}$ agent. Next, we check the case of the type-$\overline{\theta}$ agent. Note that $q(e_1)w + [x_A e_2 + y_A d(1)]\overline{\theta} - c_A(e_1, e_2)$ is strictly concave by the assumption of the second-order condition; it implies that any $e_2^{\overline{\theta}**} \neq e_2^{\overline{\theta}*}$ is not a local maximum. Hence, there exists $e_2' \in (e_2^{\overline{\theta}**} - \epsilon, e_2^{\overline{\theta}**} + \epsilon)$ such that

$$q(e_1^{\overline{\theta}**})b + [x_A e_2^{\overline{\theta}**} + y_A d(1)]\overline{\theta} - c_A(e_1^{\overline{\theta}**}, e_2^{\overline{\theta}**}) < \max_{e_1}\left\{q(e_1)b + [x_A e_2' + y_A d(1)]\overline{\theta} - c_A(e_1, e_2')\right\}.$$

That is, such an e_2' is not equilibrium dominated for the type-$\overline{\theta}$ agent. By the Intuitive Criterion, after observing e_2', the follower must conclude that the deviation comes from the type-$\overline{\theta}$ agent (i.e., $\mu_{e_2'} = 1$). But given this belief, the type-$\overline{\theta}$ agent has an incentive to deviate from $(e_1^{\overline{\theta}**}, e_2^{\overline{\theta}**})$ to the above action profile — a contradiction. This result implies that in any separating equilibrium surviving the Intuitive Criterion, either $(e_1^{\overline{\theta}**}, e_2^{\overline{\theta}**}) = e^{\overline{\theta}*}$ or (8) binds.

We now show that if $(e_1^{\overline{\theta}**}, e_2^{\overline{\theta}**}) \neq e^{\overline{\theta}*}$, then $e_2^{\overline{\theta}**}$ is the highest value among which (8) binds, i.e., $e_2^{\overline{\theta}**} = \tilde{e}_2$. Suppose, toward a contradiction, that in some separating equilibrium which survives the Intuitive Criterion, there exists $(e_1^{\overline{\theta}**}, e_2^{\overline{\theta}**})$ such that $(e_1^{\overline{\theta}**}, e_2^{\overline{\theta}**}) \neq e^{\overline{\theta}*}$ and $e_2^{\overline{\theta}**} \neq \tilde{e}_2$. By the definition of $\tilde{e}_2$, $e_2^{\overline{\theta}**} < \tilde{e}_2$ holds. Because (8) binds at both $(e_1^{\overline{\theta}**}, e_2^{\overline{\theta}**})$ and $(\tilde{e}_1, \tilde{e}_2)$,

$$q(e_1^{\underline{\theta}**})b + [x_A e_2^{\underline{\theta}**} + y_A d(0)]\underline{\theta} - c_A(e_1^{\underline{\theta}**}, e_2^{\underline{\theta}**}) = \max_{e_1} \left\{ q(e_1)b + [x_A e_2^{\overline{\theta}**} + y_A d(1)]\underline{\theta} - c_A(e_1, e_2^{\overline{\theta}**}) \right\}, \tag{15}$$

$$q(e_1^{\underline{\theta}**})b + [x_A e_2^{\underline{\theta}**} + y_A d(0)]\underline{\theta} - c_A(e_1^{\underline{\theta}**}, e_2^{\underline{\theta}**}) = \max_{e_1} \left\{ q(e_1)b + [x_A \tilde{e}_2 + y_A d(1)]\underline{\theta} - c_A(e_1, \tilde{e}_2) \right\}. \tag{16}$$

Combining (15) and (16), we have

$$\max_{e_1} \left\{ q(e_1)b + [x_A e_2^{\overline{\theta}**} + y_A d(1)]\underline{\theta} - c_A(e_1, e_2^{\overline{\theta}**}) \right\} = \max_{e_1} \left\{ q(e_1)b + [x_A \tilde{e}_2 + y_A d(1)]\underline{\theta} - c_A(e_1, \tilde{e}_2) \right\}.$$

Because $e_2^{\overline{\theta}**} < \tilde{e}_2$, $0 < x_A e_2^{\overline{\theta}**} + y_A d(1) < x_A \tilde{e}_2 + y_A d(1)$. Hence, replacing $\underline{\theta}$ with $\overline{\theta}$ in the above equality leads to

$$\max_{e_1} \left\{ q(e_1)b + [x_A e_2^{\overline{\theta}**} + y_A d(1)]\overline{\theta} - c_A(e_1, e_2^{\overline{\theta}**}) \right\} < \max_{e_1} \left\{ q(e_1)b + [x_A \tilde{e}_2 + y_A d(1)]\overline{\theta} - c_A(e_1, \tilde{e}_2) \right\}. \tag{17}$$

Note that the left hand side of (17) is none other than the equilibrium payoff of the type-$\overline{\theta}$ agent by choosing $(e_1^{\overline{\theta}**}, e_2^{\overline{\theta}**})$. Similarly to the previous paragraph, any action $e_2 > \tilde{e}_2$ is equilibrium dominated for the type-$\underline{\theta}$ agent. (17) implies that there exists $e_2' = \tilde{e}_2 + \epsilon$ with a sufficiently small $\epsilon > 0$ which is not equilibrium dominated for the type-$\overline{\theta}$ agent. Hence, the type-$\overline{\theta}$ agent can profitably deviate by choosing such an e_2' — a contradiction.

We show that in any separating equilibrium surviving the Intuitive Criterion where $(e_1^{\overline{\theta}**}, e_2^{\overline{\theta}**}) \neq e^{\overline{\theta}*}$, $\tilde{e}_2 > e_2^{\overline{\theta}*}$ must hold. Note that if $\tilde{e}_2 = e_2^{\overline{\theta}*}$, then $e_1^{\overline{\theta}**} = e_1^{\overline{\theta}*}$ and hence $e^{\overline{\theta}**} = e^{\overline{\theta}*}$. Hence, toward a contradiction, suppose $\tilde{e}_2 < e_2^{\overline{\theta}*}$. Then, as discussed in the previous paragraph, any action $e_2 > \tilde{e}_2$ is equilibrium dominated for the type-$\underline{\theta}$ agent; implying that $e_2^{\overline{\theta}*}$ is equilibrium dominated for the type-$\underline{\theta}$ agent. Because $e^{\overline{\theta}*}$ is the action profile that maximizes the type-$\overline{\theta}$ agent's payoff when $\mu_{e_2} = 1$, $e_2^{\overline{\theta}*}$ is not equilibrium dominated for the type-$\overline{\theta}$ agent. Hence, by the Intuitive Criterion, after observing $e_2^{\overline{\theta}*}$, the follower must conclude that the deviation comes from the type-$\overline{\theta}$ agent (i.e., $\mu_{e_2^{\overline{\theta}*}} = 1$). But then, the type-$\overline{\theta}$ agent can profitably deviate by choosing $e^{\overline{\theta}*}$ — a contradiction.

We also check that (10) is not binding in any separating equilibrium surviving the Intuitive Criterion. Note that the the right hand side of (10) is maximized by choosing $e^{\overline{\theta}*}$. Hence, when $e^{\overline{\theta}**} = e^{\overline{\theta}*}$, (10) is not binding. When $e^{\overline{\theta}**} = \tilde{e}$, (8) hold with equality, so by rewriting it yields:

$$q(e_1^{\underline{\theta}**})b + [x_A e_2^{\underline{\theta}**} + y_A d(0)]\underline{\theta} - c_A(e_1^{\underline{\theta}**}, e_2^{\underline{\theta}**}) = \max_{e_1} \{q(e_1)b + [x_A \tilde{e}_2 + y_A d(1)]\underline{\theta} - c_A(e_1, \tilde{e}_2)\}.$$

Because $(e_1^{\underline{\theta}**}, e_2^{\underline{\theta}**})$ maximizes the type-$\underline{\theta}$ agent's payoff given $d = d(0)$, we have

$$q(e_1^{\underline{\theta}**})b + [x_A e_2^{\underline{\theta}**} + y_A d(0)]\underline{\theta} - c_A(e_1^{\underline{\theta}**}, e_2^{\underline{\theta}**}) > q(e_1^{\overline{\theta}*})b + [x_A e_2^{\overline{\theta}*} + y_A d(0)]\underline{\theta} - c_A(e_1^{\overline{\theta}*}, e_2^{\overline{\theta}*}).$$

By combining it with the above equality, we obtain:

$$\max_{e_1} \{q(e_1)b + [x_A \tilde{e}_2 + y_A d(1)]\underline{\theta} - c_A(e_1, \tilde{e}_2)\} > q(e_1^{\overline{\theta}*})b + [x_A e_2^{\overline{\theta}*} + y_A d(0)]\underline{\theta} - c_A(e_1^{\overline{\theta}*}, e_2^{\overline{\theta}*}).$$

Because $e_2^{\overline{\theta}*} < \tilde{e}_2$, $0 < x_A e_2^{\overline{\theta}*} + y_A d(0) < x_A \tilde{e}_2 + y_A d(1)$. Hence, replacing $\underline{\theta}$ with $\overline{\theta}$ in the above inequality leads to

$$\max_{e_1} \left\{q(e_1)b + [x_A \tilde{e}_2 + y_A d(1)]\overline{\theta} - c_A(e_1, \tilde{e}_2)\right\} > q(e_1^{\overline{\theta}*})b + [x_A e_2^{\overline{\theta}*} + y_A d(0)]\overline{\theta} - c_A(e_1^{\overline{\theta}*}, e_2^{\overline{\theta}*}). \quad (18)$$

Note that the left hand side of (18) is none other than the equilibrium payoff of the type-$\overline{\theta}$ agent. Because the right hand side of (10) is maximized by choosing e^*, we have shown that (10) is not binding in any separating equilibrium surviving the Intuitive Criterion.

In summary, we have shown that in any separating equilibrium surviving the Intuitive Criterion, either $e^{\overline{\theta}**} = e^{\overline{\theta}*}$ or $e^{\overline{\theta}**} = \tilde{e}$ where $\tilde{e}_2 > e_2^{\overline{\theta}*}$. In either case, the type-$\underline{\theta}$ agent's equilibrium action profile is given by (6) and (7) and hence $e^{\underline{\theta}**} = e^{\underline{\theta}*}$.

Pooling Equilibria We next analyze the set of pooling equilibria in that both types of the agent take $e_2^{**} \geq 0$. We derive it by restricting the off-path belief that the follower always thinks that a deviating agent must be the low type: $\mu_{e_2} = 0$ for any $e_2 \neq e_2^{**}$. Then, each type of the agent's action profile must satisfy the following conditions:

$$q'(e_1^{\underline{\theta}**})b - \frac{\partial c_A(e_1^{\underline{\theta}**}, e_2^{**})}{\partial e_1} = 0, \tag{19}$$

$$q(e_1^{\underline{\theta}**})b + [x_A e_2^{**} + y_A d(\mu)]\underline{\theta} - c_A(e_1^{\underline{\theta}**}, e_2^{**}) \geq \max_{(e_1,e_2)} \{q(e_1)b + [x_A e_2 + y_A d(0)]\underline{\theta} - c_A(e_1, e_2)\}, \tag{20}$$

$$q'(e_1^{\overline{\theta}**})b - \frac{\partial c_A(e_1^{\overline{\theta}**}, e_2^{**})}{\partial e_1} = 0, \tag{21}$$

$$q(e_1^{\overline{\theta}**})b + [x_A e_2^{**} + y_A d(\mu)]\overline{\theta} - c_A(e_1^{\overline{\theta}**}, e_2^{**}) \geq \max_{(e_1,e_2)} \left\{q(e_1)b + [x_A e_2 + y_A d(0)]\overline{\theta} - c_A(e_1, e_2)\right\}. \tag{22}$$

A pooling equilibrium is characterized by a tuple of actions $((e_1^{\underline{\theta}**}, e_2^{**}), (e_1^{\overline{\theta}**}, e_2^{**}))$ that satisfies (19)-(22).

We next show that there is no pooling equilibrium that survives the Intuitive Criterion. Suppose, toward a contradiction, that there is such an equilibrium. When (20) holds with strict inequality, by the same argument as in the proof of separating equilibria, there exists $\epsilon > 0$ such that any action $e_2' \in (e_2^{**} - \epsilon, e_2^{**} + \epsilon)$ is equilibrium dominated for the type-$\underline{\theta}$ agent and the type-$\overline{\theta}$ agent has an incentive to deviate to e_2'. When (20) holds with equality, take the highest value $\bar{e}_2$ which satisfies the following equality:

$$q(e_1^{\underline{\theta}**})b + [x_A e_2^{**} + y_A d(\mu)]\underline{\theta} - c_A(e_1^{\underline{\theta}**}, e_2^{**}) = \max_{e_1} \{q(e_1)b + [x_A \bar{e}_2 + y_A d(1)]\underline{\theta} - c_A(e_1, \bar{e}_2)\}.$$

Note that $\bar{e}_2 > e_2^{**}$, as (20) holds with equality. By (19), the above equality can be written as

$$\max_{e_1} \{q(e_1)b + [x_A e_2^{**} + y_A d(\mu)]\underline{\theta} - c_A(e_1, e_2^{**})\} = \max_{e_1} \{q(e_1)b + [x_A \bar{e}_2 + y_A d(1)]\underline{\theta} - c_A(e_1, \bar{e}_2)\}.$$

Because $\overline{\theta} > \underline{\theta}$, $\bar{e}_2 > e_2^{**}$ and $d(\mu) < d(1)$, the above equality implies

$$\max_{e_1} \left\{q(e_1)b + [x_A e_2^{**} + y_A d(\mu)]\overline{\theta} - c_A(e_1, e_2^{**})\right\} < \max_{e_1} \left\{q(e_1)b + [x_A \bar{e}_2 + y_A d(1)]\overline{\theta} - c_A(e_1, \bar{e}_2)\right\}. \tag{23}$$

Note that the left hand side of (23) is none other than the equilibrium payoff of the type-$\overline{\theta}$ agent. As any action $\bar{e}_2 + \epsilon$ with $\epsilon > 0$ is equilibrium dominated for the type-$\underline{\theta}$ agent, by taking a sufficiently small $\epsilon > 0$, the type-$\overline{\theta}$ agent can profitably deviate by choosing $\bar{e}_2 + \epsilon$ — a contradiction. □

Proof of Proposition 4.

(i) When $e_2^{\bar{\theta}**} = e_2^{\bar{\theta}*}$, neither (8) nor (10) is binding; the signaling effect is not a relevant concern for each type of the agent. Hence, the optimal contract specifies $b^{**} \leq V$ (and $b^{**} < V$ if $x_P > 0$) by the same derivation with the benchmark case under symmetric information.

(ii) Suppose $e_2^{\bar{\theta}**} = \tilde{e}_2$. Let e_1^m denote the effort level of task 1 which satisfies (8) with equality (i.e., the effort level of task 1 chosen by the type-$\underline{\theta}$ agent when he *mimics* the type-$\bar{\theta}$ agent), which is characterized as:

$$q'(e_1^m)b - \frac{\partial c_A(e_1^m, \tilde{e}_2)}{\partial e_1} = 0. \tag{24}$$

Because $e_2^{\underline{\theta}**} < \tilde{e}_2$ and $\frac{\partial^2 c_A(e_1,e_2)}{\partial e_1 \partial e_2} > 0$, $e_1^m < e_1^{\underline{\theta}**}$ and hence $q(e_1^m) < q(e_1^{\underline{\theta}**})$ holds. Note also that (8) holds with equality as follows:

$$q(e_1^m)b + [x_A\tilde{e}_2 + y_A d(1)]\underline{\theta} - c_A(e_1^m, \tilde{e}_2) - \left\{q(e_1^{\underline{\theta}**})b + [x_A e_2^{\underline{\theta}**} + y_A d(0)]\underline{\theta} - c_A(e_1^{\underline{\theta}**}, e_2^{\underline{\theta}**})\right\} = 0. \tag{25}$$

In this case, the equilibrium effort levels are characterized by (6), (7), (24), (9), and (25). By applying the Implicit Function Theorem, we obtain:

$$\begin{bmatrix} q''(e_1^{\underline{\theta}**})b - \frac{\partial^2 c_A(e_1^{\underline{\theta}**}, e_2^{\underline{\theta}**})}{\partial e_1^2} & -\frac{\partial^2 c_A(e_1^{\underline{\theta}**}, e_2^{\underline{\theta}**})}{\partial e_1 \partial e_2} & 0 & 0 & 0 \\ -\frac{\partial^2 c_A(e_1^{\underline{\theta}**}, e_2^{\underline{\theta}**})}{\partial e_1 \partial e_2} & -\frac{\partial^2 c_A(e_1^{\underline{\theta}**}, e_2^{\underline{\theta}**})}{\partial e_2^2} & 0 & 0 & 0 \\ 0 & 0 & q''(e_1^m)b - \frac{\partial^2 c_A(e_1^m, \tilde{e}_2)}{\partial e_1^2} & 0 & -\frac{\partial^2 c_A(e_1^m, \tilde{e}_2)}{\partial e_1 \partial e_2} \\ 0 & 0 & 0 & q''(\tilde{e}_1)b - \frac{\partial^2 c_A(\tilde{e}_1, \tilde{e}_2)}{\partial e_1^2} & -\frac{\partial^2 c_A(\tilde{e}_1, \tilde{e}_2)}{\partial e_1 \partial e_2} \\ 0 & 0 & 0 & 0 & x_A\underline{\theta} - \frac{\partial c_A(e_1^m, \tilde{e}_2)}{\partial e_2} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial e_1^{\underline{\theta}**}}{\partial b} \\ \frac{\partial e_2^{\underline{\theta}**}}{\partial b} \\ \frac{\partial e_1^m}{\partial b} \\ \frac{\partial \tilde{e}_1}{\partial b} \\ \frac{\partial \tilde{e}_2}{\partial b} \end{bmatrix} = \begin{bmatrix} -q'(e_1^{\underline{\theta}**}) \\ 0 \\ -q'(e_1^m) \\ -q'(e_1^{\underline{\theta}**}) \\ q(e_1^{\underline{\theta}**}) - q(e_1^m) \end{bmatrix}.$$

Let A denote the above 5×5 matrix. Note that $(5,1)$, $(5,2)$, and $(5,3)$ elements of A are zero by using (6), (7), and (24), respectively. By Cramer's rule, as well as $q(e_1^m) < q(e_1^{\underline{\theta}**})$ and the assumptions, we obtain the following comparative statics:

$$\frac{\partial e_1^{\theta**}}{\partial b} = \frac{q'(e_1^{\theta**})\frac{\partial^2 c_A(e_1^{\theta**},e_2^{\theta**})}{\partial e_2^2}}{-\left(q''(e_1^{\theta**})b - \frac{\partial^2 c_A(e_1^{\theta**},e_2^{\theta**})}{\partial e_1^2}\right)\frac{\partial^2 c_A(e_1^{\theta**},e_2^{\theta**})}{\partial e_2^2} - \left(\frac{\partial^2 c_A(e_1^{\theta**},e_2^{\theta**})}{\partial e_1 \partial e_2}\right)^2} > 0,$$

$$\frac{\partial e_2^{\theta**}}{\partial b} = \frac{-q'(e_1^{\theta**})\frac{\partial^2 c_A(e_1^{\theta**},e_2^{\theta**})}{\partial e_1 \partial e_2}}{-\left(q''(e_1^{\theta**})b - \frac{\partial^2 c_A(e_1^{\theta**},e_2^{\theta**})}{\partial e_1^2}\right)\frac{\partial^2 c_A(e_1^{\theta**},e_2^{\theta**})}{\partial e_2^2} - \left(\frac{\partial^2 c_A(e_1^{\theta**},e_2^{\theta**})}{\partial e_1 \partial e_2}\right)^2} < 0,$$

$$\frac{\partial e_1^m}{\partial b} = \frac{-q'(e_1^m) + \frac{\partial^2 c_A(e_1^m,\tilde{e}_2)}{\partial e_1 \partial e_2}\frac{q(e_1^{\theta**})-q(e_1^m)}{x_A\underline{\theta} - \frac{\partial c_A(e_1^m,\tilde{e}_2)}{\partial e_2}}}{q''(e_1^m)b - \frac{\partial^2 c_A(e_1^m,\tilde{e}_2)}{\partial e_1^2}} > 0,$$

$$\frac{\partial \tilde{e}_1}{\partial b} = \frac{-q'(e_1^{\theta**}) + \frac{\partial^2 c_A(\tilde{e}_1,\tilde{e}_2)}{\partial e_1 \partial e_2}\frac{q(e_1^{\theta**})-q(e_1^m)}{x_A\underline{\theta} - \frac{\partial c_A(e_1^m,\tilde{e}_2)}{\partial e_2}}}{q''(\tilde{e}_1)b - \frac{\partial^2 c_A(\tilde{e}_1,\tilde{e}_2)}{\partial e_1^2}} > 0,$$

$$\frac{\partial \tilde{e}_2}{\partial b} = \frac{q(e_1^{\theta**}) - q(e_1^m)}{x_A\underline{\theta} - \frac{\partial c_A(e_1^m,\tilde{e}_2)}{\partial e_2}} < 0.$$

We next derive the optimal level of the bonus b^{**} when $e_2^{\overline{\theta}**} = \tilde{e}_2$. By taking the derivative of (11) with respect to b, b^{**} is characterized by:

$$\mu\frac{\partial \tilde{e}_1}{\partial b}\left[q'(\tilde{e}_1)V - \frac{\partial c_A(\tilde{e}_1,\tilde{e}_2)}{\partial e_1}\right] + \mu\frac{\partial \tilde{e}_2}{\partial b}\left[(x_P + x_A)\overline{\theta} - \frac{\partial c_A(\tilde{e}_1,\tilde{e}_2)}{\partial e_2}\right]$$
$$+ (1-\mu)\frac{\partial e_1^{\theta**}}{\partial b}\left[q'(e_1^{\theta**})V - \frac{\partial c_A(e_1^{\theta**},e_2^{\theta**})}{\partial e_1}\right] + (1-\mu)\frac{\partial e_2^{\theta**}}{\partial b}\left[(x_P + x_A)\underline{\theta} - \frac{\partial c_A(e_1^{\theta**},e_2^{\theta**})}{\partial e_2}\right] = 0.$$

By substituting (6), (7), and (9) into the above equality, we obtain:

$$\mu\frac{\partial \tilde{e}_1}{\partial b}q'(\tilde{e}_1)(V - b^{**}) + \mu\frac{\partial \tilde{e}_2}{\partial b}\left[(x_P + x_A)\overline{\theta} - \frac{\partial c_A(\tilde{e}_1,\tilde{e}_2)}{\partial e_2}\right]$$
$$+ (1-\mu)\frac{\partial e_1^{\theta**}}{\partial b}q'(e_1^{\theta**})(V - b^{**}) + (1-\mu)\frac{\partial e_2^{\theta**}}{\partial b}x_P\underline{\theta} = 0.$$

By rearranging this equality, we obtain (12). □

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Supplementary Material for "The Provision of High-powered Incentives under Multitasking" by Daido and Murooka (not intended for publication)

A Illustration with Binary Effort: Full Characterization

In the binary illustrative model with private information, Proposition 1 derives the optimal contracts under the assumption that a pure-strategy PBE is played whenever it exists. Here, we characterize PBE including all mixed-strategy equilibria, as well as applying the Intuitive Criterion (Cho and Kreps, 1987). We then argue that the optimal contracts shown in Proposition 1 remain the same, so focusing on pure-strategy PBE is without loss of generality when we derive the optimal contracts.

In addition to PBE derived in Proposition 1, there exist other two types of mixed-strategy equilibria. One kind of mixed-strategy equilibria is that the type-$\overline{\theta}$ agent randomizes his actions. In such an equilibrium, the type-$\underline{\theta}$ agent must choose $e^{\underline{\theta}} = (1, 0)$ with probability one. Given that, by Assumption 1 (ii), the follower chooses $d(1) = 1$ with probability one and $d(0) = 0$ with probability one. Then, for the type-$\overline{\theta}$ agent to be indifferent, $b = \frac{(x_A+y_A)\overline{\theta}}{q}$. In this case, any mixture of the actions of the type-$\overline{\theta}$ agent constitutes an equilibrium.

Another kind of mixed-strategy equilibria is that the type-$\underline{\theta}$ agent randomizes his actions, while the follower does not. Suppose that the type-$\underline{\theta}$ agent takes $e^{\underline{\theta}} = (0, 1)$ with probability $l \in [0, 1]$ and that the follower takes $d(1) = 1$ and $d(0) = 0$ with probability one. Then, the type-$\overline{\theta}$ agent takes $e^{\overline{\theta}} = (0, 1)$ with probability one. Under the mixed-strategy equilibrium, the type-$\underline{\theta}$ must be indifferent: $qb = (x_A + y_A)\underline{\theta} \iff b = \frac{(x_A+y_A)\underline{\theta}}{q}$. Also, the type-$\underline{\theta}$ agent must choose l which induces the follower to weakly prefer to play $d(1) = 1$: $\frac{\mu}{\mu+(1-\mu)l} y_F \overline{\theta} + \frac{(1-\mu)l}{\mu+(1-\mu)l} y_F \underline{\theta} - c_F \geq 0 \iff l \leq \frac{\mu}{1-\mu} \frac{\overline{\theta} y_F - c_F}{c_F - \underline{\theta} y_F}$.

We next show that the pooling equilibrium $e^{\overline{\theta}} = e^{\underline{\theta}} = (1, 0)$, which is described in Section 3, fails the Intuitive Criterion when $\frac{(x_A+y_A)\underline{\theta}}{q} < b < \frac{(x_A+y_A)\overline{\theta}}{q}$. For the type-$\underline{\theta}$ agent, his equilibrium payoff is $w + qb$, whereas his payoff upon deviation to $e' = (0, 1)$ is at most $w + (x_A + y_A)\underline{\theta}$. Hence, $e^{\underline{\theta}} = (1, 0)$ is equilibrium dominated for the type-$\underline{\theta}$ agent. For the type-$\overline{\theta}$ agent, his equilibrium payoff is $w + qb$, whereas his payoff upon deviation to $e' = (0, 1)$ is at most $w + (x_A + y_A)\overline{\theta}$. Hence,

$e^{\overline{\theta}} = (1,0)$ is not equilibrium dominated for the type-$\overline{\theta}$ agent. Hence, the pooling equilibrium $e^{\overline{\theta}} = e^{\underline{\theta}} = (1,0)$ survives the Intuitive Criterion when $\frac{x_A\overline{\theta}}{q} \leq b \leq \frac{(x_A+y_A)\underline{\theta}}{q}$ or $b \geq \frac{(x_A+y_A)\overline{\theta}}{q}$.

As a result, equilibrium strategies in the perfect-Bayesian equilibria that survives the Intuitive Criterion are summarized depending on the level of b as follows:

(i) (task-2 pooling) If $b \leq \frac{x_A\underline{\theta}}{q}$, then $e^{\overline{\theta}} = e^{\underline{\theta}} = (0,1)$ and $d(1) = d(0) = 0$.

(ii) (type-$\underline{\theta}$ mixed) If $b \in \left(\frac{x_A\underline{\theta}}{q}, \frac{x_A\overline{\theta}}{q}\right)$, then $e^{\overline{\theta}} = (0,1)$, the type-$\underline{\theta}$ agent takes $e^{\underline{\theta}} = (0,1)$ with probability $\frac{\mu}{1-\mu}\frac{\overline{\theta}y_F-c_F}{c_F-\underline{\theta}y_F} \in (0,1)$, and the follower takes $d(1) = 1$ with probability $\frac{qb-x_A\underline{\theta}}{y_A\underline{\theta}} \in (0,1)$ and $d(0) = 0$ with probability one.

(iii) (task-1 pooling and type-$\underline{\theta}$ mixed) If $b \in \left[\frac{x_A\overline{\theta}}{q}, \frac{(x_A+y_A)\underline{\theta}}{q}\right)$, then there exist two kinds of equilibria: (i) $e^{\overline{\theta}} = e^{\underline{\theta}} = (1,0)$ and $d(1) = d(0) = 0$. (ii) $e^{\overline{\theta}} = (0,1)$, the type-$\underline{\theta}$ agent takes $e^{\underline{\theta}} = (0,1)$ with probability $\frac{\mu}{1-\mu}\frac{\overline{\theta}y_F-c_F}{c_F-\underline{\theta}y_F} \in (0,1)$, and the follower takes $d(1) = 1$ with probability $\frac{qb-x_A\underline{\theta}}{y_A\underline{\theta}} \in (0,1)$ and $d(0) = 0$ with probability one.

(iv) (task-1 pooling, separating, and type-$\underline{\theta}$ mixed) If $b = \frac{(x_A+y_A)\underline{\theta}}{q}$, then $e^{\overline{\theta}} = (0,1)$, the type-$\underline{\theta}$ agent takes $e^{\underline{\theta}} = (0,1)$ with probability $l \in \left[0, \frac{\mu}{1-\mu}\frac{\overline{\theta}y_F-c_F}{c_F-\underline{\theta}y_F}\right]$, $d(1) = 1$, and $d(0) = 0$.

(v) (separating) If $b \in \left(\frac{(x_A+y_A)\underline{\theta}}{q}, \frac{(x_A+y_A)\overline{\theta}}{q}\right)$, then $e^{\overline{\theta}} = (0,1)$, $e^{\underline{\theta}} = (1,0)$, $d(1) = 1$, and $d(0) = 0$.

(vi) (task-1 pooling, separating, and type-$\overline{\theta}$ mixed) If $b = \frac{(x_A+y_A)\overline{\theta}}{q}$, then the type-$\overline{\theta}$ agent takes $e^{\overline{\theta}} = (0,1)$ with probability $h \in [0,1]$, $e^{\underline{\theta}} = (1,0)$, $d(1) = 1$, and $d(0) = 0$.

(vii) (task-1 pooling) If $b > \frac{(x_A+y_A)\overline{\theta}}{q}$, then $e^{\overline{\theta}} = e^{\underline{\theta}} = (1,0)$ and $d(1) = d(0) = 0$.

Next, we argue that the optimal contracts shown in Proposition 1 remains the same. In the proof of Proposition 1, we show that the principal does not induce the agent and the follower to play a mixed strategy in (ii) and (iii) above. By the similarly derivations, it is easy to check that the principal does not induce the agent and the follower to play a mixed strategy in (iv) and (vi) above.