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# Public and Private Educational Expenditure and Human Capital Accumulation

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**Public and Private Educational Expenditure and Human Capital Accumulation**[†]

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**Abstract**

This study was conducted to examine how the human capital stock of children is determined in a model for which human capital stock is produced by school education and private tutoring inputs. School education of two types is considered: public and private. Our paper presents consideration of substitution between school education and private tutoring. Although some reports of the related literature describe human capital accumulation including substitution between school education and the private tutoring, few studies have examined how education policy affects demand for school education. The human capital stock of children is determined in the model with substitution between school education and the private tutoring. Our paper presents consideration of human capital accumulation functions of several types: Constant elasticity of Substitution (CES), perfect substitution, and perfect complementarity between school education and private tutoring.

**Keywords:** Education Choice, Human Capital, Private Tutoring, Public and Private schooling, Subsidy

**JEL Classifications:** I22, H52

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## 1. Introduction

Many papers have described studies about education and school systems. A fundamental study by Glomm and Ravikumar (1992) examines how school systems affect income growth and income inequality. Glomm and Ravikumar (1992) consider school systems of two types: public and private. By public education, education investment is financed by taxation. Education investment is provided equally among all children. However, by private education, education investment is paid by households. Wealthy households can make large education investments for children of the household. Otherwise, the education investment for children remains small. Intuitively, one might infer that public education shrinks income inequality because of the lack of inequality in education investment, although inequality of education investment exists in private education systems.

After the study reported by Glomm and Ravikumar (1992), many related studies explored the manner by which education investment affects income growth and income inequality. However, because of difficulty in deriving the equilibrium of the model economy, these studies assumed some conditions in the model setting, such as a logarithmic utility function, a Cobb–Douglas human capital accumulation function, and others. These assumptions play an important role in deriving the model economy equilibrium. However, these assumptions are strong. The results ultimately depend on these assumptions.

Moreover, the related studies of the literature include no consideration of education in addition to school education such as extra lessons, private tutoring, and other points. In Japan, many children study in extra lessons, cram schools, and other modes of study after attending public or private school. How these phenomena affect human capital should be examined.

The aim of this paper is to set a model with human capital accumulation model not only with school education but also with extra lessons as education after school (private supplementary tutoring), with examination of demand for school education services and supplemental education services. Our paper sets the education selection considered by Cardak (2004). Our model economy includes households of two types: households that select public education and households that select private education. We derive different demand for private supplementary tutoring between school education systems of two types. In addition to these analyses, we derive how education policy affects not only demand for school education investment but also for additional education investment, for instance, an increase in public education investment and a subsidy for additional education investment. These analyses assume not Cobb–Douglas human capital accumulation, but Constant Elasticity of Substitution (CES) human capital accumulation. By virtue of this setting, our manuscript derives results that can not be derived by Cobb–Douglas human capital accumulation. We derive the optimal policy to maximize the social welfare function.

Moreover, our paper examines the case of Constant Relative Risk Averse (CRRA) types of utility function. Given some parametric conditions, results obtained in the model of CRRA utility function are not different from results in the model of logarithmic utility function. Concretely, the demand for private tutoring with an increase in the public school education depends on parameters of the utility function and human capital accumulation function.

The related studies of the literature assume the simple setting model for simplicity of analysis. Even if one can derive the equilibrium of model economy easily with a logarithmic utility function, the setting of Constant Elasticity substitution (CES) form and Constant Relative Risk Aversion (CRRA) form of utility function make it difficult to derive the equilibrium because deriving the reduced form is difficult. Glomm and Ravikumar (1992), Glomm and Kaganovich (2003), Cardak (2004), and others assume a logarithmic utility function.

However, by virtue of the CES or CRRA form of utility function, one can obtain rich policy implications because of substitution of consumption and demand for education investment. Studies reported by Glomm (1997), by Glomm and Ravikumar (2003), and by Bearse, Glomm and Patterson (2005) all rely on the assumption of the CRRA form of utility function. These related papers assume the same constant relative risk aversion rate between consumption and the demand for education investment. Watanabe and Yasuoka (2009) examine equilibria with different relative risk aversion rates of consumption and demand for education.

Our paper presents consideration of the school education and private tutoring for human capital accumulation. Related reports are those of studies by Bearse, Glomm and Patterson (2005) and Gamrath and Radhika (2018). These reports illustrate how the share of public school education chosen by the individuals is determined and how the dynamics of human capital accumulation is derived. Nevertheless, these analyses include no consideration of how subsidy policy for education affects the inequality of education investment and the share of public school education chosen by individuals.

Many related papers describe studies of education. Oshio and Yasuoka (2009), Andersson and Konrad (2002), and Brodaty, Gary-Bobo and Prieto (2014) set models by which education affects the expected income in the future and by which demand for education is subsequently determined. We can consider the redistribution policy for subsidy as that explained by Boadway, Marceau and Marchand (1992).

The rest of this paper is presented as follows. Section 2 explains the model. Section 3 derives education choice: how the share of public school education is chosen by individuals. Section 4 presents consideration of other types of human capital accumulation form: one for perfect substitution and the other for perfect complementarity. Section 5 describes derivation of allocations to maximize social welfare. The final section concludes our manuscript.

## 2. Model Setting

Individuals live in two periods: childhood and adulthood. In adulthood, individuals decide allocations of consumption in adulthood and of education investment for children. The utility function is assumed by the following Constant Relative Risk Aversion (CRRA) utility function as

$$u_t = \alpha \frac{c_t^{1-\sigma} - 1}{1-\sigma} + (1-\alpha)\frac{h_{t+1}^{1-\sigma} - 1}{1-\sigma}, 0 < \alpha < 1, 0 < \sigma. \tag{1}$$

In that equation, $c_t$ and $h_{t+1}$ respectively denote consumption and children human capital. Glomm and Ravikumar (1992) and others assume a logarithmic utility function for simplicity of analysis of the dynamics of human capital.[1] Studies by Bearse, Glomm and Patterson (2005), by Glomm (1997), Glomm and

[1] Some related studies rely on the assumption of the other type of utility function as a Constant Relative Risk

Ravikumar (2003), and by Watanabe and Yasuoka (2009) include assumptions of the CRRA utility function.

In this model economy, school education of two types exists: one for public education and the other for private education. If the individuals select public school education, human capital accumulation is assumed as

$$h_{t+1} = A\left(\beta E_t^{u\rho} + (1-\beta)e_t^{\rho}\right)^{\frac{1}{\rho}}, 0 < A, 0 < \beta < 1, \rho < 1. \tag{2}$$

Therein, $E_t^u$ and $e_t$ respectively denote $E_t^u$public schooling and $e_t$ private tutoring. Gamrath and Radhika (2018) set the Cobb–Douglas function form of public schooling and private tutoring. Work by Bearse, Glomm and Patterson (2005) includes the assumption of the CES function form. Public education investment $E_t^u$ is financed by taxation.

The budget constraint of households that select the public school education in period *t* is given as shown below:

$$c_t + (1-x)e_t = (1-\tau)h_t, \tag{3}$$

In that equation, $\tau$ and $h_t$ respectively represent the tax rates for public schooling and human capital (wage income). Private tutoring is subsidized by the rate of $x$.

If the individuals select the private school education, then the human capital accumulation given by (2) changes to the following.

$$h_{t+1} = A\left(\beta E_t^{r\rho} + (1-\beta)e_t^{\rho}\right)^{\frac{1}{\rho}} \tag{4}$$

Therein, $E_t^r$ denotes private schooling.

The budget constraint of households that select the private school education in *t* period is

$$c_t + (1-x)e_t + E_t^r = (1-\tau)h_t. \tag{5}$$

Even if the households select private school education, they must pay taxes for public school education funding. This paper assumes a distribution of human capital of individuals given as $[\underline{h}_t, \overline{h}_t]$, with $\underline{h}_t$ assumed to be sufficiently small for households to choose public school education. Actually, $\overline{h}_t$ is assumed to be sufficiently large to have households that choose private school education.

The government budget constraint of public school education and the subsidy for private tutoring are

$$E_t^u \int_{\underline{h}_t}^{h_t^*} f(h_t)dh_t + x\int_{\underline{h}_t}^{\overline{h}_t} e_t f(h_t)dh_t = \tau \int_{\underline{h}_t}^{\overline{h}_t} h_t f(h_t)\, dh_t, \tag{6}$$

where $f(h_t)$ represents the density function. For simplicity, we set the rule of subsidy for public school education and private tutoring as shown below.

$$E_t^u \int_{\underline{h}_t}^{h_t^*} f(h_t)dh_t = \theta\tau \int_{\underline{h}_t}^{\overline{h}_t} h_t f(h_t)\, dh_t, \tag{7}$$

$$x\int_{\underline{h}_t}^{\overline{h}_t} e_t f(h_t)dh_t = (1-\theta)\tau \int_{\underline{h}_t}^{\overline{h}_t} h_t f(h_t)\, dh_t, \tag{8}$$

In those equations, $\theta$ denotes allocations of how the tax revenue should be distributed for each policy.

## 3. Education Choice

Aversion Utility (CRRA) function. The logarithm utility function is a part of CRRA function because $\sigma = 1$.

This section presents derivation of households' optimal allocations and how education choice is determined. First, we derive the case of public schooling. Second, we derive the case of private schooling. After obtaining optimal household allocations, we can demonstrate how education choice is determined.

For public schooling, households maximize utility (1) subject to human capital accumulation (2) and the budget constraint (3). Public schooling $E_t^u$ is decided by the government. Households decide consumption $c_t$ and private tutoring $e_t$, where $e_t$ is given such that the following equation holds.

$$(1-x)e_t + \left(\frac{\alpha(1-x)e_t^{1-\rho}}{(1-\alpha)(1-\beta)A^{1-\sigma}\left(\beta E_t^{u\rho} + (1-\beta)e_t^{\rho}\right)^{\frac{1-\sigma}{\rho}-1}}\right)^{\frac{1}{\sigma}} = (1-\tau)h_t \tag{9}$$

Defining the left-hand-side and right-hand-side of (9) as L and R respectively, one can present the following figure and can obtain optimal allocation $e_t^*$.

[Insert Fig. 1 around here.]

With $1-\rho-\frac{1-\sigma-\rho}{\frac{\beta}{1-\beta}\left(\frac{E_t^u}{e_t}\right)^{\rho}} > 0$, L is the up-sloping curve. An increase in the subsidy for $x$ raises demand for the private tutoring. However, a decrease in public schooling $E_t^u$ reduces demand for private tutoring.[2] With a decrease in $E_t^u$, the marginal productivity of human capital of $e_t$ decreases. Then the individuals raise the level of consumption. The following proposition can be established.

**Proposition 1**

With $\frac{1-\sigma-\rho}{\rho} > 0$, public school education $E_t^u$ raises private tutoring $e_t$. With $\frac{1-\sigma-\rho}{\rho} < 0$, public school education $E_t^u$ reduces private tutoring $e_t$. With $\frac{1-\sigma-\rho}{\rho} = 0$, public school education $E_t^u$ does not affect private tutoring $e_t$.

If one considers the logarithmic utility function and Cobb–Douglas human capital accumulation, then $1-\sigma-\rho = 0$ because of $\rho = 0$ and $\sigma = 1$. Public school education $E_t^u$ does not affect private tutoring $e_t$.

[2] Defining $X = \frac{e_t^{1-\rho}}{\left(\beta E_t^{u\rho}+(1-\beta)e_t^{\rho}\right)^{\frac{1-\sigma}{\rho}-1}}$, one can obtain $\ln X = (1-\rho)\ln e_t - \frac{1-\sigma-\rho}{\rho}\ln\left(\beta E_t^{u\rho} + (1-\beta)e_t^{\rho}\right)$. Considering that the right hand side of (9) is constant, the left hand side of (9) should be constant. An increase in $E_t^u$ reduces $e_t$ in the case of $\frac{1-\sigma-\rho}{\rho} < 0$. With $\frac{1-\sigma-\rho}{\rho} > 0$, an increase in $E_t^u$ raises $e_t$. We can obtain $\sigma = 1-\rho$ by which public school education does not affect private tutoring. With $\rho > 0$, the condition of $\frac{1-\sigma-\rho}{\rho} > 0$ is reduced to $0 < \rho < 1-\sigma$. In the case of $\rho < 0$, we do not have the condition of $\frac{1-\sigma-\rho}{\rho} > 0$ if $1-\sigma > 0$. With $\rho < 0$ and $1-\sigma < 0$, $1-\sigma < \rho < 0$ holds for the condition of $\frac{1-\sigma-\rho}{\rho} > 0$. The condition of $\frac{1-\sigma-\rho}{\rho} < 0$ is reduced to $\rho > 1-\sigma$ in the case of $\rho > 0$. With $\rho < 0$, the condition of $\frac{1-\sigma-\rho}{\rho} < 0$ holds if $\rho < 1-\sigma$ and $1-\sigma < 0$. With $\rho < 0$ and $1-\sigma > 0$, the condition of $\frac{1-\sigma-\rho}{\rho} < 0$ holds for $\rho < 0$.

However, even if one does not consider the logarithmic utility function and Cobb–Douglas human capital accumulation, then one can obtain $1-\sigma-\rho=0$.

With $e_t^{u*}$, one can obtain optimal allocations $c_t^{u*}$ because of the budget constraint (3). Also, $e_t^{u*}$ increases with $h_t h_t$, as shown by (9). Considering (1), $c_t^{u*}$ and $e_t^{u*}$, one can obtain the following indirect utility function.

$$v_t^u = u_t(c_t^{u*}, e_t^{u*}) \tag{10}$$

For private schooling, households maximize utility (1) subject to the constraint of (4) and (5). Optimal allocations of $e_t^r$, $E_t^r$ and $c_t^r$ are shown as the following.

$$\frac{e_t^r}{E_t^r} = \left(\frac{1-\beta}{\beta(1-x)}\right)^{\frac{1}{1-\rho}} \tag{11}$$

$$\left(\left(\frac{\alpha}{(1-\alpha)\beta A^{1-\sigma}\left(\beta+(1-\beta)\left(\frac{1-\beta}{\beta(1-x)}\right)^{\frac{\rho}{1-\rho}}\right)}\right)^{\frac{1}{\sigma}} + 1 + (1-x)^{-\frac{\rho}{1-\rho}}\left(\frac{1-\beta}{\beta}\right)^{\frac{1}{1-\rho}}\right)E_t^r = (1-\tau)h_t \tag{12}$$

$$c_t = \left(\frac{\alpha E_t^{r\,1-\rho}}{(1-\alpha)\beta A^{1-\sigma}\left(\beta E_t^{r\rho}+(1-\beta)e_t^{\rho}\right)^{\frac{1-\sigma}{\rho}-1}}\right)^{\frac{1}{\sigma}} \tag{13}$$

As shown by (11), the subsidy for private tutoring $x$ raises $\frac{e_t^r}{E_t^r}$. However, the subsidy does not always reduce $E_t^r$ because of the first term of the left-hand-side of (12), which shows that an increase in subsidy $x$ raises demand for $E_t^r$.

Inserting (11)–(13) into utility function (1), one can obtain the following indirect utility function in the case of private education.

$$v_t^r = u_t(c_t^{r*}, e_t^{r*}, E_t^{r*}) \tag{14}$$

We consider education choice. If the following inequality is held, then households select public school education. Otherwise, they select private school education.

$$v_t^u > v_t^r \tag{15}$$

Defining $h_t^*$ to hold $v_t^u = v_t^r$, individuals of $[\underline{h}_t, h_t^*]$ select public school education. The share of households $[h_t^*, \overline{h}_t]$ select private school education. A household with large income will send children to the private school because the utility gained from a private education school increases with income. However, the level of public education school can not change according to household preferences.

The existence of $h_t^*$ can be shown if the gap between $\underline{h}_t$ and $\overline{h}_t$ is large. If individuals choose public education, then the marginal human capital accumulation of $e_t$ decreases. However, if the individuals choose the private school education, then even if the marginal productivity of human capital accumulation of $e_t$ decreases, an increase in private school education $E_t^r$ raises the marginal productivity of human capital $e_t$. Then the individuals can obtain large amounts of human capital stock.

**4. Human Capital Accumulation of Other Types**

For earlier discussion, the CES forms of human capital accumulation were assumed. However, obtaining the reduced form solution is difficult in the case of CES forms. Therefore, some reports of the literature have included assumptions of the specific human capital accumulation equation. With $\rho = 0$, one can obtain the solution of Cobb–Douglas form of human capital accumulation. For the analyses described in this section, we consider two cases: one with perfect substitution $\rho = 1$ and one with perfect complementarity $\rho = -\infty$.

**4.1. Perfect substitution** ($\rho = 1$)

**4.1.1. Public school education**

For perfect substitution of the public schooling and the private tutoring, human capital accumulation is given as follows because of $\rho = 1\rho = 1$:

$$h_{t+1} = A\left(\beta E_t^u + (1-\beta)e_t\right) \quad . \tag{16}$$

Based on the budget constraint (3) and human capital accumulation (16), one can obtain the following optimal allocations to maximize utility function (1).

$$e_t = \frac{(1-\tau)h_t - \beta\left(\frac{\alpha(1-x)}{A(1-\alpha)(1-\beta)}\right)^{\frac{1}{\sigma}} E_t^u}{(1-\beta)\left(\frac{\alpha(1-x)}{A(1-\alpha)(1-\beta)}\right)^{\frac{1}{\sigma}} + 1}, if\ (1-\tau)h_t > \beta\left(\frac{\alpha(1-x)}{A(1-\alpha)(1-\beta)}\right)^{\frac{1}{\sigma}} E_t^u \tag{17}$$

$$e_t = 0, if\ (1-\tau)h_t < \beta\left(\frac{\alpha(1-x)}{A(1-\alpha)(1-\beta)}\right)^{\frac{1}{\sigma}} E_t^u \tag{18}$$

$$c_t = A\left(\frac{\alpha(1-x)}{A(1-\alpha)(1-\beta)}\right)^{\frac{1}{\sigma}} \left(\beta E_t^u + (1-\beta)e_t\right). \tag{19}$$

If the income of individuals is poor because of the low level of $h_t$, then they can not allocate expenditures for private tutoring. Then, inequality of human capital acquired by children does not exist in the group for which individuals can not make expenditures for private tutoring. However, inequality prevails among individuals that can not make expenditures for private tutoring and individuals that have a high income and which can thereby make expenditures for private tutoring exists.

We define $\hat{h}_t$ such that $\hat{h}_t = \frac{\beta}{1-\tau}\left(\frac{\alpha(1-x)}{A(1-\alpha)(1-\beta)}\right)^{\frac{1}{\sigma}} E_t^u$ holds: the share of households of $[\underline{h}_t, \hat{h}_t]$ chooses public school education without private tutoring. However, the share of households of $[\hat{h}_t, h_t^*]$ chooses public school education with private tutoring.

**4.1.2 Private school education**

In the case of the private schooling, the perfect substitution form can be presented as follows.

$$h_{t+1} = A\left(\beta E_t^r + (1-\beta)e_t\right) \tag{20}$$

Based on the budget constraint (5) and human capital accumulation (20), the optimal allocations to maximize utility function (1) can be derived as shown below.

With $\frac{\beta}{1-\beta} > \frac{1}{1-x}$,

$$E_t^r = \frac{(1-\tau)h_t}{1 + \left(\frac{\alpha}{(1-\alpha)(A\beta)^{1-\sigma}}\right)^{\frac{1}{\sigma}}}, \tag{21}$$

$$e_t = 0, \tag{22}$$

$$c_t = \left(\frac{\alpha}{(1-\alpha)(A\beta)^{1-\sigma}}\right)^{\frac{1}{\sigma}} E_t^r. \tag{23}$$

With $\frac{\beta}{1-\beta} < \frac{1}{1-x}$,

$$E_t^r = 0, \tag{24}$$

$$e_t = \frac{(1-\tau)h_t}{1 - x + \left(\frac{\alpha(1-x)}{(1-\alpha)\left(A(1-\beta)\right)^{1-\sigma}}\right)^{\frac{1}{\sigma}}}, \tag{25}$$

$$c_t = \left(\frac{\alpha(1-x)}{(1-\alpha)\left(A(1-\beta)\right)^{1-\sigma}}\right)^{\frac{1}{\sigma}} e_t. \tag{26}$$

If public school education $E_t^u$ is pulled up by an increase in $\theta$, which reduces the subsidy for private tutoring $x$, then the share of the individuals choosing public school education increases. Demand increases not for private school education, but for private tutoring because the household chooses public school education with private tutoring. Then, the following proposition can hold.

**Proposition 2**

With $\frac{\beta}{1-\beta} > \frac{1}{1-x}$, the share of households $[\underline{h}_t, \hat{h}_t]$ chooses public school education without private tutoring. However, the share of households $[\hat{h}_t, h_t^*]$ chooses public school education with private tutoring. The share of households $[h_t^*, \overline{h}_t]$ chooses private school education without private tutoring. With $\frac{\beta}{1-\beta} < \frac{1}{1-x}$, no household chooses the private school system.

It is noteworthy that $\hat{h}_t = \frac{\beta}{1-\tau}\left(\frac{\alpha(1-x)}{A(1-\alpha)(1-\beta)}\right)^{\frac{1}{\sigma}} E_t^u$ depends on the preference parameters of the utility function. For instance, a decrease in $\alpha$, that is, an increase in the preference for the human capital of children, reduces $\hat{h}_t$ Then the share of households that chooses public school education without private tutoring shrinks.

### 4.2. Perfect complementarity ($\rho = -\infty$)

#### 4.2.1. Public school education

Human capital accumulation is given by the following form because $\rho = -\infty$,[3]

$$h_{t+1} = A\, min\big(\beta E_t^u, (1-\beta)e_t\ \big) \tag{27}$$

In this case, the optimal allocations of $e_t$ are given as $\beta E_t^u = (1-\beta)e_t$ :

$$e_t = \frac{\beta E_t^u}{1-\beta}. \tag{28}$$

$$c_t = (1-\tau)h_t - \frac{\beta(1-x)}{1-\beta}E_t^u. \tag{29}$$

However, if the households have insufficient income for private tutoring (supplemental education services), then

$$e_t = \frac{(1-\tau)h_t}{1-x+\left(\frac{\alpha(1-x)}{(1-\alpha)\big(A(1-\beta)\big)^{1-\sigma}}\right)^{\frac{1}{\sigma}}}, if\ \frac{(1-\tau)h_t}{1-x+\left(\frac{\alpha(1-x)}{(1-\alpha)\big(A(1-\beta)\big)^{1-\sigma}}\right)^{\frac{1}{\sigma}}} < \frac{\beta E_t^u}{1-\beta} \tag{30}$$

$$c_t = \left(\frac{\alpha(1-x)}{(1-\alpha)\big(A(1-\beta)\big)^{1-\sigma}}\right)^{\frac{1}{\sigma}} e_t. \tag{31}$$

Compared to the case of perfect substitution, even if individuals have sufficient income to expend for private tutoring $e_t$, because of complementarity they do not increase the level of private tutoring beyond the level of $e_t = \frac{\beta E_t^u}{1-\beta}$. Then, among high income individuals, inequality of human capital of children does not exist. Defining $\tilde{h}_t = \frac{\beta E_t^u\left(1-x+\left(\frac{\alpha(1-x)}{(1-\alpha)(A(1-\beta))^{1-\sigma}}\right)^{\frac{1}{\sigma}}\right)}{(1-\tau)(1-\beta)}$, the demand for private tutoring rises proportionally in the share of households of $[\underline{h}_t, \tilde{h}_t]$. However, in the share of $[\tilde{h}_t, h_t^*]$, demand for private tutoring does not change according to the household income.

#### 4.2.2. Private school education

Human capital accumulation is given by the following form because $\rho = -\infty$,

$$h_{t+1} = A\, min\big(\beta E_t^r, (1-\beta)e_t\ \big) \tag{32}$$

Based on the budget constraint (5) and human capital accumulation (32), the optimal allocations to maximize the utility function (1) are presented below.

$$E_t^r = \frac{\beta}{1-\beta}e_t, \tag{33}$$

$$e_t = \frac{(1-\tau)h_t}{\left(\frac{\alpha(1-\beta x)}{\beta(1-\alpha)\big(A(1-\beta)\big)^{1-\sigma}}\right)^{\frac{1}{\sigma}} + \frac{1-\beta x}{\beta}} \tag{34}$$

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[3] Strictly, with $\rho = -\infty$, the human capital accumulation function is given by $h_{t+1} = A\, min\big(E_t^u, e_t\ \big)$ . However, because we consider the productivity of the education investment, we assume (27) as the case of $\rho = -\infty$.

$$c_t = \left( \frac{\alpha(1-\beta x)}{\beta(1-\alpha)\big(A(1-\beta)\big)^{1-\sigma}} \right)^{\frac{1}{\sigma}} e_t \tag{35}$$

As shown for the substitution case in private education, if individuals have higher income, then the education investment of private school education and private tutoring is greater. Among the group in which individuals choose private school education, inequality exists. Therefore, the following proposition can be established.

**Proposition 3**

There exist households of three types. The share of households of $[\underline{h}_t, \tilde{h}_t]$ chooses public school education with income proportional to private tutoring. The share of households of $[\tilde{h}_t, h_t^*]$ chooses public school education with private tutoring that is not correlated with household income. The share of households of $[h_t^*, \overline{h}_t]$ chooses private school education.

## 5. Welfare Policy

Generally, the tax rate chosen by the median voter is equal to the tax rate to maximize social welfare.

$$W = \Omega \int_{h_t^*}^{\overline{h}_t} v_{pri}(\tau, x, h_t) f(h_t) dh_t + (1-\Omega) \int_{\underline{h}_t}^{h_t^*} v_{pub}(\tau, E_t^u, x, h_t) f(h_t) dh_t, \tag{36}$$

In that equation, $\Omega$ denotes the weight parameter of the welfare of the individuals that choose the private school education. We can show optimal policy $\tau$ and $\theta$ as

$$\begin{aligned} &\frac{\partial \Omega \int_{h_t^*}^{\overline{h}_t} v_{pri}(\tau, x, h_t) f(h_t) dh_t}{\partial \tau} + \frac{\partial (1-\Omega) \int_{\underline{h}_t}^{h_t^*} v_{pub}(\tau, E_t^u, x, h_t) f(h_t) dh_t}{\partial \tau} \\ &\quad + \frac{\partial (1-\Omega) \int_{\underline{h}_t}^{h_t^*} v_{pub}(\tau, E_t^u, x, h_t) f(h_t) dh_t}{\partial E_t^u} \frac{\partial E_t^u}{\partial \tau} \\ &\quad + \frac{\partial \Omega \int_{h_t^*}^{\overline{h}_t} v_{pri}(\tau, x, h_t) f(h_t) dh_t}{\partial x} \frac{\partial x}{\partial \tau} \\ &\quad + \frac{\partial (1-\Omega) \int_{\underline{h}_t}^{h_t^*} v_{pub}(\tau, E_t^u, x, h_t) f(h_t) dh_t}{\partial x} \frac{\partial x}{\partial \tau} = 0, \end{aligned} \tag{37}$$

$$\begin{aligned} &\frac{\partial (1-\Omega) \int_{\underline{h}_t}^{h_t^*} v_{pub}(\tau, E_t^u, x, h_t) f(h_t) dh_t}{\partial E_t^u} \frac{\partial E_t^u}{\partial \theta} + \frac{\partial \Omega \int_{h_t^*}^{\overline{h}_t} v_{pri}(\tau, x, h_t) f(h_t) dh_t}{\partial x} \frac{\partial x}{\partial \theta} \\ &\quad + \frac{\partial (1-\Omega) \int_{\underline{h}_t}^{h_t^*} v_{pub}(\tau, E_t^u, x, h_t) f(h_t) dh_t}{\partial x} \frac{\partial x}{\partial \theta} = 0, \end{aligned} \tag{38}$$

where (37) represents the optimal tax rate to maximize the welfare function. An increase in the income tax rate reduces the disposable household income, which reduces welfare. This effect is demonstrated by the first and the second terms of (37). An increase in the income tax rate raises the subsidy for public school education (the third term) and the private tutoring (the fourth and fifth terms). This is the positive effect for welfare. Especially with low $\Omega$, the positive effect, as shown by third term of (37) for welfare, is large. Then, the income tax rate to maximize the welfare is large.

Also, (38) presents the allocation rule to maximize welfare. With low $\Omega$ and $\beta$, a positive effect for welfare as shown by the first term of (38) is large. The allocation for public school education is large. However, even if $\Omega$ is low, then low $\beta$ can bring about a decrease in $\theta$ because the subsidy for private tutoring raises the level of social welfare.

## 6. Conclusions

Our paper presents examination of human capital accumulation that is inputted not only by school education but also by private tutoring. Some cases of substitution between school education and private tutoring are considered. Being different from a Cobb–Douglas human capital accumulation function and a logarithmic utility function, which are invariably assumed in the related literature, the study presented herein has produced many policy implications. For instance, with substitution between school education and private tutoring, households which choose public school education and which have low income are adversely affected by the inequality of the education investment for children. Demand for private tutoring by households that choose public school education depends on the level of public school education and parameters of the utility function and the human capital accumulation function.

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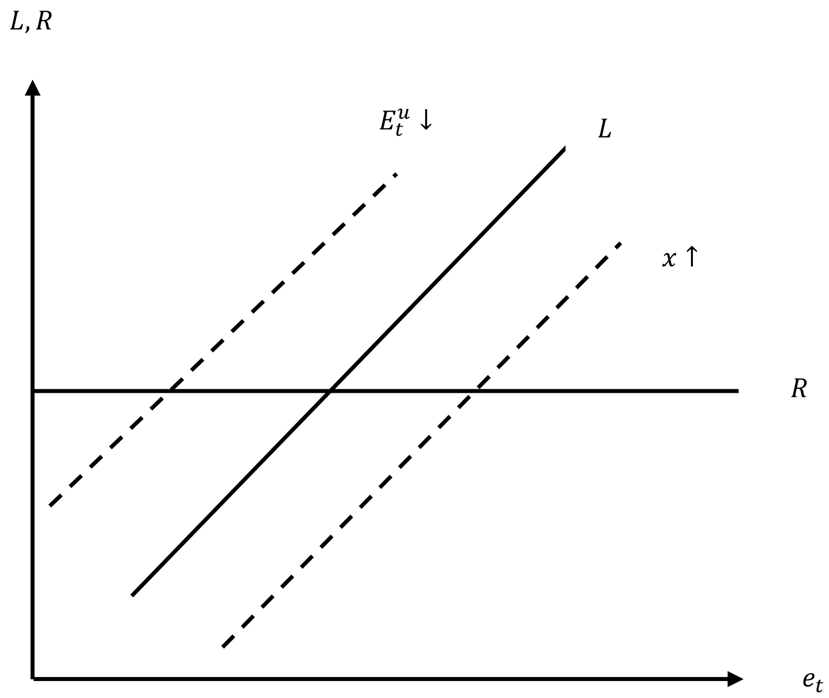


Fig. 1 Optimal allocation in the case of public education.