DISCUSSION PAPER SERIES

Discussion paper No. 230

How Do the Relative Superiority of a High-quality Good and Cost Inefficiency between Firms Affect Product Lines in Multiproduct Firms?

Tetsuya Shinkai
School of Economics, Kwansei Gakuin University

Ryoma Kitamura
Faculty of Economics, Otemon Gakuin University

October 14,2021

SCHOOL OF ECONOMICS

KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho
Nishinomiya 662-8501, Japan

How Do the Relative Superiority of a High-quality Good and Cost Inefficiency between Firms Affect Product Lines in Multiproduct Firms?*

Tetsuya Shinkai†

School of Economics, Kwansei Gakuin University

1-155, Uegahara Ichiban-cho, Nishinomiya, Hyogo 662-8501, Japan

Ryoma Kitamura‡

Faculty of Economics, Otemon Gakuin University

2-1-15, Nishiai, Ibaraki, Osaka 567-8502, Japan

Abstract

We consider the product line strategies of duopolistic firms, each of which can supply two vertically differentiated products under nonnegative output constraints and expectations of their rival's product line reaction. Considering a game of firms with heterogeneous (homogeneous) unit costs for high- (low-) quality products, we derive the equilibria of the game and explore the effects of the relative superiority of the high-quality product and relative cost efficiency on the equilibrium outcomes and illustrate the result using the production substitution of differentiated goods within a firm and the high-quality good between firms.

Keywords: Multiproduct firm; Product line; Vertical product differentiation;
JEL Classification Codes: D21, D43, L13, L15

---

*The authors are grateful to Tommaso Valletti, Federico Etro, Noriaki Matsushima, Toshihiro Matsumura, Naoshi Doi, Dan Sasaki, participants in the Industrial Organization Seminar at the University of Tokyo, and especially to Professor John Sutton for their useful comments on an earlier version of this manuscript. The first author was supported by Grants-in-Aid for Scientific Research (Nos. 23330099 and 24530255) MEXT, and the Special Research Fund 2017, Kwansei-Gakuin University.

†Corresponding author. School of Economics, Kwansei Gakuin University, 1-155, Uegahara Ichiban-cho, Nishinomiya, Hyogo 662-8501, Japan. E-mail: shinkai@kwansei.ac.jp. Phone: +81-798-54-6967. Fax: +81-798-51-0944.

‡E-mail: r-kitamura@otemon.ac.jp, Phone +81-72-641-9608, Fax +81-72-643-9414.

1 Introduction

Real-world economies often include oligopolistic competition in the same market segment, in which firms supply multiple vertically differentiated products. In the mobile phone market, for example, Apple supplies the iPhone 12 to the first line segment, and Samsung competes with Apple by supplying the Galaxy S21 to businesses. In the second line segment, Apple sends the iPhone 10, and Samsung responds by forwarding the Galaxy S10.

In the existing literature on vertical product differentiation, the quality of the goods that the firms produce is treated as an endogenous variable. For example, in Bonanno (1986) and Motta (1993), firms initially choose the quality level and then compete in a Cournot or Bertrand fashion in an oligopolistic market. Shaked and Sutton (1987) consider a two-stage game model in which each of horizontally and vertically differentiated multiproduct firms pays a fixed sunk cost for R&D or advertising expenditure to improve the (perceived) quality of its products in the first stage and chooses its respective prices in the second stage. Mussa and Rosen (1978) analyze monopoly price discrimination in a monopoly model in which the quality level and the quantity consumed by individuals are distorted downward from the socially efficient level. Gal-Or (1983) extends their monopoly model to a symmetric Cournot oligopoly model and explores the effect of quality on the market equilibrium and the impact of increased competition on the quality levels, price and welfare.

In a horizontally differentiated multiproduct model, Bental and Spiegel (1984) consider an optimal set of product varieties in a monopoly and analyze the relationship between the degree of differentiation between any two varieties and the variety price, or the cost of installing an additional variety. Shaked and Sutton (1990) consider a two-stage price game model in which each of horizontally differentiated multiproduct firm (potential entrant) chooses in the first stage which product(s) it will produce and incurs a "sunk cost" per product entered and chooses its respective prices in the second stage. Then they *graphically* characterize the market structure in equilibrium using two parameters that measure expansion and competition effects. Our study's results are also related to those of marketing studies on product segmentation and product distribution strategies. Calzada and Valletti (2012) study a model of film distribution and consumption. They consider a film studio that can release two versions of one film—one for theaters and one for video—although they do not consider oligopolistic competition between film studios. They show that the optimal strategy for the studio is to introduce versioning (the simultaneous release of the film with one version for theaters and another version for video) if its goods are not close substitutes for one another. We established a result that indirectly supports the result of Calzada and Valletti (2012). In their model, "versioning" and "sequencing" correspond to the simultaneous supply and sequential supply, respectively, of high- and low-quality goods, as in our model. In the case of sequential supply, the film studio supplies a high-quality film version to theaters and then launches a low-quality DVD version in the same market.

By taking an "upgrade approach," in which a monopolist chooses not the output quantity of actual products but instead upgrades one to be equivalent to another, Johnson and Myatt (2003) consider monopoly and duopoly models in which a firm (or firms) sells (sell) multiple quality-differentiated products and frequently changes is product lines when a competitor enters the market in the duopoly model. They provided an explanation for the common strategies of using "fighting brands" and "pruning" product lines. In particular, they endogenized not only the quality level of each good but also the number of goods that firm supplied to the market.

In markets where firms supply multiple vertically differentiated products, they sometimes compete with rivals that supply one or more vertically differentiated products (i.e., the rival chooses a single product line) to the same market segment. The seminal works in the literature on the product lines of multiple vertically differentiated products in an oligopoly setting are Johnson and Myatt (2006) and (2018). Using the same "upgrade approach," in Johnson and Myatt (2006), they show that the results for the single-product Cournot equilibrium supply carry over to the supply of updates but not necessary to the full set of complete products (that is, all vertically differentiated products are supplied by all firms in equilibrium). In their most recent work (2018), they extend their analysis to allow for cost asymmetries among firms and differences in product lines configurations among rivals and derive equilibrium product lines and explore their determinants. The results derived in Johnson and Myatt (2018) are general, but some results, propositions 3 and 4 and a corollary in section 5 are closely related to the results in our paper. In section 2, we explore how our results correspond to those of propositions 3 and 4 and the corollary in Johnson and Myatt (2018). We confirm that in equilibrium E that we derive in our model, the inequality conditions in proposition 3 in their paper are strictly satisfied, so there exists an equilibrium (equilibrium E) in which all firms offer complete product lines (they offer both high- and low-quality goods).

The first contribution of this paper is to identify the existence of equilibria that correspond to those in proposition 4 in Johnson and Myatt (2018). However, in our model, one of the conditions whereby the cost advantage of firm 1, the sum of output with a higher quality level than V_L *is strictly increasing in* quality in proposition 4 in their paper (2018) is not satisfied. Our second contribution is that we find an equilibrium (equilibrium C in proposition 1) in which the high-cost firm 2 supplies only low-quality good, but the low-cost firm 1 supplies both the high- and low-quality goods in the market. Thus, there is a gap between the firms' two product lines, although the quality increment from the low- to the high-quality good is sufficiently large (in proposition 4 in Johnson and Myatt (2018), the condition for the quality increment from the low- to the high-quality good is sufficiently small).

In Kitamura and Shinkai (2015a), we considered a game that includes heterogeneous unit production costs between firms for high-quality goods but homogeneous costs for low-quality products. We described the firms' product line strategies based on the relative quality of the products and on the cost-efficiency ratios of the firms producing the high-quality good. Unlike most previous studies, in our model (in Kitamura and Shinkai(2015a) and in this study), both the quality level and the number of differentiated goods that each firm supplies are exogenously given, and we also do not explicitly consider the stage of product line choice with a fixed "sunk cost" as Shaked and Sutton (1987, 1990) did. We first derived equilibria by assuming that, in any equilibrium, each rival firm chooses positive outputs for both the high- and the low-quality good. Consequently, these equilibria included cases in which a firm chooses negative output for one of the goods for some parameter ranges (firms' relative quality ratio or cost-inefficiency ratio for the high-quality good). We then retroactively excluded the ranges of parameters in equilibrium that result in any negative outputs, and we graphically described the firms' product line strategies based on the relative quality of the products and on the cost-efficiency ratios between the firms in the case of high-quality goods.

Although Kitamura and Shinkai (2015a) assumed that each rival firm chooses positive outputs for both goods in duopolistic competition, it is crucial that each firm considers its rivals' product line strategies when choosing its own strategy. In these cases, it is important that each firm chooses its own product line strategies for multiple products, given their expectations of their rivals' product line reactions. Therefore, in this study, we consider the product line strategies of duopolistic firms that each supply two vertically differentiated products under nonnegative output constraints and an expectation regarding their rivals' product line reactions. This study differs from our earlier study, Kitamura and Shinkai (2015a) in the following respects.

First, in this study, we *explicitly* examine the product line strategies of duopolistic firms that supply two vertically differentiated products under a nonnegative output constraint and an expectation with respect to rivals' product line reactions. We show that there are five nontrivial equilibria with positive outputs for one or both products and that both firms have positive profits in each equilibrium. In these equilibria, the ranges of the two ratio parameters for which positive equilibrium outputs exist for the two firms differ. We graphically describe the firms' product line strategies in equilibrium, based on the relative quality of the products and on the firms' relative cost efficiency for the high-quality good (Figure 1).

Second, in each of the nontrivial equilibria, we illustrate how the changes in the relative superiority of high-quality good, μ, and relative cost inefficiency of the high-quality good of firm 2, c_{2H}, affect the product lines of firms in each equilibrium through production substitution *between* goods *within* a firm and that of goods *between* firms[1].

The remainder of this paper is organized as follows. In Section 2, we present our model and derive the duopoly equilibrium product lines with two vertically differentiated products in the same market under a nonnegative output constraint and an expectation

[1]Professor John Sutton suggested this analysis to us in his comment on our presentation of an earlier version of this study, Shinkai and Kitamura (2015b), at EARIE 2015, the Annual Conference of the European Association for Research Industrial Economics, in Munich, Germany. His comment and suggestion have much improved our study.

with respect to rivals' product line reactions. Furthermore, we graphically describe the firms' product line strategies in equilibrium, based on the relative quality of the differentiated products and on the firms' relative cost efficiency for the high-quality good and explore how the changes in the relative superiority of high-quality good, μ, and relative cost inefficiency for the high-quality good of the ineffective firm (which has a higher unit cost than its rival) affect product lines of firms in each equilibrium (Figure 1). In addition, we describes the relationship of the results of this proposition with those of propositions 3 and 4 and the corollary in Johnson and Myatt (2018) and present the equilibrium price and profits of the firms for each of five equilibria. In Section 3, we illustrate how the changes in the relative superiority of high-quality good, μ, and the relative cost inefficiency for the high-quality good of firm 2, c_{2H}, affect the product lines and profits of firms in each equilibrium through product substitution *between* goods *within* a firm and that of goods *between* firms. Finally, Section 4 concludes this paper.

2 The Model and the Equilibria of the Game

Suppose that there are two firms ($i = 1, 2$) in a duopoly, each of which produces two goods (H and L), which differ in terms of quality. We assume a continuum of consumers, represented by a taste parameter, θ, which is uniformly distributed between 0 and r (> 0) with density 1. We further assume that a consumer is of type $\theta \in [0, r]$, for $r > 0$. The consumers' preferences are the standard Mussa and Rosen preferences. Thus, the utility (net benefit) of consumer θ who buys good α ($= H, L$) from firm i ($= 1, 2$) is given by

$$U_{i\alpha}(\theta) = V_\alpha \theta - p_{i\alpha} \qquad i =, 1, 2 \quad \alpha = H, L. \tag{1}$$

To maximize his/her surplus, each consumer decides whether to buy nothing or one unit of good α from firm i.

Let V_H and V_L denote the quality of the high-quality and the low-quality good, respectively. Then, the maximum amount that consumers are willing to pay for each good is assumed to be $V_H = \mu V_L = \mu > V_L = 1$. Thus, for simplicity, we normalize the quality of the low-quality good by setting $V_L = 1$ and assume that the quality of the high-quality good is μ times that of the low-quality good.

Note that the consumers' preferences and the utility of each consumer *never change* when *the quality of both products changes exogenously.*

Good α ($= H, L$) is assumed to be homogeneous for all consumers. Suppose that there always exists a consumer $\underline{\theta}_{iL}, i = 1, 2$ who is indifferent between purchasing good L and purchasing nothing in a monopoly or a duopoly. For this consumer, $\underline{\theta}_{iL}$ satisfies

$$\begin{aligned} & U_{iL}(\underline{\theta}_L) = 0 \\ \Leftrightarrow \ & \underline{\theta}_{iL} = \frac{p_{iL}}{V_L} = p_{iL}, i = 1, 2. \end{aligned} \tag{2}$$

We can derive the demand for good H as $Q_H = r - \widehat{\theta}$ and that for good L as $Q_L = \widehat{\theta} - \underline{\theta}_{iL}$, as shown in Figure 1, where $Q_\alpha = q_{i\alpha} + q_{j\alpha}$ for $\alpha = H, L$ and $j = 1, 2$. Without loss of generality, we set $r = 1$. Here, $\widehat{\theta}$, the threshold between the demand for H and that for L, is given by

$$\widehat{\theta} = (p_H - p_L)/(\mu - 1). \tag{3}$$

Then, as in Kitamura and Shinkai (2015a), we derive the following inverse demand functions:

$$\begin{cases} p_H = V_H(1 - Q_H) - Q_L = \mu(1 - Q_H) - Q_L \\ p_L = V_L - Q_H - Q_L = 1 - Q_H - Q_L, \end{cases} \tag{4}$$

where $Q_\alpha = q_{i\alpha} + q_{j\alpha}$ and p_α and $q_{i\alpha}$ denote the price of good α and firm i's output of good α, respectively, for $\alpha = H, L$ and $i, j = 1, 2$.

Moreover, suppose that each firm has constant returns to scale and that $c_{iH} > c_{iL} = c_{jL} = c_L = 0$, where $c_{i\alpha}$ is firm i's marginal and average cost of good α. This implies that a high-quality good incurs a higher cost of production than a low-quality good. Here, without loss of generality, we assume that $c_{2H} > c_{1H} = 1 > c_{iL} = 0$, which means that firm 1 is more efficient than firm 2 at producing the high-quality good, but as for low-quality good, there is fierce cost competition between the two firms. Under these assumptions, each firm's profit is defined in the following manner:

$$\pi_i = (p_H - c_{iH})q_{iH} + p_L q_{iL} \qquad i = 1, 2. \tag{5}$$

Firm $i(= 1, 2)$ chooses the outputs for H and L to maximize its profit function in Cournot fashion under nonnegative output constraints, provided that firm $j(\neq i)$ chooses *any given* product line strategy $\mathbf{s}_j \in \mathbf{S}_j \equiv \{(0, 0), (+, 0), (0, +), (+, +)\}$, where $(0, 0)$ implies $(q_{jH} = 0, q_{jL} = 0)$, $(+, 0)$ implies $(q_{jH} > 0, q_{jL} = 0)$, and so forth. Thus, for any given $\mathbf{s}_j \in \mathbf{S}_j$

$$\max_{q_{iH},q_{iL}} \ \pi_i = \{\mu(1 - q_{iH} - q_{jH}) - q_{iL} - q_{jL} - c_{iH})\}q_{iH} + (1 - q_{iH} - q_{jH} - q_{iL} - q_{jL})q_{iL} \quad (6)$$

$$s.t. \ \ q_{iH} \geq 0, q_{iL} \geq 0, \ i \neq j, \ i, j = 1, 2.$$

The necessary and complementary conditions for this maximization problem are

$$\frac{\partial \pi_i}{\partial q_{iH}} \leq 0, \ \frac{\partial \pi_i}{\partial q_{iL}} \leq 0, \tag{7}$$

$$q_{iH} \cdot \frac{\partial \pi_i}{\partial q_{iH}} = q_{iL} \cdot \frac{\partial \pi_i}{\partial q_{iL}} = 0, \tag{8}$$

$$q_{iH} \geq 0, \ q_{iL} \geq 0, \ \ i = 1, 2. \tag{9}$$

Each firm chooses its product line strategy for the two vertically differentiated products, that is, whether to produce positive (zero) quantities of products H and L given the rival firm's product line strategy.

Note that each inequality $\partial \pi_i / \partial q_{i\alpha} \leq 0$ in (7) and the corresponding complementary slackness condition $q_{i\alpha} \cdot \partial \pi_i / \partial q_{i\alpha} = 0$ in (8) imply that if the marginal revenue of firm i for product $\alpha (= H, L)$ is below (the same as) its marginal cost, then firm i does not produce (does produce) a positive quantity of the product.

In the following, we present the equilibria of a Cournot duopoly game in which each firm can choose its product line and outputs for the two vertically differentiated goods. The firms operate under a nonnegative output constraint. After presenting the equilibrium, we describe the firms' product line strategies based on the products' relative quality and on the firms' relative cost efficiency with respect to the high-quality good in equilibrium.

There are 15 cases to be solved, based on each firm's product line strategies, given the firm's expectation of its rival's product line strategies, which excludes the trivial case in which neither firm produces H or L.

- **Case A**: $\mathbf{s}_1 = (0, +),\ \mathbf{s}_2 = (0, +)$

  In this case, a duopoly market for the low-quality good is realized in equilibrium.

- **Case B**: $\mathbf{s}_1 = (+, 0),\ \mathbf{s}_2 = (0, +)$

  In this case, each firm specializes in the product that is more cost efficient for it.

- **Case C**: $\mathbf{s}_1 = (+, 0),\ \mathbf{s}_2 = (+, +)$

  In case C, firm 2 (which has a higher unit cost for the high-quality product H) produces both products, but firm 1, which is efficient in the production of product H, specializes in product H.

- **Case D**: $\mathbf{s}_1 = (+, +),\ \mathbf{s}_2 = (0, +)$

  In this case, in contrast to case C, firm 1 is efficient in producing product H and supplies both products. However, the inefficient firm 2 specializes in product L.

- **Case E**: $\mathbf{s}_1 = (+, +),\ \mathbf{s}_2 = (+, +)$

  In case E, both firms produce both products.

Now, we present the following proposition on the equilibria of our game. For the sketch of the derivation and proofs, see the appendix.

Proposition 1 *In the duopoly equilibrium of the game, given the rival's expectation of a nonnegative quantity, there exist five types of nontrivial Nash equilibria in the following:*

$$(q_{1H}^{*A}, q_{1L}^{*A}, q_{2H}^{*A}, q_{2L}^{*A}) = \left(0,\ \frac{1}{3},\ 0,\ \frac{1}{3}\right),\ \text{where } 1 < \mu < 2, \tag{10}$$

$$(q_{1H}^{*B}, q_{1L}^{*B}, q_{2H}^{*B}, q_{2L}^{*B}) = \left(\frac{2\mu - 3}{4\mu - 1}, 0,\ 0,\ \frac{\mu + 1}{4\mu - 1}\right), \tag{11}$$

$$\text{where} \qquad 4 \leq \mu \leq \frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}), \tag{12}$$

$$(q_{1H}^{*C}, q_{1L}^{*C}, q_{2H}^{*C}, q_{2L}^{*C}) = \left(\frac{\mu + c_{2H} - 2}{3\mu},\ 0,\ \frac{2\mu^2 - 4c_{2H}\mu + c_{2H} - 2}{6\mu(\mu - 1)},\ \frac{c_{2H}}{2(\mu - 1)}\right), \tag{13}$$

$$\textit{where, } c_{2H} \geq 2 \textit{ and } 4 \leq \frac{1}{2}(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}) < \mu \textit{ for } q_{2H}^{*C} > 0, \tag{14}$$

$$(q_{1H}^{*D}, q_{1L}^{*D}, q_{2H}^{*D}, q_{2L}^{*D}) = \left(\frac{\mu - 2}{2(\mu - 1)},\ \frac{4 - \mu}{6(\mu - 1)},\ 0,\ \frac{1}{3}\right),\ \textit{where}\ \ 2 < \mu < 4 \textit{ and } \mu \leq 2c_{2H}, \tag{15}$$

and

$$(q_{1H}^{*E}, q_{1L}^{*E}, q_{2H}^{*E}, q_{2L}^{*E}) = \left(\frac{\mu + c_{2H} - 3}{3(\mu - 1)},\ \frac{2 - c_{2H}}{3(\mu - 1)},\ \frac{\mu - 2c_{2H}}{3(\mu - 1)},\ \frac{2c_{2H} - 1}{3(\mu - 1)}\right)$$

$$\textit{where } 1 < c_{2H} < 2 \text{ and } \mu \geq 2c_{2H}. \tag{16}$$

The last inequalities followed since these equilibrium outputs must hold due to both the *positive output condition and the necessary condition. A, B, C, D, and E indicate the area in the c_{2H}-μ plane in Figure 1. As the parameters μ, c_{2H} change, except for case A, the equilibrium product lines of firms change through their substitution of the production of goods as illustrated by the directions of the solid and broken arrows in Figure 1.*

[Insert Figure 1 here]

Note that each equilibrium output presented in proposition 1 is that of a duopoly game, given the firms' expectations about their rival's nonnegative output(s). Note that we assume that $c_{2H} > c_{1H} = 1$ and $V_H = \mu V_L = \mu > V_L = 1$. The horizontal and vertical axes in Figure 1 show the relative cost ratio c_{2H} and the quality ratio μ, respectively. Here, we present the equilibrium prices and equilibrium profits of both firms in 5 nontrivial equilibria presented in proposition 1.

From (4), (5), and (10), the equilibrium price and each firm's profit are

$$(p_H^{*A}, p_L^{*A}) = \left(\frac{3\mu - 2}{3}, \frac{1}{3}\right) \tag{17}$$

$$(\pi_1^{*A}, \pi_2^{*A}) = \left(\frac{1}{9}, \frac{1}{9}\right). \tag{18}$$

In Figure 1, area B corresponds to this case. In area B, the relative cost inefficiency of the high-quality good for firm 2, c_{2H}, is relatively strong compared to μ, the relative quality superiority of the high-quality product H. From (4), (5), and (11), we obtain the corresponding equilibrium prices and the profit of each firm:

$$(p_H^{*B}, p_L^{*B}) = \left(\frac{(\mu+1)(2\mu-1)}{4\mu-1}, \frac{\mu+1}{4\mu-1} \right) \tag{19}$$

$$(\pi_1^{*B}, \pi_2^{*B}) = \left(\frac{\mu(2\mu-3)^2}{(4\mu-1)^2}, \frac{(\mu+1)^2}{(4\mu-1)^2} \right). \tag{20}$$

For $q_{1H}^{*C} > 0$, the inequality $\mu > 2 - c_{2H}$ holds because $c_{2H} \geq 2$. In Figure 1, areas C.1 and C.2 correspond to this case. From (4), (5), and (13), the corresponding equilibrium prices and profit for each firm are

$$(p_H^{*C}, p_L^{*C}) = \left(\frac{\mu + c_{2H} + 1}{3}, \frac{2\mu - c_{2H} + 2}{6\mu} \right), \tag{21}$$

$$\begin{aligned} \pi_1^{*C} &= \frac{(\mu + c_{2H} - 2)^2}{9\mu}, \\ \pi_2^{*C} &= \frac{4\mu^3 - 4(4c_{2H} - 1)\mu^2 + 4(2c_{2H} - 1)(2c_{2H} + 1)\mu - (7c_{2H} - 2)(c_{2H} - 2)}{36\mu(\mu - 1)}. \end{aligned} \tag{22}$$

The relative superiority, μ, of the high-quality good is small compared to the relative cost inefficiency, c_{2H}, of the high-quality good for firm 2 in case D.

From (4), (5), and (15), the corresponding equilibrium prices and profit for each firm are

$$(p_H^{*D}, p_L^{*D}) = \left(\frac{3\mu + 2}{6}, \frac{1}{3} \right) \tag{23}$$

$$(\pi_1^{*D}, \pi_2^{*D}) = \left(\frac{9\mu^2 - 32\mu + 32}{36(\mu - 1)}, \frac{1}{9} \right). \tag{24}$$

We obtain the corresponding equilibrium price and profit of each firm from (4), (5), and (16):

$$(p_H^{*E}, p_L^{*E}) = \left(\frac{\mu + c_{2H} + 1}{3}, \ \frac{1}{3} \right) \tag{25}$$

$$\begin{aligned}
\pi_1^{*E} &= \frac{1}{9(\mu-1)} \left(\mu^2 + (2c_{2H} - 5)\,\mu + (c_{2H} - 2)(c_{2H} - 4) \right), \\
\pi_2^{*E} &= \frac{1}{9(\mu-1)} \left(\mu^2 - (4c_{2H} - 1)\,\mu + 4c_{2H}^2 - 1 \right).
\end{aligned} \tag{26}$$

Regarding the equilibrium profits of the firms for these five nontrivial equilibria, we can easily see that $\pi_1^{*A} = \pi_2^{*A}$, $\pi_1^{*k} > \pi_2^{*k}$, $k = B, C, D$ and E.

In words, the firm that is cost efficient in producing the high-quality product earns more than the inefficient firm does in all equilibria except for equilibrium A.

2.1 Difference of the Results of Proposition 1 from those in Johnson and Myatt (2018)

In this section, we describe the relationship of the results of this proposition with those of propositions 3 and 4 and the corollary in Johnson and Myatt (2018).

In the following, we explore how our results correspond to those of propositions 3 and 4 and the corollary in Johnson and Myatt (2018).

In subsection 'Linear Demand Specification 2', in section 5 of their paper, the following notations are used: $c_r(q) = \gamma_r c(q), c(q) = \frac{1}{R}\sum_{r=1}^{R} c_r(q), c'(q) > 0, c''(q) \geq 0, c_r(q)$:

production cost of firm r for the good with the i'th upgrade quality $q_i(q_1 < q_2 < \cdots < q_I)$, $r = 1, ..., R$. $c(q_i)$: the industrial average production cost of the good with the i'th upgrade quality q_i. $\frac{1}{R}\sum_{r=1}^{R}\gamma_r = 1, \Delta c_r(q_i) = c_r(q_i) - c_r(q_{i-1}), \Delta c(q_i) = \frac{1}{R}\sum_{r=1}^{R}\Delta c_r(q_i), \Delta q_i = q_i - q_{i-1}$

Applying the notations in this paper to the above, $R = 2, q_L = 1 < q_H = \mu, c_r(L) = \gamma_r c(L) = 0 = c_{rL} = 0, r = 1, 2. c_1(H) = \gamma_1 c(H) = c_{1H} = 1,$

$c_2(H) = \gamma_2 c(H) = c_{2H} > 1, q_H = \mu, q_L = 1, \Delta q_H = q_H - q_L = \mu - 1$

$c(L) = \frac{1}{2}(c_{1L} + c_{2L}) = \frac{1}{2}(0 + 0) = 0, c(H) = \frac{1}{2}(c_{1H} + c_{2H}) = \frac{1}{2}(1 + c_{2H}),$ $c_2(H) = \gamma_2 c(H) = c_{2H}$ so from $\gamma_1 c(H) = \gamma_1 \cdot \frac{1}{2}(1 + c_{2H}) = c_{1H} = 1$ and $\gamma_2 c(H) = \gamma_2 \cdot \frac{1}{2}(1 + c_{2H}) = c_{2H}$,

we obtain

$$\gamma_1 = \frac{2}{1 + c_{2H}}$$

and

$$\gamma_2 = \frac{2c_{2H}}{1 + c_{2H}}.$$

Thus, we see that $\frac{1}{2}(\gamma_1 + \gamma_2) = 1$ holds. From notations in our model and in Johnson and Myatt (2018), we have $\Delta c_{H1} = c_{1H} - c_{1L} = 1 - 0 = 1, \Delta c_{H2} = c_{2H} - c_{2L} = c_{2H} - 0 = c_{2H}$. In our model, because there exist only two qualities $V_H = \mu(> 1)$,$V_L = 1$, there is thus no variety with quality below $q_L = V_L$, and from our assumption that $c_{1L} = c_{2L} = 0$, denoting a fictional quality level below $q_L = V_L$, by $q_o(\equiv 0)$, without loss of generality, we can assume that $c_1(q_o) = c_2(q_o) \equiv 0$. Hence, we have $\Delta c_1(q_L) = c_1(V_L) - c_1(q_o) = \Delta c_{L1} \equiv c_{1L} - 0 = c_{1L} = 0$ and $\Delta c_2(q_L) = \Delta c_{L2} \equiv c_{2L} - 0 = c_{2L} = 0$.

Thus, we obtain

$$\Delta c_H = \frac{1}{2}(\Delta c_{H1} + \Delta c_{H2}) = \frac{1}{2}(1 + c_{2H})$$

and

$$\Delta c_L = \frac{1}{2}(\Delta c_{L1} + \Delta c_{L2}) = \frac{1}{2}(0 + 0) = 0.$$

Furthermore, from notations in our model and in Johnson and Myatt (2018), we have, $\Delta q_{H1} = q_{H1} - q_{L1} = \mu - 1, \Delta q_{H2} = q_{H2} - q_{L2} = \mu - 1, \Delta q_{L1} \equiv q_{L1} - q_o = V_L - 0 = 1$ and $\Delta q_{L2} \equiv q_{L2} - q_o = V_L - 0 = 1$.

Thus, we obtain

$$\Delta q_H = \frac{1}{2}(\Delta q_{H1} + \Delta q_{H2}) = \mu - 1$$

and

$$\Delta q_L = \frac{1}{2}(\Delta q_{L1} + \Delta q_{L2}) = 1.$$

Applying these to condition (17) in proposition 3 of Johnson and Myatt (2018),

$\frac{2}{2+1} = \frac{2}{3} < \min\{\gamma_1, \gamma_2\} = \min\{\frac{2}{1+c_{2H}}, \frac{2c_{2H}}{1+c_{2H}}\} = \frac{2}{1+c_{2H}}$ and $\max\{\frac{2}{1+c_{2H}}, \frac{2c_{2H}}{1+c_{2H}}\} = \frac{2c_{2H}}{1+c_{2H}} < \frac{2}{3} + \frac{1}{3\Delta c_H/\Delta q_H} = \frac{2}{3} + \frac{2(\mu-1)}{3(1+c_{2H})} = \frac{2(\mu+c_{2H})}{3(1+c_{2H})}$ from our assumption that $c_{2H} > 1$ and $\mu > c_{2H}$ in equilibrium E in this paper. Hence, we can confirm that our model setting strictly satisfies the inequalities in (17), so there exists an equilibrium (equilibrium E) in which all firms offer complete product lines (they offer both positive quantities of goods H and L). Thus, proposition 3 in Johnson and Myatt (2018) holds in our model.

Next, we explore whether proposition 4 in Johnson and Myatt (2018) holds in our model. From equation (14) in Johnson and Myatt (2018), we obtain

$$Z_{L2}^{\dagger} = \frac{1 + [2 - 3\gamma_2(\Delta c_L/\Delta q_L)}{1+2} = \frac{1 + [2 - 3 \cdot \frac{2c_{2H}}{1+c_{2H}}(0/1)]}{3} = 1,$$

$$Z_{H2}^{\dagger} = \frac{1 + [2 - 3\gamma_2(\Delta c_H/\Delta q_H)]}{1+2} = \frac{1 + [2 - 3 \cdot \frac{2c_{2H}}{1+c_{2H}}(\frac{1}{2}(1+c_{2H})/(\mu - 1)]}{3} = \frac{\mu - (1 + c_{2H})}{\mu - 1} < 1 = Z_{L1}^{\dagger}.$$

Therefore, for the disadvantaged firm 2, $\{Z_{i2}^{\dagger}\}$ is strictly decreasing in $i(= L, H\}$. However, we see that

$$Z_{L1}^{\dagger} = \frac{1 + [2 - 3\gamma_1(\Delta c_L/\Delta q_L)}{1+2} = \frac{1 + [2 - 3 \cdot \frac{2}{1+c_{2H}}(0/1)]}{3} = 1,$$

and

$$Z_{H1}^{\dagger} = \frac{1 + [2 - 3\gamma_1(\Delta c_H/\Delta q_H)]}{1+2} = \frac{1 + [2 - 3 \cdot \frac{2}{1+c_{2H}}(\frac{1}{2}(1+c_{2H})/(\mu - 1)]}{3} = \frac{\mu - 2}{\mu - 1} < 1 = Z_{L1}^{\dagger}.$$

For the advantaged firm 1, $\{Z_{i1}^{\dagger}\}$ *is not* strictly increasing but strictly decreasing in $i(= L, H\}$ in our model. Thus, our model setting *does not satisfy one of the conditions* that for the advantaged firm 1, $\{Z_{i1}^{\dagger}\}$ *is strictly increasing in* $i(= L, H\}$ in proposition 4 in Johnson and Myatt (2018). Nevertheless, from the result of proposition 1 in this paper, we find that the low-cost firm 1 sells only (the higher-quality) good H in equilibrium case C, the high-cost firm 2 sells only (the lower-quality) good L in equilibrium case D, and the two firms split the market in equilibrium case B. These results exactly imply statements (a), (b) and (c), respectively, in proposition 4 in Johnson and Myatt (2018).

Furthermore, in proposition 1, we show that in equilibrium case C, the high-cost firm 2 supplies only good L, but the low-cost firm 1 supplies both goods H and L in the market so there is a gap between the firms' two product lines, although the quality increment, $\mu - 1$ between goods H and L is sufficiently large! This result is different from statement (d) in proposition 4 in Johnson and Myatt (2018).

The reason that $\{Z_{i1}^{\dagger}\}$ *is not* strictly increasing but strictly decreasing in $i(= L, H\}$ in our model is our assumption that $c_{iL} = 0, i = 1, 2$, and in our model, there are only two fixed qualities, so there exists only one 'upgrade'! Therefore, we set $\Delta c_{Li} \equiv c_{iL} - 0 = c_{iL} = 0, i = 1, 2$ so that $\Delta c_L = \frac{1}{2}(\Delta c_{L1} + \Delta c_{L2}) = 0$.

3 Effects of Changes in the Relative Superiority of the High-quality Good and its Cost Inefficiency for Firms on Product Lines

In this section, we illustrate how the changes in relative superiority for high-quality good, μ, and relative cost inefficiency for the high-quality good of firm 2, c_{2H}, affect product lines and profits of firms in each equilibrium through production substitution *between* goods *within* a firm and that of goods *between* firms.

In case A, taking into account the results of proposition 1, we find that the relative superiority μ of the high-quality good is too small compared to the unit costs of good H. Therefore, both firms specialize in good L, and the market for good L becomes a Cournot duopoly. Hence, the two firms' equilibrium profits are identical, and any change in μ and c_{2H} in area A has no effect on the outputs and profits of the firms.

At any point (c_{2H}, μ) in equilibrium E in area E in Figure 1, the relative cost ratio c_{2H} is between one and two, so the difference between the unit costs of the two firms is small. In the lower part of area E, the relative superiority of the high-quality good μ is not very large. Thus, both firms are likely to supply high- and low-quality goods. Naturally, the efficient firm 1 produces more of the high-quality good H than of the low-quality good L because its production costs for H are lower than those of the rival firm; its profitability is high owing to the superiority in producing the high-quality good μ. As μ becomes sufficiently large, we find substitution of production away from the low-quality good and toward the high-quality good by both firms as the point (c_{2H}, μ) moves from the lower part to the upper part of area E in Figure 1, and this substitution effect is stronger for the efficient firm than for the inefficient firm. Hence, the profit of the efficient firm 1 increases as degree of superiority in producing the high-quality product μ and the relative cost ratio c_{2H} increase. In this equilibrium, each of the total outputs of the two firms, Q_1^{*E} and Q_2^{*E}, remains unchanged when μ (c_{2H}) increases because the production substitution quantities from one good to another offset each other. Note that in equilibrium E, $Q_1^{*E} = Q_2^{*E} = 1/3$ from (16). This implies that both goods H and L are perfect substitutes in each firm, so changing μ or c_{2H} causes *direct production substitution* between goods H and L *within* each firm, and *subsequently* generates *indirect production substitution of each good between the cost-efficient firm 1 and the cost-inefficient firm 2*. The former direct effect, however, is weaker than the latter effect. Consequently, the equilibrium profits of both firms increase as μ increases because the increase in μ enhances the production substitution from good L to good H in both firms. However, the equilibrium profit of firm 1 (firm 2) increases (decreases) as c_{2H} increases since the increase in c_{2H} enhances the production substitution from good L to good H in firm 1 but does so from good H to good L in firm 2 and the efficient firm 1's markup on good

H is larger than that of the inefficient firm 2.

At any point (c_{2H}, μ) in area C, the relative superiority μ is large compared to the relative cost ratio c_{2H}. Because the margin of the efficient firm 1 for the high-quality good H, $p_H^{*C} - 1$, is very high, firm 1 entirely substitutes the production of good L by that of good H and specializes in good H, with its relatively large margin compared to that for the low-quality good L (that is p_L^{*C}) . As c_{2H} increases, the inefficient firm 2 substitutes its production of the high-quality good for that of the low-quality good. The total (i.e. ,the high-quality good's) output at the efficient firm 1 $Q_1^{*C}(= q_{1H}^{*C})$ decreases (increases) but the high-quality good's output at inefficient firm 2 q_{2H}^{*C} increases (decreases) as μ (c_{2H}) becomes large because the decrease (increase) in $Q_1^{*C}(= q_{1H}^{*C})$ outweighs the increase (decrease) in the resultant total output of firm 2 Q_2^{*C} due to the production substitution from good L (H) to good H (L) *within* firm 2. From (13) and (22), we can obtain the following proposition derived from the characteristics that both (i) the marginal costs of the high-quality good produced by both firms differ between firms $(c_{1H} \neq c_{2H})$ and (ii) each firm chooses a different production line $(\mathbf{s}_1 = (+, 0),\ \mathbf{s}_2 = (+, +))$. For the proof, see the appendix.

Proposition 2 *In equilibrium C, the difference in the profit between firms* $\pi_1^{*C} - \pi_2^{*C}$ *increases as* μ *or* c_{2H} *increases, but an increase in the* μ *reduces the difference in the firms' market shares* $Q_1^{*C} - Q_2^{*C}$.

We can easily confirm this reasoning by considering the reaction functions in case C:

$$\begin{aligned} q_{1H} &= \frac{1}{2} - \frac{1}{2}q_{2H} - \frac{1}{2\mu}q_{2L} \\ q_{2H} &= \frac{1}{2} - \frac{1}{2}\frac{c_{2H}}{\mu} - \frac{1}{2}q_{1H} - \frac{1}{\mu}q_{2L} \\ q_{2L} &= \frac{1}{2} - \frac{1}{2}q_{1H} - q_{2H}. \end{aligned}$$

Regarding the slope of these reaction functions, an increase in μ leads to a smaller effect of q_{2L} on both $(q_{1H},\ q_{2H})$ and directly leads to expansion q_{2H} because of the decrease in the cost-quality ratio c_{2H}/μ. As a result, an increase in μ causes a large increase in q_{2H} and a large decrease in q_{1H}, so that $\Delta Q_{12}^{*C} = Q_1^{*C} - Q_2^{*C}$ decreases. On the other hand, because the markup for the high-quality good H of efficient firm 1 is the larger than those of the high- and low-quality goods of firm 2, firm 1 earns more than firm 2 does irrespective of changes in μ and c_{2H}. Then, an increase in μ or c_{2H} brings about a larger markup for the efficient firm 1, which expands the difference in profits between firms.

In equilibrium B, μ, the relative superiority of the high-quality good, is larger and the relative cost inefficiency c_{2H} is not as high as in equilibrium D. Hence, firm 1 stops producing the low-quality good L and specializes in producing the high-quality good H. In contrast, the inefficient firm 2 continues to specialize in producing good L because its relative cost inefficiency of good H over firm 1's is high in this area. However, when μ is sufficiently high, firm 1 stops supplying good L and increases its output of good H, so firm 2 increases its output of good L due to the strategic substitution on good L . The profits of both firms increase as μ increases, but the profit of the efficient firm 1 specializing in good H is larger than that of the inefficient firm 2 specializing good L since the markup of the high-quality good H is larger than that of the low-quality good. In area B, the relative superiority μ is at a moderate level but is smaller than those in area C, and the relative cost ratio c_{2H} is larger than those in area C. Hence, firm 2, with its inefficient production technology for the high-quality good, stops producing good H and specializes in the low-quality good L. Two monopoly markets appear in this case.

In equilibrium D, the relative superiority of the high-quality good μ is relatively small but the relative cost ratio of the high-quality good c_{2H} is not as small. The inefficient firm

2 stops producing the high-quality good and specializes in supplying the low-quality good L, but the efficient firm 1 supplies the high-quality good H to the market, as well as the low-quality good L. Hence, increasing the relative superiority of the high-quality good μ in area D yields production substitution from the low-quality good L to the high-quality-good H in firm 1's product line, but there is no change in firm 2's product line. However, the total output Q_1^{*D} of firm 1 for both goods L and H does not change because they offset each other by perfect production substitution from good L to good H within the efficient firm 1 as μ increases. Consequently, if the relative superiority of the high-quality good μ increases, then only π^{*D} increases. Of course, the change in c_{2H} has no effect because the inefficient firm 2 never produces the high-quality good H in equilibrium D. As a matter of course, the efficient firm 1 specializing in good H earns more than firm 2 specializing in the low-quality good L.

4 Conclusion

In this study, we consider a duopoly game with two vertically differentiated products under nonnegative output constraints and an expectation with respect to the rival's product line strategies. We derive an equilibrium for the game and describe the firms' product line strategies and their realized profits in each equilibrium, based on the goods' quality superiority and relative cost efficiency. In these equilibria, the ranges of the two ratio parameters for which positive equilibrium outputs exist for the two firms differ. We graphically describe the firms' product line strategies in equilibrium, based on the relative quality of the products and on a firm's relative cost efficiency for the high-quality good (Figure 1). Thus, in proposition 1, we find that there exist equilibria that correspond to those in proposition 4 in Johnson and Myatt (2018), although in our model, one of the conditions regarding the cost advantage firm 1,the sum of output with a higher quality level than V_L *is strictly increasing in* quality in proposition 4 in their paper (2018) is not satisfied. Furthermore, we find an equilibrium (equilibrium C in proposition 1) in which the high-cost firm 2 supplies only good L, but the low-cost firm 1 supplies both goods H and L in the market, and so there is gap between the two firms' product lines, despite that the quality increment from good L to good H is sufficiently large (in proposition 4 in Johnson and Myatt (2018), the condition is that the quality increment from good L to H is sufficiently small).

We also show that the cost-efficient firm producing the high-quality good earns more than the inefficient firm does, except in the special case in which the relative superiority of the high-quality good μ is too small compared to the unit cost of the high-quality good H. In this case, both firms specialize in good L, and the market for good L becomes a Cournot duopoly. Thus, both firms' profits are the same. In Section 3, we illustrate how the changes in the relative superiority of the high-quality good, μ, and the relative cost inefficiency for high-quality good of firm 2, c_{2H}, affect the product lines and profits of firms at each equilibrium through production substitution *between* goods *within* a firm and that of goods *between* firms.

5 References

Bental, B. and Spiegel, M. (1984). Horizontal Product Differentiation, Prices and Quantity Selection of a Multi-product Monopolist. *International Journal of Industrial Organization,* 2 (2), 99-104.

Bonanno, G. (1986). Vertical Differentiation with Cournot Competition. *Economic Notes,* 15 (2), 68-91.

Calzada, J. and Valletti, T. (2012). Intertemporal Movie Distribution: Versioning when Customers Can Buy Both Versions. *Marketing Science,* 31 (4), 649-667.

Gal-Or, E. (1983). Quality and Quantitiy Competiton, *The Bell Journal of Economics,* 14, 590-600.

Johnson, J. P. and Myatt, D. (2018). The Determinants of Product Lines. *Rand Journal of Economics,* 49, 541-573.

________ and ________ (2006). Multiproduct Cournot Oligopoly. *Rand Journal of Economics,* 37, 583-601.

________ and ________ (2003). Multiproduct Quality Competition: Fighting Brands and Product Line Pruning. *American Economic Review,* 93 (3), 3748-3774.

Katz, M. and Shapiro, C. (1985). Network Externalities, Competition, and Compatibility. *American Economic Review,* 75 (3), 424-440.

Kitamura, R. and Shinkai, T. (2016). Corrigendum to 'Kitamura, R. and Shinkai, T. (2015) Product Line Strategy within a Vertically Differentiated Duopoly. *Economics Letters* 137, 114-117.', http://www-econ2.kwansei.ac.jp/~shinkai/ELCorrigendumfinalRenew2016.pdf.

Kitamura, R. and Shinkai, T. (2015a). Product Line Strategy within a Vertically Differentiated Duopoly. *Economics Letters* 137, 114-117.

Kitamura, R, Shinkai T. (2015b). Cannibalization within the Single Vertically Differentiated Duopoly. *Paper presented at the EARIE 2015, Annual Conference of European Association for Research Industrial Economics, Munich, Germany August 28-30,* 1-23.

Kitamura, R. and Shinkai, T. (2013). The Economics of Cannibalization: A Duopoly in which Firms Supply Two Vertically Differentiated Products. *Paper presented at the EARIE 2013, Annual Conference of European Association for Research Industrial Economics, Evora, Portugal August 30–September 1.* 1-22.

Lahiri, S. and Ono, Y. (1988). Helping Minor Firms Reduces Welfare. *The Economic Journal,* 98 (393), 1199-1202.

Motta, M. (1993) Endogenous Quality Choice: Price vs. Quantity Competition. *Journal of Industrial Economics,* 41 (2), 113-131.

Mussa, M. and Rosen, S. (1978). Monopoly and Product Quality. *Journal of Economic Theory, 18, 301-317.*

Shaked, A. and Sutton, J. (1990). Multiproduct Firms and Market Structure. *The Rand Journal of Economics* 21 (1), 45-62.

Shaked, A. and Sutton, J. (1987). Product Differentiation and Industrial Structure. *The Journal of Industrial Economics,* 36 (2), 131-146.

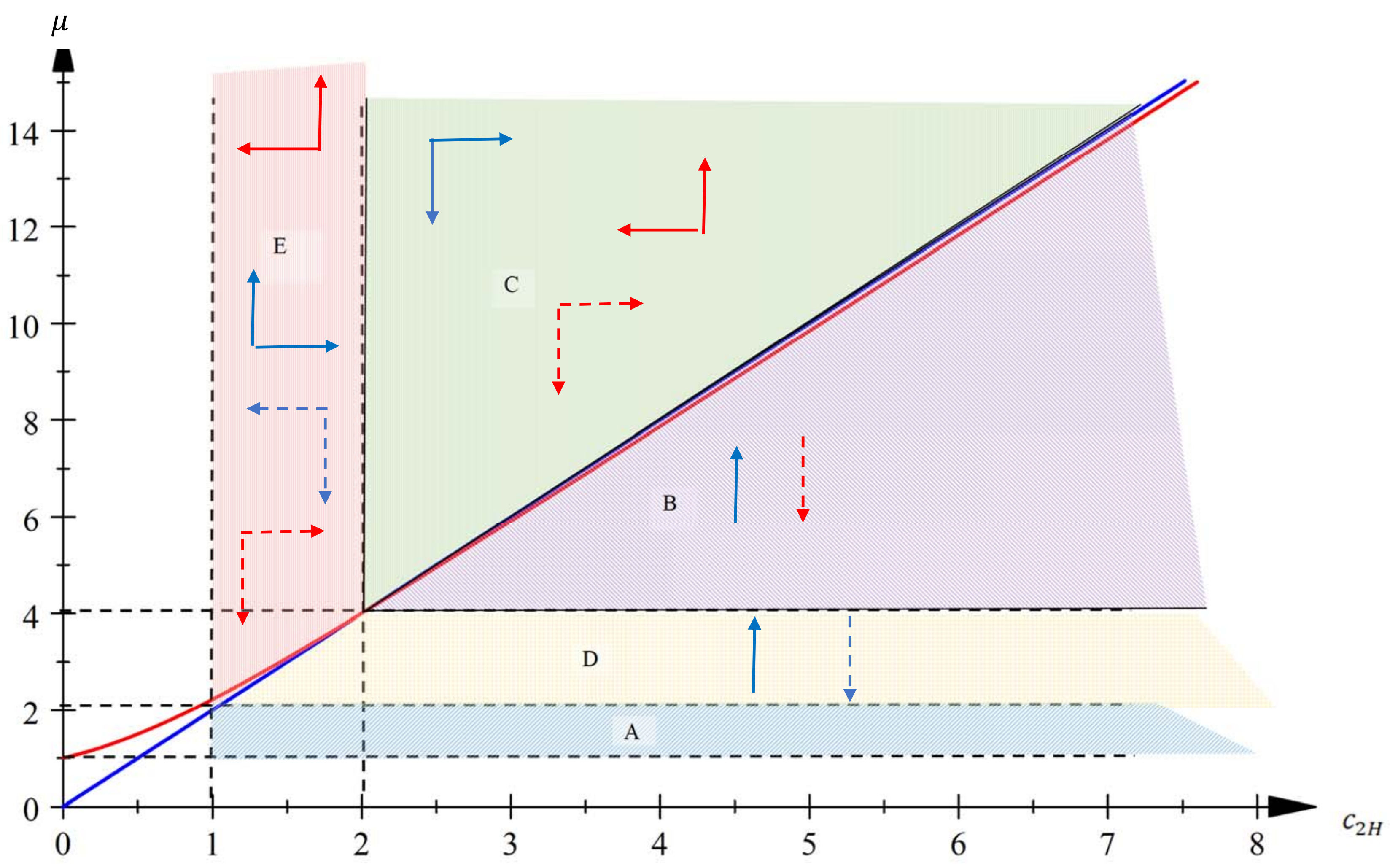

Figure 1 Five Equilibria in $c_{2H} - \mu$ plane

The light blue line stands for $\mu = 2c_{2H}$, the light red curve stands for $\mu = \frac{1}{2}\left(2c_{2H} + \sqrt{4c_{2H}^2 - 2c_{2H} + 4}\right)$. , ↑ and ↓ imply $\partial q_{1H}^*/\partial\mu > 0, \partial q_{1H}^*/\partial\mu < 0$, respectively. ↑ implies $\partial q_{2H}^*/\partial\mu > 0$. ↓ (dashed) and → (dashed) imply $\partial q_{2L}^*/\partial\mu < 0, \partial q_{2L}^*/\partial c_{2H} < 0$, respectively. ↓ (dashed) and ← (dashed) imply $\partial q_{1L}^*/\partial\mu < 0, \partial q_{1L}^*/\partial c_{2H} < 0$, respectively.