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Free entry under an output-cap constraint

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Abstract

This study considers a peer-to-peer market with capacity-constrained suppliers. We examine a free-entry market of individual suppliers and discuss the welfare consequences of free entry. We show that the number of entries is socially optimal.

Keywords:sharing economy, Cournot competition, excess entry theorem, private lodging businesses, capacity constraint

JEL Classification:D43, L13, K25

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1 Introduction

The sharing economy, globally prevalent across various markets and industries, has grown rapidly with the development of information technology, enabling individuals to use their idle or underutilized capacities efficiently (Schlagwein et al., 2019). Examples of underutilized capacities include private rooms, parking lots, automobiles, and, potentially, human capital. These capacities, invested for an individual's own use in the past, are underutilized. As a result of using these installed capacities without further new investments, there is usually an upper limit to each individual's supply of services.

The private lodging business is a typical example of such a sharing economy. In Japan, the new law regulating private lodging businesses took effect in 2018, excluding illegal private lodging businesses and bringing legal businesses under the local government's supervision and regulation. The law allows anyone to lease their rooms for a maximum of 180 days a year on advance registration with their local government. As individuals lease their own underutilized rooms, the marginal cost for providing such services is low. However, registration and supervision impose non-negligible market entry costs. Thus, the number of suppliers entering the new regulated market is fewer than the earlier market where individuals would engage in illegal businesses with little or no entry costs. Many of these individuals have now exited the private lodging market. This implies that the number of individuals entering this market varies with the entry costs.

A distinct property of this market, which is overlooked by the literature, is that the quantity each individual can supply is restricted. Individual suppliers lease their own rooms, built in the past for their own use, but are now underutilized. Therefore, there is an upper limit to the quantity of serviceable rooms they can supply. Moreover, the law allows them to lease their rooms for a maximum of 180 days a year only, which can be another supply constraint.

In this article, we focus on this property of a sharing economy and investigate the welfare consequence of free entry under the output-cap constraint. We find that even under strategic interaction among suppliers (imperfect competition), the number of entering individual suppliers is efficient, in contrast to the literature, which suggests that the number of entering firms is excessive (Mankiw and Whinston, 1986; Suzumura and Kiyono, 1987).

2 The model

We consider a market where n suppliers with an output-cap constraint compete in Cournot fashion. The maximal output each supplier can choose is $\bar{q} > 0$. Let the quantity of supplier $i = 1, \ldots, n$ be q_i and $Q = \sum_{i=1}^{n} q_i$. The inverse demand of this market is represented by a continuous function $P(Q) : \mathbb{R}_+ \mapsto \mathbb{R}_+$ with $P' < 0$ when $P > 0$. The production cost of a supplier is $C(q_i) : \mathbb{R}_+ \mapsto \mathbb{R}_+$ with $C' \geq 0$, $C'' \geq 0$, and entry cost of each supplier is $f > 0$. We assume the regular conditions usually adopted in the literature (Vives, 1999).

Assumption 0 *The inverse demand function satisfies the following conditions: (i)* $P(0) > C'(0)$*, (ii)* $\exists Q$, $P(Q) = 0$*, and (iii)* $P'(Q) + P''(Q)q < 0$ *for all* Q *such that* $P(Q) > 0$ *and for all* $q \in [0, Q]$.

We consider the two-stage game that runs as follows. In the first stage, each supplier decides to enter the market. In the second stage, the suppliers simultaneously choose their quantities under the output-cap constraint.

The game is solved by backward induction. In the second stage, for given n, the profit maximization problem of supplier i is

$$\max_{q_i} \pi_i = P(Q)q_i - C(q_i) - f \text{ subject to } q_i \leq \bar{q}.$$

Let $q^*(n)$ be the equilibrium output of a supplier with this constraint, which is common for all i by symmetry. At this market equilibrium, each supplier's profit is $\pi^*(n) = P(nq^*(n))q^*(n) - C(q^*(n)) - f$. In the first stage, the equilibrium number of firms, n^*, is determined by the zero-profit condition, $\pi^*(n^*) = 0$.[1]

If the constraint does not bind (or does not exist), each supplier produces $q^U(n)$, such that

$$\left.\frac{\partial \pi_i}{\partial q_i}\right|_{q_i = q^U(n)} = P(nq^U(n)) + P'(nq^U(n))q^U(n) - C'(q^U(n)) = 0, \quad (1)$$

[1]For simplicity, we neglect the integer problem.

for given n, where the superscript U means (u)nconstrained. Assumption 0 assures the well-performed equilibrium quantities,[2] which are obtained by differentiating (1) as

$$\frac{\partial q^U(n)}{\partial n} = -\frac{q^U(P' + P''q^U)}{n(P' + P''q^U) + (P' - C'')} < 0, \tag{2}$$

$$\frac{\partial nq^U(n)}{\partial n} = \frac{q^U(P' - C'')}{n(P' + P''q^U) + (P' - C'')} > 0. \tag{3}$$

The equilibrium number of suppliers without the constraint n^U satisfies

$$\pi_i|_{q_i=q^U(n^U)} = P(n^U q^U(n^U))q^U(n^U) - C(q^U(n^U)) - f = 0. \tag{4}$$

We focus on the cases where the free-entry equilibrium with $n^U > 0$ and $q^U(n^U) > 0$ exists if there is no output-cap constraint. Then, this equilibrium is unique (Ino and Matsumura, 2012).

We assume the following.

Assumption 1 *Each supplier's output cap $\bar{q}$ is strictly smaller than the equilibrium output without the constraint $q^U(n^U)$.*

The constraint binds and thus each supplier produces $\bar{q}$ if and only if[3]

$$\left.\frac{\partial \pi_i}{\partial q_i}\right|_{q_i=\bar{q}} = P(n\bar{q}) + P'(n\bar{q})\bar{q} - C'(\bar{q}) \geq 0, \tag{5}$$

for given n. If each supplier produces $\bar{q}$, the number of entering suppliers is n^C such that

$$\pi_i|_{q_i=\bar{q}} = P(n^C\bar{q})\bar{q} - C(\bar{q}) - f = 0, \tag{6}$$

where superscript C denotes (c)onstrained.

We do not assume the constraint binds a priori, but assume the following.

Assumption 2 $n^C > 0$.

Note that n^C is uniquely determined by (6) under this assumption[4] whenever the constraint binds or not.

[2]Along with (1) ($P - C' > 0$ at the equilibrium), (2) and (3) satisfies the assumptions of excess entry theorem provided by Mankiw and Whinston (1986). Thus, without the constraint, our model results in socially excessive market entry.

[3]From Assumption 0(iii), the second-order condition is globally met, that is π_i is strictly concave in q_i. Thus, the following is also the sufficient condition.

[4]The left-hand side of (6) strictly decreases in n^C for given $\bar{q}$. See also the 2nd paragraph of the proof of Theorem 1.

Example: linear demand and cost. To understand the assumptions and model performance of our free-entry market, we provide a linear example. Suppose that $P(Q) = a - Q$ and $C(q) = cq$ with $a > c \geq 0$. Then, by (1) and (4), we obtain $q^U(n) = (a-c)/(n+1)$ and $q^U(n^U) = \sqrt{f}$. Thus, Assumption 1 corresponds to $\bar{q} < \sqrt{f}$. By (6), we obtain

$$n^C = \frac{(a-c)\bar{q} - f}{\bar{q}^2}.$$

Thus, Assumption 2 corresponds to $\bar{q} > f/(a-c)$. The upper-left panel of Figure 1 depicts the level of n^C for $\bar{q} \in (f/(a-c), \sqrt{f})$, which satisfies Assumptions 1 and 2. As seen in the lower-left panel, under the assumptions, $\bar{q}$ is smaller than the unconstrained outcome $q^U(n^C)$ for each n^C.[5] Hence, the constraint binds in the free entry equilibrium (i.e., $q^*(n^*) = \bar{q}$ with $n^* = n^C$). The equilibrium market price is $P(n^*q^*(n^*)) = c + f/\bar{q}$, which is depicted in the upper-right panel of Figure 1. The equilibrium market size $n^*q^*(n^*) = n^C\bar{q}$ is also depicted in the lower-right panel.

3 Efficient entry theorem

Social welfare for given n is

$$W^*(n) = \int_0^{nq^*(n)} P(s)ds - nC(q^*(n)) - nf.$$

Let n^o be the welfare maximizing number of firms. In contrast to the well-known "Excess entry theorem" in oligopoly markets (Mankiw and Whinston, 1986; Suzumura and Kiyono, 1987), under the output-cap constraint, the number of firms to enter the market is socially optimal.[6]

Theorem 1 *Under Assumptions 0-2, the entry is efficient (i.e., $n^* = n^o$).*

[5]Explicitly, we have

$$q^U(n^C) = \frac{(a-c)\bar{q}^2}{(a-c)\bar{q} - f + \bar{q}^2} \quad \therefore q^U(n^C) - \bar{q} = \frac{(f - \bar{q}^2)\bar{q}}{(a-c)\bar{q} - f + \bar{q}^2} > \frac{(f - \sqrt{f}^2)\bar{q}}{(a-c)\bar{q} - f + \bar{q}^2} = 0,$$

where we use Assumptions 1 and 2 to obtain the last inequality.

[6]The literature shows that the number of entering firms can be insufficient (Ghosh and Morita, 2007a,b; Gu and Wenzel, 2009; Sato and Matsumura, 2020). In these cases, the number of entering firms is inefficient under plausible conditions, and then too, our result contrasts with their results.

Proof. First, we show that $\bar{q} < q^U(n^C)$ for any $\bar{q}$ that satisfies Assumptions 1 and 2. By $\bar{q} < q^U(n^U)$ (Assumption 1) and $n^U > 0$, we obtain

$$\begin{aligned} 0 &= P(n^U q^U(n^U)) + P'(n^U q^U(n^U))q^U(n^U) - C'(q^U(n^U)) \quad (\because (1) \text{ with } n = n^U) \\ &< P(n^U \bar{q}) + P'(n^U \bar{q})\bar{q} - C'(\bar{q}). \quad (\because \text{Assumption 0(iii), } P' < 0, \text{ and } C'' \geq 0) \end{aligned} \tag{7}$$

Assumption 0(i,ii) implies that there exists $\bar{Q} > 0$ such that $P(\bar{Q}) = C'(\bar{Q})$. Since $q^U(n^U) < \bar{Q}$ holds, we restrict our attention to the case where $\bar{q} < \bar{Q}$ by Assumption 1.

We investigate the left-hand side of (6) by the function

$$\pi(q; n) = P(nq)q - C(q) - f.$$

We show that $n^C > 0$ if and only if (i) $\pi(\bar{q}; 0) = P(0)\bar{q} - C(\bar{q}) - f > 0$, which is equivalent to (ii) $\bar{q} > q^0$ such that $\pi(q^0; 0) = P(0)q^0 - C(q^0) - f = 0$ as long as $\bar{q} < \bar{Q}$. To show (i), suppose that $n^C > 0$. Then, $\pi(\bar{q}; 0) > 0$ holds since $\pi(\bar{q}; n^C) = 0$ by (6) and $\partial\pi(\bar{q}; n)/\partial n = P'\bar{q}^2 < 0$. Conversely, suppose that $\pi(\bar{q}; 0) > 0$. When n is sufficiently larger than $\bar{Q}/\bar{q} > 0$, $\pi(\bar{q}; n) < 0$ by Assumption 0(ii). Thus, there uniquely exists $n^C > 0$ such that $\pi(\bar{q}; n^C) = 0$ by $\partial\pi(\bar{q}; n)/\partial n < 0$. To show (ii), observe that from $P(0) > C'(0)$ (Assumption 0(i)) and $P(0) > P(\bar{Q}) = C'(\bar{Q})$, $\partial\pi(q; 0)/\partial q = P(0) - C'(q) > 0$ for all $q \in (0, \bar{Q})$ by $C'' \geq 0$. Moreover, $\pi(0; 0) = -C(0) - f < 0$. Thus, $\pi(\bar{q}; 0) > 0$ if and only if there uniquely exists $q^0 > 0$ that satisfies $\pi(q^0; 0) = 0$ and $\bar{q} > q^0$.

Hence, $\bar{q}$ that satisfies Assumptions 1 and 2 is $\bar{q} \in (q^0, q^U(n^U))$. By the continuity of n^C (i.e., n^C that satisfies (6) is continuous in $\bar{q}$ by the uniqueness and the implicit function theorem), $n^C \searrow 0$ as $\bar{q} \searrow q^0$ because $\pi(q^0; 0) = 0$. Accordingly, we can pick up $\bar{q} \in (q^0, q^U(n^U))$ (sufficiently close to q^0) such that it satisfies $n^C \leq n^U$ under Assumptions 1 and 2 (note that $q^U(n^U) > 0$ and $n^U > 0$ is independent of $\bar{q}$). For such $\bar{q}$, suppose that $\bar{q} \geq q^U(n^C)$ contrarily. Then, since $n^C q^U(n^C) \leq n^C \bar{q}$ by using $n^C > 0$ (Assumption 2), we obtain

$$\begin{aligned} 0 &= P(n^C q^U(n^C)) + P'(n^C q^U(n^C))q^U(n^C) - C'(q^U(n^C)) \quad (\because (1) \text{ with } n = n^C) \\ &\geq P(n^C \bar{q}) + P'(n^C \bar{q})\bar{q} - C'(\bar{q}). \quad (\because \text{Assumption 0(iii), } P' < 0, \text{ and } C'' \geq 0) \end{aligned} \tag{8}$$

In contrast, since $n^C \bar{q} \leq n^U \bar{q}$ by $n^C \leq n^U$ (now we are taking such $\bar{q}$), we obtain

$$P(n^C \bar{q}) + P'(n^C \bar{q})\bar{q} - C'(\bar{q}) \geq P(n^U \bar{q}) + P'(n^U \bar{q})\bar{q} - C'(\bar{q})$$

by Assumption 0(iii). This contradicts (7) and (8). Thus, for the picked-up $\bar{q}$, $\bar{q} < q^U(n^C)$ holds.

If there exists another $\bar{q} \in (q^0, q^U(n^U))$ such that $\bar{q} \geq q^U(n^C)$, because of the continuity of $q^U(n^C)$ in $\bar{q}$ (i.e., $q^U(n)$ is continuous in n by the maximum theorem and n^C is continuous in $\bar{q}$ as mentioned above), considered with the result of the previous paragraph, we must have $\bar{q} \in (q^0, q^U(n^U))$ that satisfies $\bar{q} = q^U(n^C)$. Such $\bar{q}$ and n^C satisfies the conditions for free-entry equilibrium without the constraint (i.e., (1) and (4)) by $\bar{q} = q^U(n^C)$ and (6). However, since $\bar{q} < q^U(n^U)$, this contradicts the uniqueness of the free-entry equilibrium without the constraint. Thus, $\bar{q} < q^U(n^C)$ holds for any $\bar{q}$ that satisfies Assumptions 1 and 2.

Next, we show that $n^* = n^o$. Since $q^U(n^C) > \bar{q}$, as shown above, there exists $\epsilon > 0$, for all $n \in (n^C - \epsilon, n^C + \epsilon)$, $q^U(n) > \bar{q}$. Thus, by taking such ϵ, for all $n \in (n^C - \epsilon, n^C + \epsilon)$,

$$\begin{aligned} 0 &= P(nq^U(n)) + P'(nq^U(n))q^U(n) - C'(q^U(n)) \quad (\because (1)) \\ &< P(n\bar{q}) + P'(n\bar{q})\bar{q} - C'(\bar{q}). \quad (\because \text{Assumption 0, } P' < 0, \text{ and } C'' \geq 0) \end{aligned} \tag{9}$$

Thus, (5) is satisfied. In other words, the constraint binds (i.e., $q^*(n) = \bar{q}$ for all $n \in (n^C - \epsilon, n^C + \epsilon)$). Therefore, we can differentiate $W^*(n)$ at $n = n^C$ as

$$\frac{\partial W^*(n^C)}{\partial n} = P(n^C \bar{q})\bar{q} - C(\bar{q}) - f = \pi(\bar{q}; n^C). \tag{10}$$

Since $q^*(n^C) = \bar{q}$ by $n^C \in (n^C - \epsilon, n^C + \epsilon)$ and thus $n^* = n^C$, $\pi(q^*; n^*) = \pi(\bar{q}; n^C) = 0$ by (6). This and (10) imply that n^* satisfies the necessary condition for the welfare maximizing number of firms (i.e., $\partial W^*(n^*)/\partial n = 0$).

For sufficiency, consider the cases where $\bar{q} < q^U(n)$ and $\bar{q} \geq q^U(n)$, respectively. Since $q^U(n)$ is strictly decreasing in n by (2) and $\bar{q} < q^U(n^C)$, by taking $n^1 > n^C$ such that $\bar{q} = q^U(n^1)$, the former case corresponds to the case of $n < n^1$ and the latter to the case of $n \geq n^1$.

When $n < n^1$ ($\bar{q} < q^U(n)$), similar to (9), the constraint always binds in this case (i.e.,

$q^*(n) = \bar{q}$ for all $n < n^1$). Therefore, for all $n < n^1$,

$$\frac{\partial^2 W^*(n)}{\partial n^2} = P'(n\bar{q})\bar{q}^2 < 0. \tag{11}$$

This concavity and the necessary condition, $\partial W^*(n^C)/\partial n = 0$ by (10), imply that n^C maximizes the welfare in the range of $n < n^1$.

When $n \geq n^1$ ($\bar{q} \geq q^U(n)$), the constraint does not bind and $q^*(n) = q^U(n)$. Thus,

$$\frac{\partial W^*(n)}{\partial n} = \pi^U(n) + n[P(nq^U(n)) - C'(q^U(n))]\frac{\partial q^U(n)}{\partial n} < \pi^U(n) < 0,$$

where the last inequality holds because (1)–(2) and

$$\begin{aligned}
\pi^U(n) &= P(nq^U(n))q^U(n) - C(q^U(n)) - f \\
&< P(n^1 q^U(n^1))q^U(n) - C(q^U(n)) - f \quad (\because nq^U(n) > n^1 q^U(n^1) \text{ by (3), and } P' < 0) \\
&< P(n^C\bar{q})q^U(n) - C(q^U(n)) - f \quad (\because n^1 q^U(n^1) > n^C\bar{q} \text{ by } q^U(n^1) = \bar{q} \text{ and } n^1 > n^C) \\
&\leq P(n^C\bar{q})\bar{q} - C(\bar{q}) - f = 0. \quad (\because \bar{q} \geq q^U(n) \text{ and } P(n^C\bar{q}) - C'(q) > 0 \text{ for } q < \bar{q} \text{ by (9)})
\end{aligned}$$

Thus, the welfare decreases as the number of firms increases in the range of $n \geq n^1$. Consequently, n^C maximizes the welfare globally (i.e., $n^o = n^C = n^*$). **Q.E.D.**

Under moderate assumptions (Assumptions 0–2), the output-cap constraint in fact binds, and thus, there is no business stealing effect in this market. Therefore, the number of entering suppliers is efficient.

4 Concluding remarks

In this article, we have focused on the quantity cap constraint—an important property of a sharing economy, and yet overlooked by the literature. We show that under plausible conditions, the entry of individual suppliers is efficient. Therefore, additional entry regulations and taxes on individual suppliers are unnecessary.

The sharing economy may crowd out traditional businesses. For example, new private lodging businesses may reduce the business opportunities for traditional hotel businesses, which may create additional distortions or welfare gains. Investigating this problem using a multi-product model is a potential area for future research.

References

Ghosh, A., Morita, H., 2007a. Free entry and social efficiency under vertical oligopoly. RAND Journal of Economics 38(2),541–554.

Ghosh, A., Morita, H., 2007b. Social desirability of free entry: a bilateral oligopoly analysis. International Journal of Industrial Organization 25(5), 925–934.

Gu, Y., Wenzel, T., 2009. A note on the excess entry theorem in spatial models with elastic demand. International Journal of Industrial Organization 27(5), 567–571.

Ino, H., Matsumura, T. 2012, How many firms should be leaders? Beneficial concentration revisited. *International Economic Review* 53, 1323–1340.

Mankiw, N.G., Whinston, M.D., 1986, Free entry and social inefficiency. *Rand Journal of Economics* 17(1), 48–58.

Sato, S., Matsumura, T., 2020. Free entry under common ownership. Economics Letters 195, 109489.

Schlagwein, D., Schoder, D., Spindeldreher, K. 2019. Consolidated, systemic conceptualization, and definition of the "sharing economy". *Journal of the Association for Information Science and Technology* 71(7),817–838.

Suzumura, K., Kiyono, K., 1987. Entry barriers and economic welfare. *Review of Economic Studies* 54(1), 157–167.

Vives, X., 1999. *Oligopoly Pricing: Old Idea and New Tools*, The MIT Press, Cambridge, MA, London, England.

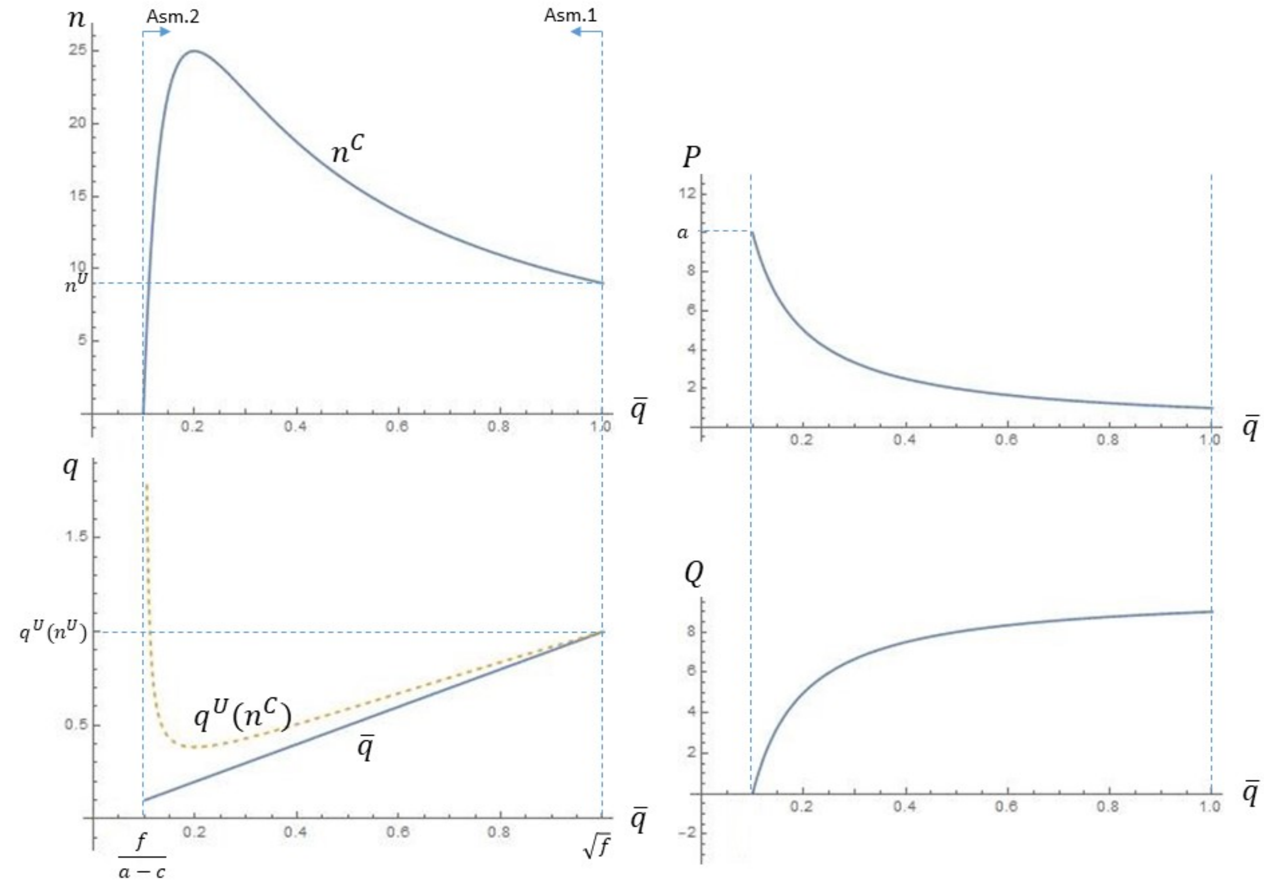

Figure 1: The model performance of a linear model. The graphs are depicted in the case where $a = 10$, $c = 0$, and $f = 1$ for $0 < \bar{q} < 1$ (horizontal axis).