DISCUSSION PAPER SERIES

Discussion paper No. 231

Tax Incidence and Fiscal Sustainability in DSGE Model

Junko Doi
Kansai University

Kota Yamada
Kansai University

Masaya Yasuoka
Kwansei Gakuin University

November 2021

MASTERY FOR SERVICE

SCHOOL OF ECONOMICS

KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho
Nishinomiya 662-8501, Japan

Tax Incidence and Fiscal Sustainability in DSGE Model[†]

Junko Doi[‡], Kota Yamada[§], Masaya Yasuoka[¶]

Abstract

The aims of our study are to set a Dynamic Stochastic General Equilibrium (DSGE) model and to examine how increased income or consumption tax rates affect the ratio of public debt to GDP and other macroeconomic parameters. We consider taxation of three types, on labor income, capital income, and consumption. Results derived from our simulation show that an increase in income tax rates of these forms of taxation raises the ratio of public debt to GDP because GDP and tax revenues decrease. An increase in consumption tax rate can reduce the ratio of public debt to GDP because of an increase in the aggregate demand that is pulled up by the investment. Our study shows that a decrease in the income tax rate reduces the ratio of public debt to GDP.

Key words: DSGE Model, Fiscal Sustainability, Taxation.
JEL Classification: E60.

[†] We would like to thank Yasuyuki Nishigaki and participants of the conference of JEPA 2021 for helpful comments. Nevertheless, any remaining errors are the authors' responsibility.

[‡] Kansai University
[§] Kansai University
[¶] Corresponding to: Kwansei Gakuin University, E-mail: yasuoka@kwansei.ac.jp

Tax Incidence and Fiscal Sustainability in DSGE Model

1. Introduction

In 2020, COVID-19 brought with it a worldwide pandemic. Many people, especially elderly people, have become afflicted with COVID-19. Among them, many have died. In addition, in many countries, lock-down policies were launched to stop the pandemic. Nevertheless, it remains unknown when the pandemic will cease. Along with medical and public health challenges, economic difficulties have become daunting. Firms have had to halt production activities, leaving many people unemployed. In some countries, unemployment has increased sharply. Figure 1 portrays recent gross domestic product (GDP) growth rates in context: in 2020, the GDP growth rate decreased sharply because of halted economic activity caused by lockdowns and other anti-COVID-19 initiatives.

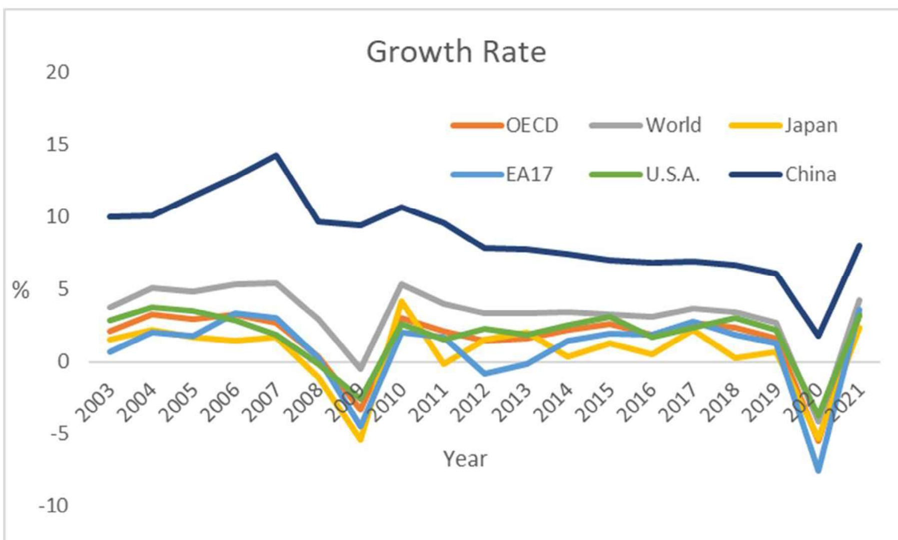

Fig. 1 GDP growth rate (Data: OECD Statistics).

Because of decreased economic growth, the employment situation gradually worsened. The unemployment rate increased. Wage rates decreased. Working generation people have been adversely affected by the bad employment situation. Governments have provided active policies for people to whom government gives benefits for compensation of lost wage income.

Many countries have considered economic policies for COVID-19. For instance, in Japan, great amounts of cash benefits have been provided for people. Because of active economic policies, public debt in Japan has reached a high level: the ratio of public debt to GDP has reached to more than 200%. Figure 2 exhibits the ratio of fiscal surplus to GDP worldwide. Negative value shows the fiscal deficit.

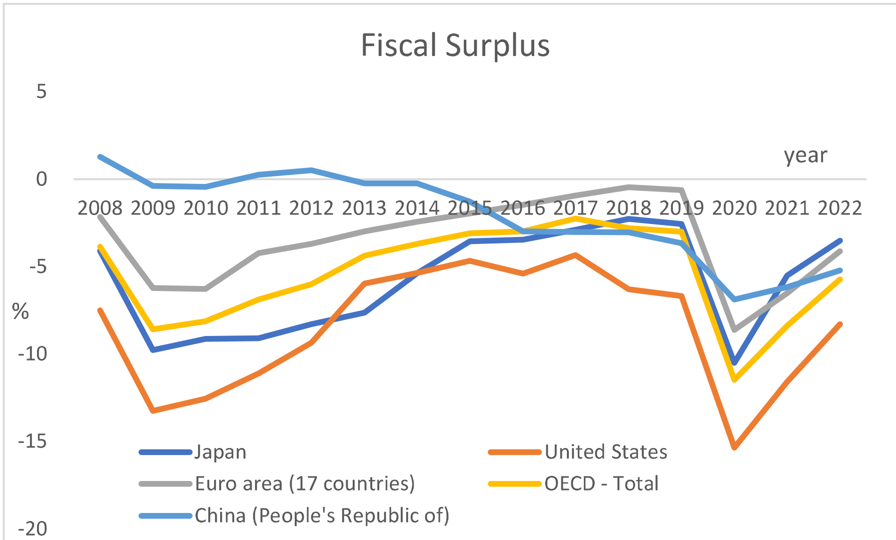

Fig. 2 Fiscal surplus (Data: OECD Economic Outlook 108).

The aims of our study are to set a Dynamic Stochastic General Equilibrium (DSGE) model and to examine how increased income or consumption tax rates affect the ratio of public debt to GDP and other macroeconomic parameters. We consider taxation of three types, on labor income, capital income, and consumption. Results derived from our simulation show that an increase in income tax rates of these forms of taxation raises the ratio of public debt to GDP because GDP and tax revenues decrease. Therefore, our study demonstrates that a decrease in tax rates applied for these forms of taxation ultimately reduces the ratio of public debt to GDP. Thereby, fiscal sustainability can be maintained. The parameter settings of our DSGE model simulation are based partially on Eguchi (2011). Some parameters are derived through calibration based on actual data.

Some studies examine how taxation affects macroeconomic parameters. Doi (2010) and Hayashida, Yasuoka, Nanba and Ono (2019) examine how taxation affects income distribution and income inequality. Doi (2010) and Hayashida, Yasuoka, Nanba and Ono (2019) respectively examine the cases of corporate tax and consumption tax and derive effects of the distribution rates of capital income and labor income. Nevertheless, these models do not consider the public debt that our manuscript considers. Considering the fiscal sustainability in the model with public debt is important.

Watanabe, Miyake and Yasuoka (2015) compare income and consumption taxes in terms of income growth.[1] According to the consumption tax, the decrease occurring in capital stock is smaller than it is the income tax case. Then the consumption tax should be used to finance government expenditures.

Herein, we present an examination of taxation effects on public debt. There exist many related studies in the literature. Futagami, Iwaisako and Ohdoi (2008), Yakita (2008), Arai (2011), and Teles and Mussolini (2014) consider productive government expenditures financed by public debt. Ono (2003) derive correlation with an aging population with public debt stock. Chalk (2000), Greiner (2007, 2008), and Moraga and Vidal (2008) all derive fiscal sustainability using models with public debt. Oguro and Sato (2014) demonstrate that an increase in the income tax rate does not always bring about fiscal sustainability.

The paper is presented as follows. In section 2, a simple DSGE model with public debt is set. Section 3 describes derivation of the equilibrium of this model economy. The steady state equilibrium is derived in section 4. In section 5, we explain the parameter settings. Section 6 shows results of policy shocks. Section 7 explains additional simulation results. The final section concludes with a presentation of the results.

2. Model

Based on Eguchi (2011), Kato (2008), and Hayashida, Yasuoka, Nanba and Ono (2019), we set the following DSGE model with public debt. This model economy has agents of three types: households, firms, and government.

2.1. Households

The instantaneous utility function of household is assumed as

$$u_t = \frac{c_t^{1-\theta}}{1-\theta} + \frac{m_t^{1-\mu}}{1-\mu} - \frac{l_t^{1+\kappa}}{1+\kappa}, 0 < \theta, 0 < \mu, 0 < \kappa. \tag{1}$$

In that equation, c_t, m_t, and l_t respectively denote the consumption, real money, and labor time in t period. This function form is a constant relative risk averse (CRRA) utility function.

The budget constraint in t period is shown as presented below.

$$\begin{aligned} m_t + b_t + (1+\tau_{ct})c_t + I_t \\ = \frac{1}{1+\pi_t}[(1+i_t)b_{t-1} + m_{t-1}] + \varphi_t + (1-\tau_{wt})w_t l_t \\ + (1-\tau_{rt})r_t K_{t-1}. \end{aligned} \tag{2}$$

Therein, b_t represents a bond issued by both firms and the government; I_t denotes investment in real capital stock K_t. Also, i_t, r_t, and π_t respectively stand for the nominal interest rate, the real interest rate, and inflation rate. w_t is the wage rate. Households own the firms, from which they

[1] Barro (1990) derive that an increase in productive government expenditure financed by income tax raises GDP.

obtain firm profits φ_t. Also, τ_{wt}, τ_{rt} and τ_{ct} respectively denote the tax rate of labor income, capital income, and consumption.

An equation for capital stock accumulation dynamics is shown below:

$$K_t = I_t + (1-\delta)K_{t-1} - S\left(\frac{I_t}{I_{t-1}}\right)I_t, 0 < \delta < 1, S' > 0, S(1) = S'(1) = 0. \tag{3}$$

Therein, δ denotes the depreciation rate. The regulation cost of investment is $S\left(\frac{I_t}{I_{t-1}}\right)$.

Households maximize lifetime utility as

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\theta}}{1-\theta} + \frac{m_t^{1-\mu}}{1-\mu} - \frac{l_t^{1+\kappa}}{1+\kappa}\right], 0 < \beta < 1. \tag{4}$$

In that equation, E_0 and β respectively denote the expectation operator and discount rate. One can derive the optimal allocations of household to maximize the lifetime utility (4) subject to the budget constraint (2) and the dynamics of capital stock accumulation (3). Lagrange equation L can be set as presented below.

$$\begin{aligned} L = {} & E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\theta}}{1-\theta} + \frac{m_t^{1-\mu}}{1-\mu} - \frac{l_t^{1+\kappa}}{1+\kappa}\right] \\ & + E_0 \sum_{t=0}^{\infty} \lambda_t \Big[m_t + b_t + (1+\tau_{ct})c_t + I_t - \frac{1}{1+\pi_t}\left[(1+i_t)b_{t-1} + m_{t-1}\right] - \varphi_t \\ & \qquad - (1-\tau_{wt})w_t\, l_t - (1-\tau_{rt})r_t K_{t-1}\Big] \\ & + E_0 \sum_{t=0}^{\infty} \gamma_t \left[K_t - I_t - (1-\delta)K_{t-1} + S\left(\frac{I_t}{I_{t-1}}\right)I_t\right] \end{aligned} \tag{5}$$

In those expressions, λ_t and γ_t are the Lagrange multipliers. Because of the first order conditions, one can obtain the following allocations:

$$\frac{c_t^{-\theta}}{1+\tau_{ct}} = \beta E_t \frac{c_{t+1}^{-\theta}}{1+\tau_{ct+1}} \frac{1+i_{t+1}}{1+\pi_{t+1}}, \tag{6}$$

$$\frac{(1-\tau_{wt})w_t}{1+\tau_{ct}} = \frac{l_t^{\kappa}}{c_t^{-\theta}}, \tag{7}$$

$$1 = q_t\left(1 - S\left(\frac{I_{t-1}}{I_{t-2}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right) + E_t q_{t+1} \frac{1+\pi_{t+1}}{1+i_{t+1}} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2, \tag{8}$$

$$E_t\big((1-\tau_{rt+1})r_{t+1} + q_{t+1}(1-\delta)\big) = q_t E_t \frac{1+i_{t+1}}{1+\pi_{t+1}}. \tag{9}$$

Therein, $q_t = \frac{\gamma_t}{\lambda_t}$ is the ratio of Lagrange multiplier.

2.2 Firm

There exist firms of two types: final goods firms and intermediate goods firms. In the former, final goods are produced using intermediate goods in a perfectly competitive market.

2.2.1. Final goods firms

The product function is assumed to be

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}, 1 < \varepsilon. \tag{10}$$

Therein, Y_t and Y_{jt} respectively represent the final goods and the intermediate goods produced in the *j*-th firm. The profit function of final goods firm π_t^f is

$$\pi_t^f = p_t Y_t - \int_0^1 p_{jt} Y_{jt} dj, 0 \le j \le 1. \tag{11}$$

In that equation, p_t and p_{jt} respectively stand for the price of final goods and the price of *j*-th intermediate goods. Maximizing the profit of (11) can be reduced to the following demand for *j*-th intermediate goods.

$$Y_{jt} = \left(\frac{p_{jt}}{p_t}\right)^{-\varepsilon} Y_t. \tag{12}$$

Moreover, one can obtain

$$p_t = \left(\int_0^1 p_{jt}^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}, \tag{13}$$

$$p_t Y_t = \int_0^1 p_{jt} Y_{jt}\, dj. \tag{14}$$

Equations (13) and (14) respectively show the price index and nominal GDP.

2.2.2. Intermediate goods firm

The production function of the *j*-th firm that produces intermediate goods is assumed as

$$Y_{jt} = K_{jt-1}^{\alpha} N_{jt}^{1-\alpha}, 0 < \alpha < 1. \tag{15}$$

Therein, K_{jt} and N_{jt} respectively represent the capital stock and labor input for production of *j*-th intermediate goods.

Given by the total cost of the firm as $C_j = w_{jt} N_{jt} + r_{jt} K_{jt-1}$, one can set the following Lagrange equation to minimize the cost as

$$\Lambda = w_{jt} N_{jt} + r_{jt} K_{jt-1} + \omega_{jt}\left(Y_{jt} - K_{jt-1}^{\alpha} N_{jt}^{1-\alpha}\right), \tag{16}$$

where ω_{jt} is the Lagrange multiplier. Then, the optimization of K_{jt-1} and N_{jt} can be reduced to the following equations.

$$w_{jt} = \omega_{jt}(1-\alpha) K_{jt}^{\alpha} N_{jt}^{-\alpha} \tag{17}$$

$$r_{jt} = \omega_{jt} \alpha K_{jt-1}^{\alpha-1} N_{jt}^{1-\alpha} \tag{18}$$

Because of (17) and (18), the total cost can be shown as

$$C_j = w_{jt} N_{jt} + r_{jt} K_{jt-1} = \omega_{jt} Y_{jt}, \tag{19}$$

where ω_{jt} represents the marginal cost.

The profit function of j-th firm π_{jt} can be shown as

$$\pi_{jt} = \frac{p_{jt}}{p_t}\left(\frac{p_{jt}}{p_t}\right)^{-\varepsilon} Y_t - \omega_{jt}\left(\frac{p_{jt}}{p_t}\right)^{-\varepsilon} Y_t. \quad (20)$$

Profit maximization of p_{jt} can be shown as presented below.

$$\omega_{jt} = \frac{\varepsilon - 1}{\varepsilon}\frac{p_{jt}}{p_t} \quad (21)$$

With homogenous firms, we obtain $\omega = \frac{\varepsilon-1}{\varepsilon}$.

Log linearization of (17) and (18) are the following.

$$\widehat{w}_t = \widehat{\omega}_t + \hat{A}_t + \alpha\widehat{K}_{t-1} + \alpha\hat{l}_t \quad (22)$$

$$\hat{r}_t = \widehat{\omega}_t + \hat{A}_t + (\alpha - 1)\widehat{K}_{t-1} + (1-\alpha)\hat{l}_t \quad (23)$$

In those expressions, $\widehat{w}_t$ and $\widehat{w}_t$ respectively represent the rates of change of w_t and $\hat{r}_t$. In addition, $\hat{A}_t$ stands for the rate of change of TFP. $\widehat{K}_t$ and $\hat{l}_t$ respectively denote the rates of change of K_t and l_t. The equilibrium condition of the labor market is $l_t = N_t$. $\widehat{\omega}_t$ denotes the rate of change of ω_t.

2.2.3. Sticky price

Based on Calvo (1983) price setting, we consider the firm pricing rule. In the Calvo (1983) model, some firms are unable to set the optimal price to maximize profit. Given ρ as the probability that the firm can set an optimal price level at t period, we can obtain the following the dynamics of the inflation rate as

$$\tilde{\pi}_t = E_t\tilde{\pi}_{t+1} + \frac{\rho^2}{1-\rho}\widehat{\omega}_t, \quad (24)$$

where $\tilde{\pi}_{\mathrm{t}}$ represents the level of change π_t.

2.3 Policy

Our study is designed to consider monetary policy and fiscal policy.

2.3.1. Monetary Policy

Based on the Taylor rule, the monetary policy is provided as

$$\tilde{\iota}_t = \chi\tilde{\iota}_{t-1} + (1-\chi)\left(\psi_1 E_t\tilde{\pi}_{t+1} + \psi_2\widehat{Y}_t\right), 0 < \chi < 1, 0 < \psi_1, 0 < \psi_2. \quad (25)$$

Therein, $\tilde{\iota}_t$ and $\widehat{Y}_t$ respectively denote the level of change i_t and the rate of change Y_t,.

2.3.2 Fiscal policy

The government provides lump-sum benefits as the economic policy. If the government is allowed to finance the benefits with public debt B_t, then the government budget constraint can be represented as

$$B_{t+1} = (1 + i_t)B_t + G_t - \tau_{wt}w_t l_t - \tau_{rt}r_t K_{t-1} - \tau_{ct}c_t. \quad (26)$$

In that equation, G_t denotes the non-productive government expenditure.

3. Equilibrium

This section presents derivation of the equilibrium of this model economy. Linearization of (6) can be reduced as follows.

$$\hat{c}_{t+1} = \hat{c}_t + \frac{1}{\theta}\left(E_t(\tilde{\imath}_{t+1} - \tilde{\pi}_{t+1}) - E_t \frac{\hat{\tau}_{ct+1}}{\frac{1}{\tau_c}+1} + \frac{\hat{\tau}_{ct}}{\frac{1}{\tau_c}+1} \right) \tag{27}$$

In that equation, $\hat{c}_t$ denotes the rate of change of c_t. In addition, $\tilde{\imath}_{t+1}$ and $\tilde{\pi}_{t+1}$ respectively stand for the level of change of i_{t+1} and π_{t+1}. τ_c is a steady state variable. The Fisher equation (9) can be shown as

$$\hat{q}_t = E_t\tilde{\pi}_{t+1} - \tilde{\imath}_t + \frac{E_t(-\tau_r\hat{\tau}_{rt+1} + r\hat{r}_{t+1} + (1-\delta)\hat{q}_{t+1})}{(1-\tau_r)r + (1-\delta)}, \tag{28}$$

where $\hat{q}_t$ represents the rate of change q_t. Also, τ_r and r denote the steady state variables. At the steady state, we obtain $q = 1$.

With linearization of (7), the rate of change of labor $\hat{l}_t$ can be presented as

$$\kappa\hat{l}_t + \theta\hat{c}_t = \widehat{w}_t - \frac{\hat{\tau}_{wt}}{\frac{1}{\tau_w}+1} - \frac{\hat{\tau}_{ct}}{\frac{1}{\tau_c}+1}. \tag{29}$$

Also, τ_w denotes the steady state variables. The rate of change of investment $\hat{I}_t$ is

$$\hat{I}_t = \frac{1+i}{2+i+\pi}\hat{I}_{t-1} + \frac{1+i}{2+i+\pi}E_t\hat{I}_{t+1} + \frac{1+i}{(2+i+\pi)S''(1)}\hat{q}_t. \tag{30}$$

In that equation, $\widehat{K}_t$ is given by the linearization of (3).

$$\widehat{K}_t = \delta\hat{I}_t + (1-\delta)\widehat{K}_{t-1}. \tag{31}$$

Because of market clearing condition of goods and service market, one can obtain the following equation:

$$\hat{Y}_t = \frac{c}{Y}\hat{c}_t + \frac{I}{Y}\hat{I}_t + \frac{G}{Y}\hat{G}_t. \tag{32}$$

The production function (15) can be shown as linearization as

$$\hat{Y}_t = \alpha\widehat{K}_{t-1} + (1-\alpha)\hat{l}_t. \tag{33}$$

Therein, $\widehat{w}_t$ and $\hat{r}_t$ are given by (22) and (23).

Because of log linearization at the steady state, the government budget constraint is

$$\widehat{B}_{t+1} = \tilde{\imath}_t + (1+i_t)\widehat{B}_t + \frac{G}{B}\hat{G}_t - \frac{T}{B}\hat{T}_t \tag{34}$$

where

$$\hat{T}_t = \frac{\tau_w wl}{T}\left(\hat{\tau}_{wt} + \widehat{w}_t + \hat{l}_t\right) + \frac{\tau_r rk}{T}\left(\hat{\tau}_{rt} + \hat{r}_t + \widehat{K}_{t-1}\right) + \frac{\tau_c c}{T}(\hat{\tau}_{ct} + \hat{c}_t). \tag{35}$$

In those equations, $\widehat{B}_t$ and $\hat{G}_t$ respectively denote the rates of change of B_t and G_t. Also, $\hat{T}_t$ represents the rate of change of aggregate tax revenue T_t. $\hat{\tau}_{wt}$, $\hat{\tau}_{rt}$, and $\hat{\tau}_{ct}$ express the rates of change of tax rates. The variables of no period are variables at the steady state. Finally, B, G, and T respectively denote the steady state variables.

4. Steady State

The equilibrium at the steady state is explained using the equations below.

Factor price

$$w = \omega(1-\alpha)K^{\alpha}l^{-\alpha}. \tag{36}$$

$$r = \omega\alpha K^{\alpha-1}l^{1-\alpha}. \tag{37}$$

Marginal cost

$$\omega = \frac{\varepsilon - 1}{\varepsilon}. \tag{38}$$

Final goods market

$$Y = c + I + G \tag{39}$$

Production function

$$Y = AK^{\alpha}l^{1-\alpha}. \tag{40}$$

Capital accumulation

$$\delta = \frac{I}{K}. \tag{41}$$

Intertemporal allocation of consumption (Euler equation)

$$1 = \beta\frac{1+i}{1+\pi} \tag{42}$$

Fisher equation

$$r + 1 - \delta = \frac{1+i}{1+\pi}. \tag{43}$$

Marginal rate substitution of consumption and labor (labor supply)

$$\frac{l^{\kappa}}{c^{-\theta}} = \frac{(1-\tau_w)w}{(1+\tau_c)c}. \tag{44}$$

Government budget constraint

$$B = (1+i)B + G - \tau_w wl - \tau_r rK - \tau_c c. \tag{45}$$

5. Parameter Settings

Based on an explanation by Eguchi (2011), we set the parameters as shown below.

θ	1.5
δ	0.06
α	0.33
ρ	0.25
κ	2
$S''(1)$	1/7

Table 1 Parameter Setting

The following parameters were derived through calibration.[2]

ψ_1	0.1885
ψ_2	0.1628
χ	0.2826

Table 2 Parameter Setting with Calibration

The steady state variables are set as shown below.

$\frac{c}{Y}$	0.55
$\frac{I}{Y}$	0.25
$\frac{G}{Y}$	0.2
π	0
q	1
i	0.004
r	0.064
$\frac{\tau_w wl + \tau_r rK}{T}$	0.576
$\frac{\tau_c c}{T}$	0.426
$\frac{G}{B}$	0.00193
$\frac{T}{B}$	0.1446
τ_w, τ_r	0.0494
τ_c	0.06667

Table 3 Parameter settings at the steady state[3]

[2] The calibration is based on the Data: Cabinet Office, Government of Japan, SNA (National Accounts of Japan). We use the data of GDP, consumption, investment, government expenditure, the working population size, nominal interest rate, and inflation rate at 1994–2019. Except for nominal interest rate and inflation rate, we change these variables to the logarithm variables. With the HP filter, we derive the variance from the trend about these variables. The calibration code and the setting for the calibration are presented in our paper.

[3] c/Y, I/Y, and G/Y are derived by the annual consumption, investment, government expenditure and GDP data at 1994–2019 as the average value (Data: Cabinet Office, Government of Japan). In recent years, the inflation rate is about constant level. Then we set $\pi = 0$ (Data: Cabinet Office, Government of Japan). Considering the nominal interest rate at recent years (2010–2019), we set $i = 0.04$ (Data: Cabinet Office, Government of Japan). Then, because of the Fisher equation at the steady state, we can obtain $r = 0.064$.

$(\tau_w wl + \tau_r rK)/T$ and $\tau_c c/T$ are given by data of the ratio of income taxation to consumption taxation in fiscal year 2021 (Data: Ministry of Finance, Japan). Also, T/B are average data of the ratio of total tax revenue to public debt stock level in 1994–2019 (Data: Ministry of Finance, Japan). Furthermore, G/B are data of fiscal year 2019 (Data:

6. Results

Our paper presents an examination of how an increase in the tax rates of labor income, capital income, and consumption affect the ratio of public debt stock to GDP and other parameters to ascertain whether these policies affect fiscal sustainability.

6.1 Increased labor income tax rate

First, we verify the results of an increase in the income tax rate. The government budget constraint is

$$\hat{B}_{t+1} = \tilde{\iota}_t + (1+i)\hat{B}_t - \frac{T}{B}\left(\frac{\tau_w wl}{T}\left(\hat{\tau}_{wt} + \hat{w}_t + \hat{l}_t\right) + \frac{\tau_r rK}{T}\left(\hat{r}_t + \hat{K}_{t-1}\right) + \frac{\tau_c c}{T}\hat{c}_t\right) \quad (46)$$

where

$$\tilde{\tau}_{wt} = \phi\tilde{\tau}_{wt-1} + f\ . \quad (47)$$

After setting $\phi = 0.5$, we apply a shock as $f = 1$. Figure 3 presents simulation results.[4] "tax" represents an increase in the income tax rate. At the first period, tax shows 1: an increase in income tax rate is 1%. Therefore, if an increase in income tax rate by 1%, then GDP (y) decreases because of a decrease in the aggregate demand, as shown by a decrease in consumption. The ratio of public debt stock to GDP (b) increases by 0.35% because GDP decreases and the tax revenue decreases because of a decrease in labor supply and consumption.

Cabinet Office, Government of Japan, Ministry of Finance, Japan). At fiscal year 2019, GDP and the aggregate consumption are, respectively, 559 trillion JPY and 304 trillion JPY. Assuming $\tau_w = \tau_r$ and considering income tax revenues (including private and corporate tax) 276 trillion JPY=$\tau_w \times 559\ trillion$ JPY and consumption tax revenues of 202 trillion JPY=$\tau_c \times 304\ trillion$ JPY, we obtain $\tau_w = \tau_r = 0.0494$ and $\tau_c = 0.06667$. Because of the share of capital income $\alpha = 0.33$, we set $\tau_w wl/T = 0.576 \times 0.67$ and $\tau_r rk/T = 0.576 \times 0.33$, respectively.

[4] The intertemporal government budget constraint is set as $\hat{B}_t = \tilde{\iota}_{t-1} + (1+i)\hat{B}_{t-1} - T/B\left(\tau_w wl\left(\hat{\tau}_{wt-1} + \hat{w}_{t-1} + \hat{l}_{t-1}\right)/T + \tau_r rK\left(\hat{r}_{t-1} + \hat{K}_{t-2}\right)/T + \tau_c c\hat{c}_{t-1}/T\right)$ to derive the equilibrium of the simulation model.

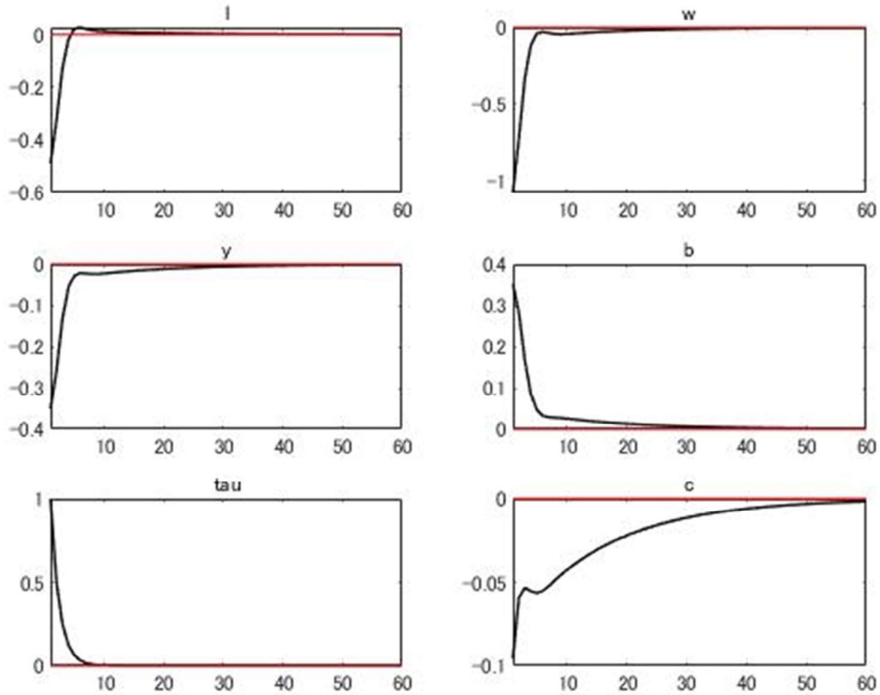

Fig. 3 Increased labor income tax rate. [5]

6.2 Increased Capital Income Tax Rate

The government budget constraint is

$$\hat{B}_{t+1} = \tilde{\iota}_t + (1+i)\hat{B}_t - \frac{T}{B}\left(\frac{\tau_w wl}{T}\left(\hat{w}_t + \hat{l}_t\right) + \frac{\tau_r rK}{T}\left(\hat{\tau}_{rt} + \hat{r}_t + \hat{K}_{t-1}\right) + \frac{\tau_c c}{T}\hat{c}_t\right) \quad (48)$$

where

$$\tilde{\tau}_{rt} = \phi\tilde{\tau}_{rt-1} + f\ . \quad (49)$$

After setting $\phi = 0.5$, we apply a shock as $f = 1$. Figure 4 portrays results of the simulation. An increase in the capital income tax rate reduces the capital stock (k) because of decreased investment. Because of a decrease in aggregate demand including consumption and investment, GDP decreases. As a result, the ratio of public debt stock to GDP rises by 0.2% for an increase in capital income tax rate of 1%.

[5] Our paper presents the program code as an example.

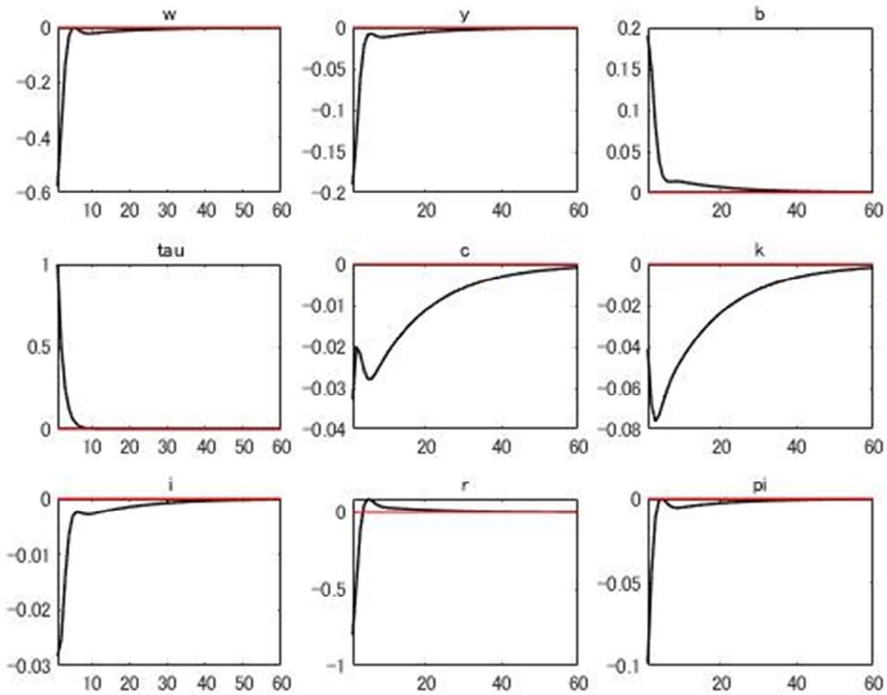

Fig. 4 Increased capital income tax rate.

Nominal interest rate reduces because of a decrease in GDP and inflation rate. This effect raises the consumption at present period. However, a decrease in wage income reduces the consumption over time.

6.3 Increased Consumption Tax Rate

The government budget constraint is presented as

$$\hat{B}_{t+1} = \tilde{\imath}_t + (1 + i\ \ )\hat{B}_t - \frac{T}{B}\left(\frac{\tau_w wl}{T}(\hat{w}_t + \hat{l}_t) + \frac{\tau_r rK}{T}(\hat{r}_t + \hat{K}_{t-1}) + \frac{\tau_c c}{T}(\hat{\tau}_{ct} + \hat{c}_t)\right), \quad (50)$$

where

$$\tilde{\tau}_{ct} = \phi\tilde{\tau}_{ct-1} + f. \quad (51)$$

The Euler equation of consumption (27) and the marginal substitution rate of consumption and labor (29) should be changed as

$$\hat{c}_{t+1} = \hat{c}_t + \frac{1}{\theta}\left(E_t(\tilde{\imath}_{t+1} - \tilde{\pi}_{t+1}) - E_t\frac{\hat{\tau}_{ct+1}}{\frac{1}{\tau_c} + 1} + \frac{\hat{\tau}_{ct}}{\frac{1}{\tau_c} + 1}\right). \quad (52)$$

$$\hat{w}_t - \frac{\hat{\tau}_{ct}}{\frac{1}{\tau_c} + 1} = \kappa \hat{l}_t + \theta \hat{c}_t. \tag{53}$$

Figure 5 shows the case of a 1% increase in the consumption tax rate. Intuitively, an increase in consumption tax reduces consumption at the first stage. However, the consumption in the future period rises because of a decrease in consumption tax. Then, the consumption increases in the future. There is no consumption tax for investment. Then, the demand for investment increases. Thanks to this effect, the aggregate demand increases and GDP increases. Then, the tax revenue increases and then the ratio of public debt stock to GDP decreases.

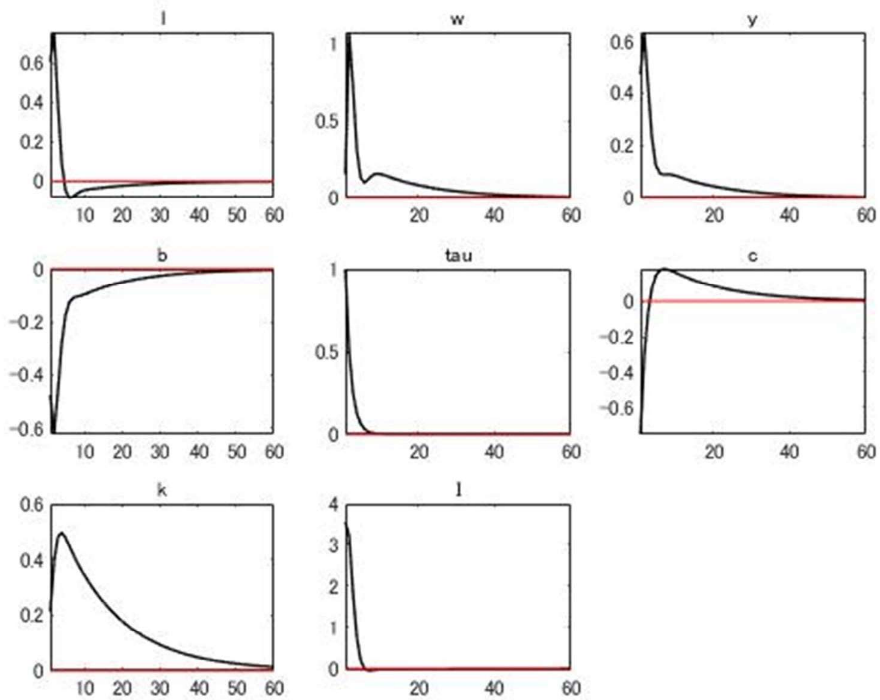

Fig. 5 Increased consumption tax rate.

7. Discussion

Section 6 presents an examination of how an increase in the tax rate affects the ratio of public debt stock to GDP and others. This section presents an examination of two additional cases with simulation. One is a simulation of a decrease in Total Factor Productivity (TFP). The other is that of an increase in government expenditure.

7.1 Decrease in TFP

First, we verify the case of a decrease in TFP. A decrease in TFP shows that A decreases by 1%. As shown by Fig. 6, a 1% decrease in A decreases GDP by 1.5%. During the COVID-19 pandemic, governments of some countries managed lockdown policies that compulsorily stopped economic activity by means such as travel restrictions and prohibitions against public drinking at night. A lockdown policy reduces aggregate consumption. This effect is the same as that with the case of a decrease in A.

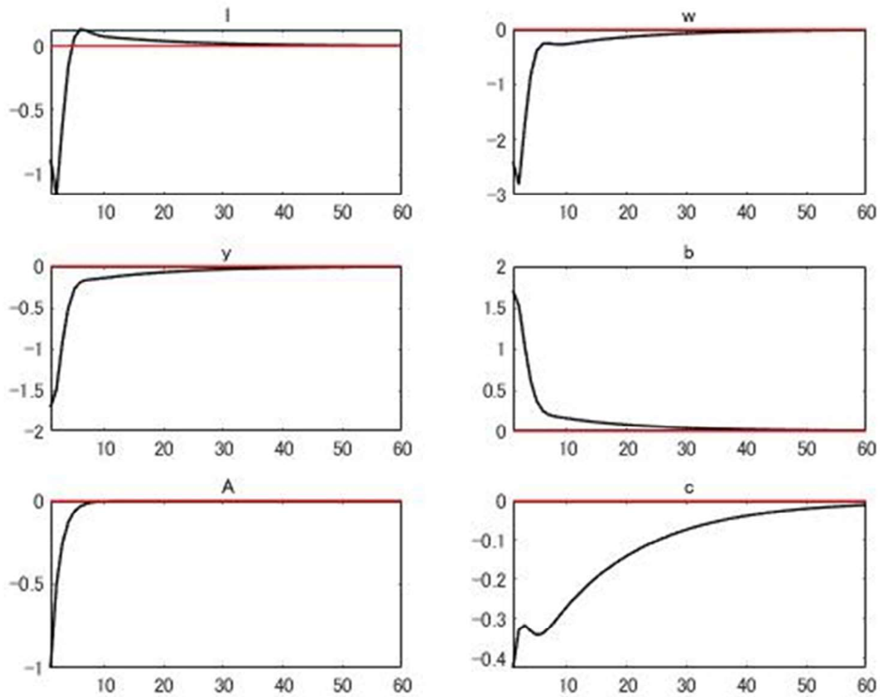

Fig. 6 Decreased TFP.

7.2 Increased Government Expenditure

An increase in government expenditures raises GDP by 0.04% by virtue of an increase in aggregate demand as an instantaneous effect. The ratio of public debt stock to GDP can be decreased by government expenditure. However, considering the effect in the long run, GDP and aggregate demand reduce and then the ratio of public debt to GDP increases.

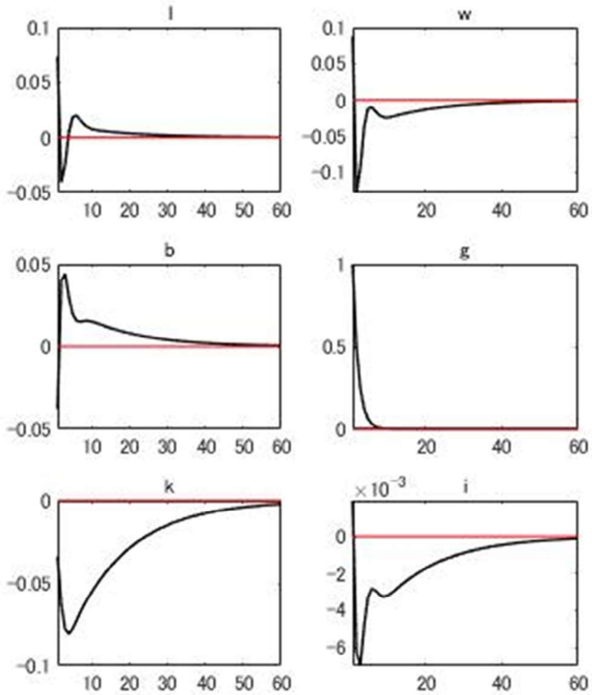

Fig. 7 Increased government expenditure.

8. Conclusions

Our paper sets a DSGE model and examines how tax policy affects the public debt stock and other macroeconomic variables. Fiscal sustainability is considered in OECD countries because these OECD countries continue increasing the public debt stock. Especially, because of COVID-19 pandemic policies, the ratio of public debt stock to GDP has increased sharply. Fiscal sustainability must be considered.

However, as illustrated by results derived from our study, an increase in the income tax rate and capital income tax rate raises the ratio of public debt stock to GDP because an increase in the tax burden reduces GDP. A decrease in GDP decreases tax revenues even if the tax rate is constant over time. An increase in government expenditures as an increase in aggregate demand reduces the public debt stock to GDP because of increased tax revenues brought about by increased GDP. Therefore, as shown by results of our study, policies undertaken to avoid shrinking aggregate demand should be considered to ensure fiscal sustainability and an acceptable ratio of public debt stock to GDP.

References

Arai R. (2011) "Productive Government Expenditure and Fiscal Sustainability," FinanzArchiv: Public Finance Analysis, vol. 67(4), pp. 327-351.

Barro R.J. (1990) "Government Spending in a Simple Model of Endogenous Growth," Journal of Political Economy, vol. 98(5), pp. 103-126.

Calvo G.A. (1983) "Staggered Prices in a Utility-Maximizing Framework," Journal of Monetary Economics, Elsevier, vol. 12(3), pp. 383-398.

Chalk N.A. (2000) "The Sustainability of Bond-Financed Deficits: An Overlapping Generations Approach," Journal of Monetary Economics, vol. 45, pp. 293-328.

Doi T. (2010) "Houjinzeino Kichakunikansuru Dougakutekibunseki-Kansonamoderuniyoru Bunseki-(in Japanese)," RIETI D.P.S. 10-J-034

Eguchi M. (2011) "Dougakuteki Ippan Kinkou Moderu Ni Yoru Zaisei Seisaku No Bunseki (Analysis of Fiscal Policy Through Dynamic General Equilibrium Modeling), (in Japanese)," Mitsubishi Economic Research Institute

Futagami K., Iwaisako T. and Ohdoi R. (2008) "Debt Policy Rule, Productive Government Spending, and Multiple Growth Paths," Macroeconomic Dynamics, vol. 12, pp. 445-462.

Greiner A. (2007) "An Endogenous Growth Model with Public Capital and Sustainable Government Debt," Japanese Economic Review, vol. 58-3, pp. 345-361.

Greiner A. (2008) "Human Capital Formation, Public Debt and Economic Growth," Journal of Macroeconomics, vol. 30, pp. 415-427.

Hayashida M., Yasuoka M., Nanba R. and Ono H. (2019) “DSGE Moderu Ni Yoru Sozeikichaku No Bunseki, (Tax Incidence Analysis in DSGE Model), (in Japanese),” Studies in Applied Economics, vol. 12, pp. 57-78.

Kato R. (2008) "Gendai Makurokeizaigaku Kougi (Lecture on Modern Macroeconomics), (in Japanese)," Toyo Keizai Inc.

Moraga J.F.H. and Vidal J.-P. (2010) "Fiscal Sustainability and Public Debt in an Endogenous Growth Model," Journal of Pension Economics and Finance, vol. 9-2, pp. 277-302.

Oguro K. and Sato M. (2014) "Public Debt Accumulation and Fiscal Consolidation," Applied Economics, vol. 46-7, pp. 663-673.

Ono T. (2003) "Social Security Policy with Public Debt in an Aging Economy," Journal of Population Economics, vol. 16, pp. 363-387.

Teles V.K. and Mussolini C.C. (2014) "Public Debt and the Limits of Fiscal Policy to Increase Economic Growth," European Economic Review, vol. 66, pp. 1-15.

Watanabe M., Miyake Y. and Yasuoka M. (2015) "Public Investment Financed by Consumption Tax In An Aging Society," Singapore Economic Review, vol. 60(05), pp. 1-17.

Yakita A. (2008) "Sustainability of Public Debt, Public Capital Formation, and Endogenous Growth in an Overlapping Generations Setting," Journal of Public Economics, vol. 92, pp. 897-914.

Appendix

Our paper presents derivation of the parameters with calibration. Because of the prior distribution, we set the parameters as shown by the table.

Parameter	Distribution	Average	Standard Error
χ	beta_pdf	0.6	0.1
φ_1	normal_pdf	0.2	0.01
φ_2	normal_pdf	0.2	0.01
ρ_a	beta_pdf	0.85	0.1
ρ_g	beta_pdf	0.85	0.1
e_i	inv_gamma_pdf	0.1	Inf
e_a	inv_gamma_pdf	0.1	Inf
e_g	inv_gamma_pdf	0.1	Inf
u_y	inv_gamma_pdf	0.1	Inf
u_c	inv_gamma_pdf	0.1	Inf
u_I	inv_gamma_pdf	0.1	Inf
u_g	inv_gamma_pdf	0.1	Inf
u_l	inv_gamma_pdf	0.1	Inf
u_i	inv_gamma_pdf	0.1	Inf
u_π	inv_gamma_pdf	0.1	Inf

Table A.1: Prior Distribution

We set the following equations to consider the calibration.

$$A_t = \rho_a A_{t-1} + e_a \quad \text{(A.1)}$$
$$G_t = \rho_g G_{t-1} + e_g \quad \text{(A.2)}$$
$$Y_{obs} = Y + u_y \quad \text{(A.3)}$$
$$c_{obs} = c + u_c \quad \text{(A.4)}$$
$$I_{obs} = I + u_I \quad \text{(A.5)}$$
$$G_{obs} = G + u_g \quad \text{(A.6)}$$
$$l_{obs} = l + ul \quad \text{(A.7)}$$
$$i_{obs} = i + u_i \quad \text{(A.8)}$$
$$\pi_{obs} = \pi + u_\pi \quad \text{(A.9)}$$

obs shows the data values.

Using Bayesian Estimation, one can obtain the following posterior distribution of parameters.

Parameter	Average	Confidence Interval (90%)	
χ	0.2826	0.1788	0.3782
φ_1	0.1885	0.1721	0.2047
φ_2	0.1628	0.1485	0.1767
ρ_a	0.5852	0.4148	0.7677
ρ_g	0.8825	0.7681	0.9964
e_i	0.0162	0.0125	0.0194
e_a	0.0210	0.0156	0.0261
e_g	0.0282	0.0195	0.0369
u_y	0.0210	0.0157	0.0261
u_c	0.0196	0.0150	0.0239
u_I	0.0333	0.0211	0.0449
u_g	0.0289	0.0218	0.0360
u_l	0.0224	0.0164	0.0282
u_i	0.0168	0.0129	0.0204
u_π	0.0178	0.0137	0.0218

Table A.1: Posterior Distribution

Program Code

Calibration

```
var pi A k l y c I g q i r w B y_obs c_obs I_obs g_obs l_obs i_obs pi_obs;

varexo ea eg uy uc uI ug ul ui upi ei;

parameters rho alpha delta theta kappa kai psi_1 psi_2 S_1 rhoa rhog;

parameters cy Iy;

rho = 0.25; %parameters
alpha =0.33;
delta =0.06;
theta =1.5;
kappa =2;
S_1=1/7;
cy =0.55;
Iy=0.25;

model(linear);
pi=pi(+1)+rho^2/(1-rho)*(w-A-alpha*k+alpha*l);
y = A + alpha * k + (1 - alpha) * l;
y=cy*c+Iy*I+g;
q=pi(+1)-i+(0.064*r(+1)+(1-0.06)*q(+1))/((1-0.0494)*0.064+(1-0.06));
r = (w-A-alpha*k+alpha*l) +A+ (alpha - 1) * k+ (1-alpha) * l ;
l=(-theta*c+w)/kappa;
c(+1)=c+1/theta*i(+1)-1/theta*pi(+1);
k = (1 - delta) * k(-1)+ delta * I;
i=kai*i(-1)+(1-kai)*(psi_1*pi(+1)+psi_2*y)+ei;
I=1.004/2.004*I(-1)+1/1.004*I(+1)+1.004/(2.004*S_1)*q;
B=i(-1)+1.004*B(-1)-0.1446*(0.574*(0.67*(w(-1)+l(-1))+0.33*(r(-1)+k(-2)))+0.426*(c(-1)));
A=rhoa*A(-1)+ea;
g=rhog*g*(-1)+eg;

y_obs=y+uy;
c_obs=c+uc;
```

```
I_obs=I+uI;
g_obs=g+ug;
l_obs=l+ul;
i_obs=i+ui;
pi_obs=pi+upi;
end;

estimated_params;
kai, beta_pdf, 0.6, 0.1;
psi_1, normal_pdf, 0.2, 0.01;
psi_2, normal_pdf, 0.2, 0.01;
rhoa, beta_pdf, 0.85, 0.1;
rhog, beta_pdf, 0.85, 0.1;
stderr ei, inv_gamma_pdf, 0.1, inf;
stderr ea, inv_gamma_pdf, 0.1, inf;
stderr eg, inv_gamma_pdf, 0.1, inf;
stderr uy, inv_gamma_pdf, 0.1, inf;
stderr uc, inv_gamma_pdf, 0.1, inf;
stderr uI, inv_gamma_pdf, 0.1, inf;
stderr ug, inv_gamma_pdf, 0.1, inf;
stderr ul, inv_gamma_pdf, 0.1, inf;
stderr ui, inv_gamma_pdf, 0.1, inf;
stderr upi, inv_gamma_pdf, 0.1, inf;
end;

varobs y_obs c_obs I_obs g_obs l_obs i_obs pi_obs;

estimation(datafile = jpdat, mode_check, mh_replic =500000, mh_nblocks =2, mh_drop =0.5,
mh_jscale =0.5, bayesian_irf);
```

Program Code

Income taxation

```
//1.variables
var pi w k l y c I i r q B b tau;
varexo f;

//2.1.parameters
parameters delta rho psi_1 psi_2 theta kai alpha kappa S_1 phi A;
theta = 1.5;
delta = 0.06;
rho = 0.25;
psi_1 =0.1885;
psi_2 =0.1628;
kai =0.2826;
alpha =0.33;
kappa =2;
S_1=1/7;
phi =0.5;
A=0;

//3.model
model(linear);
pi=pi(+1)+rho^2/(1-rho)*(w-A-alpha*k+alpha*l);
y = A + alpha * k + (1 - alpha) * l;
y=0.55*c+0.25*I;
q=pi(+1)-i+(0.064*r(+1)+(1-0.06)*q(+1))/((1-0.0494)*0.064+(1-0.06));
r = (w-A-alpha*k+alpha*l) +A+ (alpha - 1) * k+ (1-alpha) * l ;
l=(-theta*c+w-tau/(1/0.0494+1))/kappa;
c(+1)=c+1/theta*i(+1)-1/theta*pi(+1);
k = (1 - delta) * k(-1)+ delta * I;
i=kai*i(-1)+(1-kai)*(psi_1*pi(+1)+psi_2*y);
I=1.004/2.004*I(-1)+1/1.004*I(+1)+1.004/(2.004*S_1)*q;
B=i(-1)+1.004*B(-1)-0.1446*(0.574*(0.67*(w(-1)+l(-1)+tau(-1))+0.33*(r(-1)+k(-
2)))+0.426*c(-1));
b=B-y;
tau=phi*tau(-1)+f;
```

```
end;

//steady state check
steady;
check;

//5. simulation
shocks;
var f=1;
end;

//results
stoch_simul(irf=60) l w y b tau c
```