DISCUSSION PAPER SERIES

Discussion paper No.183

International R&D Spillovers, Innovation by Learning from Abroad and Medium-Run Fluctuations

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September 2018

MASTERY FOR SERVICE

SCHOOL OF ECONOMICS

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International R&D Spillovers, Innovation by Learning from Abroad and Medium-Run Fluctuations*

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Abstract

Many developed economies experienced large and correlated fluctuations in the medium run during the postwar period. A good number of industrialized countries experienced high productivity growth during the 1960s and low growth between the early 1970s and the early 1980s. This paper develops a model of medium-run fluctuations incorporating research and development (R&D)-based endogenous growth and international R&D spillovers from a technologically leading country to a technologically lagging country. An important feature of the model is that a key role of the lagging country's R&D is innovation by leaning (IBL) from abroad. After calibration using U.S. and Japanese data, the model shows that changes in U.S. R&D expenditure alone can substantially explain Japan's medium-run fluctuations. The paper argues that the diffusion of U.S. innovations (generated by U.S. R&D) to Japan plays an important role in determining Japan's medium-run fluctuations.

JEL: E32, O19, O33, O41,
Key words: International R&D spillovers, technology diffusion, endogenous growth, medium-run fluctuations.

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* I would like to thank R. Anton Braun and Nao Sudo for helpful discussions. Part of this paper was written while the author was a visiting scholar at the University of California, Santa Cruz and the author thanks them and in particular Carl Walsh for their outstanding hospitality and support.

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1 Introduction

Over the postwar period, developed economies have experienced large and correlated decade-to-decade fluctuations, i.e., medium-run fluctuations. For example, a good number of industrialized countries experienced high productivity growth during the 1960s, which was followed by low growth between the early 1970s and the early 1980s. Medium-run comovements have been reported by several studies, e.g., Heathcote and Perri (2003); Kose, Otrok and Whiteman (2003); Pakko (2004); and Stock and Watson (2005).[1]

This would lead one to consider whether a common factor is driving internationally correlated fluctuations in the medium run. There are some empirical evidences for this. Kose, Otrok and Whiteman (2003) find that a common world factor serves as an important source of business cycle fluctuations (i.e., short-run fluctuations) in most countries, with the U.S. representing an important source of these fluctuations. More importantly, they find that the common factor is very persistent when compared with country specific factors. In other words, they find that low frequency comovements across economies are captured by this common world factor. Klenow and Rodriguez (2005) report evidence that the productivity slowdown experienced in the 1970s was a common global phenomenon, and based on this fact, they argue that important knowledge spillovers occur across countries.

It is natural to think that technology is a prominent candidate for the common factor observed to generate the correlated medium-run fluctuations.[2]

[1] Heathcore and Perri (2003) report some decline in medium-run comovements in the 1980-2002 period when compared with those in the 1960-1981 period.

[2] A common shock is obviously not necessary for correlated economic fluctuations. For example, in the international business cycle literature, it is empirically well known that tighter trade linkages result in higher business cycle correlations across countries. The literature both empirically and theoretically focuses on trade's role in transmitting shocks (see, for example, Backus et al. (1995); Kose and Yi (2006); Ng (2010); and Giovanni and Levchenko (2010), among many others). The trade-business cycle link does not depend on the existence of a common shock. However, the literature shows that high TFP shock co-movement is needed to generate business cycle models that quantitatively match data: see Backus et al (1995) and Kose and Yi (2006). In other words, even in the short run, the common TFP (technology) shock plays some important role in the correlated fluctuations. This role of technology is most probably more important in the medium run.

Figure 1 gives a rough foundation for this argument by plotting total factor productivity (TFP) growth rates for eight developed countries (G7 countries plus Australia) for the 1950-2010 period. The growth rates are averaged over 10-year intervals.[3] The figure clearly demonstrates the existence, in general, of a medium-run international TFP comovement. TFP growth rates in all depicted countries except in the U.K. were high in the 1960s, decreased in the 1970s, and stayed low in the 1980s before increasing in the 1990s and then decreasing back in the 2000s. I proceed to consider technology as the common factor and examine a technology-related explanation for these correlated medium-run fluctuations.

This paper asks whether technology diffusion from abroad has quantitatively important effects on medium-run fluctuations. To this end, the paper builds a two-country and two-sector version of a real business cycle model that has been extended to incorporate research and development (R&D)-based endogenous technological change and international R&D spillovers. Assuming that one country is a technological leader and the other is a follower, the model shows that the leader country's R&D has an effect on the follower country. This impact occurs because the leader's innovations generated by its R&D diffuse to the follower. However, the effect on the follower economy due to R&D spillovers from the leader (i.e., the technology diffusion effect) does not emerge automatically or exogenously. Rather, it depends on R&D conducted by the follower. The follower's R&D plays an absorptive role for learning frontier technologies, termed "*innovation by learning (IBL) from abroad.*" The follower country learns from the leader to produce technologies (i.e., new types of goods) that are suitable for use in its own environment. In other words, firms in the follower country, *at least to some extent*, need to change the leader country's new goods (technologies) to fit with their domestic customers.

IBL from abroad is assumed to play a key role in the follower country's R&D in this paper. IBL from abroad implies that R&D costs in the follower

[3] First, TFP growth rates are linearly detrended, and then the detrended growth rates are averaged over 10-year intervals. Next, the averaged growth rates are divided by their respective standard deviations. The data on TFP are taken from Penn World Table ("rtfpna" in Penn World Table 9.0).

country depend on the technology gap between leader and follower (this cost decreases with an increase in the gap because the larger the gap is, the more the follower country can learn from the leader). This leads to a close connection between the leader's R&D and the follower's R&D. Consequently, changes in the leader's R&D cause fluctuations in the follower's technology level as well as in other variables.

To quantitatively assess the model, this paper assumes R&D spillovers from the U.S. to Japan and considers R&D linkages between these two countries. This assumption is supported by empirical studies. Bernstein and Mohnen (1998) apply growth accounting methods to R&D intensive industries and estimate R&D spillovers between the U.S. and Japan. They find no evidence of spillovers from Japan to the U.S.; in contrast, approximately 46 percent of Japanese TFP growth is owing to spillovers from U.S. R&D. Branstetter and Ug (2004) use firm level data and find evidence of R&D spillovers to Japanese R&D from scientific ideas that originated in the U.S. [4]

Assuming that the U.S. is a technological leader and Japan is a technological follower, this paper assesses the present model's ability to generate Japanese medium-run macroeconomic fluctuations. With exogenous U.S. R&D, the model can successfully reproduce medium-run fluctuations in Japanese TFP, output, R&D, consumption, investment, and labor. Changes in U.S. R&D spending explain Japan's medium-run fluctuations to a great extent. This finding is consistent with the data fact found by Braun, Okada, and Sudo (2006), which show that slow (fast) economic growth in Japan over the postwar period was preceded by a persistent decline (increase) in U.S. R&D.

This paper is based on several important contributions made by previous studies. First, some studies have endogenized technological change to analyze medium-run macroeconomic fluctuations or the persistent effect of a temporary shock.[5] For example, the seminal paper by Comin and Gertler (2006) incorpo-

[4] In addition, Braun, Okada and Sudo (2006) use industry data and show that U.S. R&D is a strong driving force of Japanese TFP.

[5] See, for example, Francois and Lloyd-Ellis (2003); Comin and Gertler (2006); Bilbiie, Ghironi and Melitz (2012); Anzoategui, Comin, Gertler, and Martinez (2016); and Bianchi, Kung and Morales (2017).

rates R&D-based endogenous technological progress to analyze medium-term business cycles. Comin and Gertler (2006) show that non-technological shocks could produce most of the cyclical fluctuations in productivity both at high and medium frequencies. In keeping with Comin and Gertler (2006), the present paper also uses R&D-based endogenous technological change. However, in departure from Comin and Gertler (2006) and other studies that introduce endogenize technological progress into a dynamic (stochastic) general equilibrium model, the present paper incorporates international diffusion of technology (i.e., international R&D spillovers) to examine internationally correlated medium-run fluctuations. The present paper's model is also built on previous research on economic growth with international technology diffusion, including Nelson and Phelps (1966); Parente and Prescott (1994); Barro and Sala-i-Martin (1997); Eaton and Kortum (1999); and Cordoba and Ripoll (2008). These papers, however, focus on (long-run) economic growth rather than fluctuations.

In addition to the aforementioned papers, the present paper is closely related to studies by Hayashi and Prescott (2002, 2006) and Chen, İmrohoroğlu and İmrohoroğlu (2006), which consider the medium-run Japanese economy. Hayashi and Prescott (2002) show that the model with *exogenous* changes in TFP and the workweek of labor can explain Japan's persistent stagnation since the mid-1990s. Hayashi and Prescott (2006) find that Japan's depressed output level during the pre-WWII period can be explained by a two-sector neoclassical growth model with *exogenous* TFP and a barrier that held agricultural employment constant. Chen et. al. (2006) argue that one can explain the variations in postwar Japanese savings rates using a neoclassical growth model with *exogenous* TFP and initially low capital stock. Differently from these studies, the present paper considers an R&D-based endogenous growth model and shows that *endogenous* technological change induced by R&D spillovers from the U.S. can largely explain Japan's medium-run fluctuations.

This paper is also closely related to the study by Comin, Loayza, Pasha and Serven (2014). On medium-run fluctuations in developing countries, they use a two-country dynamic stochastic general equilibrium model extended to include endogenous technological change to examine the effect of technology diffusion from developed to developing economies. Although the present paper studies the effect of international technology diffusion, as in Comin, et. al. (2014), it considers a different kind of international technology diffusion process. In Comin, et. al. (2014), the follower (*developing* economy) does not perform R&D and new technologies transfer from the leader (developed economy) to the follower (*developing* economy). In contrast, in the present paper, the follower (*developed* economy) does perform R&D and innovate new technologies suitable for use in its own environment by learning from technologies created by the leader (*developed* economy). That is, the follower performs *IBL from abroad*. The present paper is, thus, complementary to Comin, et. al. (2014).

The remainder of this paper is structured as follows. Section 2 describes the model. Section 3 explains the calibration and simulation procedure and provides the results. Section 4 summarizes and concludes the paper.

2 Model

The paper considers a two-sector version of a real business cycle model extended to include R&D-based endogenous knowledge production as modeled by Romer(1990) and Jones(1995). The model also introduces international R&D spillovers. The model is deterministic as those considered by Hayashi and Prescott (2002 and 2006) and Chen et. al. (2006). The key assumptions are as follows: (1) there are two economies (technologically leading country and technologically lagging country) that are symmetric except for some aspect of R&D; (2) there are two types of firms: final goods firms and intermediate goods firms; (3) firms are owned by homogeneous households; (4) final goods firms produce goods competitively; (5) intermediate goods firms conduct R&D to produce product blueprints (ideas, i.e., technologies) of differentiated intermediate goods (a product blueprint is needed to produce intermediate goods) and finance their R&D expenditures by borrowing money from households; (6) intermediate goods firms rent capital from households; (7) product blueprints of intermediate goods become obsolete in the next period with the probability $(1-\psi)$; (8) when a product blueprint for intermediate goods becomes obsolete, final goods firms no longer demand those goods; (9) the two economies are closed except for their R&D: firms in the lagging country learn from product blueprints (ideas) made by firms in the leading country; and (10) firms in both the leading country as well as the lagging one need their own product blueprints to produce goods suitable to their own country's environment.

In what follows, I focus mainly on the lagging country because the two economies are symmetric except for their R&D. A subscript of L is used to denote variables for the leading country where necessary.[6] The words, "blueprint," "idea," and "technology" are used interchangeably in the following.

2.1 Firms

2.1.1 Final goods firms

Final goods firms produce Y_t using intermediate goods $Y_t(j)$. The production function is given by

$$Y_t = \left[\int_0^{A_{t-1}} Y_t(j)^{\frac{\phi-1}{\phi}} dj \right]^{\frac{\phi}{\phi-1}}, \phi > 1, \tag{1}$$

where A_{t-1} is the number of types of intermediate goods at time $t-1$, i.e., the number of intermediate goods blueprints in the economy at time $t-1$. A_{t-1} rather than A_t enters equation (1) because it is assumed that an intermediate goods firm that innovates a blueprint of goods at time $t-1$ can produce the goods only from time t onward (i.e., after the innovation, it takes one period to start production of goods). Another explanation is that for stock variables, the "stock at the end of the period" concept is used.

The maximization problem is shown by (the price of a final good is taken

[6] A detailed derivation of some of the equations that follow is provided in Model Appendix of the paper, which is available upon request.

as numeraire and is normalized to one)

$$\max_{Y_t(j)} \left[\int_0^{A_{t-1}} Y_t(j)^{\frac{\phi-1}{\phi}} di \right]^{\frac{\phi}{\phi-1}} - \int_0^{A_{t-1}} P_t(j) Y_t(j) dj.$$

2.1.2 Intermediate goods firms' goods production decisions

The innovator of intermediate good j's blueprint retains a monopoly right over the production and sales of $Y(j)$. Firm j has the following production function:

$$Y_t(j) = T_t K_{t-1}(j)^{\theta} H_t(j)^{1-\theta} \tag{2}$$

where K is capital (K_{t-1} denotes capital stock at the end of period $t-1$), H is the labor input (number of (quality-adjusted) workers times hours worked, i.e., $H = hN$, where h is hours worked and N is the number of (quality-adjusted) workers), and T denotes the level of technology. T represents knowledge such as basic scientific knowledge, social knowledge, and other kinds of knowledge or technology that are freely available to firms in a country. T is unrelated to R&D. In contrast, as is shown later, A is determined by costly R&D process. I call T "general" technology and A "applied" technology.

General technology T is given by

$$T_t = (\kappa N_t)^{\beta} G_t, \; 0 < \kappa, \; 0 < \beta < 1. \tag{3}$$

Equation (3) shows that T consists of two components: κN and G. The first component is related to human capital. It is assumed that there is a kind of technology (knowledge) that evolves in proportion to the population of a country.[7] This technology is freely available to all firms in a country and is shown by κN_t. The second component denoted by G is exogenous and is

[7] I assume that human capital (HC) is accumulated in the following way: $HC_t = \sum_{l=0}^{l=t} p \ (v \ \widetilde{N}_l)$, where $\widetilde{N}_l$ is the total quantity of workers in an economy at time l; v is the quality (knowledge) of a worker; and p shows a proportion of $v\widetilde{N}$ ($v\widetilde{N}$ = the aggregate knowledge of workers) stocked as economy-wide knowledge in every period. HC_t can then be rewritten as $HC_t = \kappa N_t$, where $\kappa \equiv p(1+n)/n$; n is the growth rate of $\widetilde{N}$; and $N_t = v\widetilde{N}_t$. For simplicity, I assume that $v = 1$. This assumption, however, does not affect the following simulation results because the analysis considers fluctuations of an economy from its trend.

assumed to be the same across firms and countries. The growth rate of G is constant at the rate of g_G (i.e., $G_{t+1}/G_t = 1 + g_G$).[8] Note that I include κN_t (human capital externality) to make N appear in the R&D cost specification shown later. As shown later, this approach can guarantee the existence of a balanced growth path, i.e., a steady state (in the following, I use the words "balanced growth path (BGP)" and "steady state" interchangeably).

Firm j, which sets a price of $Y(j)$ facing the demand curve (i.e., the first order condition of the final goods firms), chooses the price $P_{t+l}(j)$ to maximize

$$\sum_{l=0}^{\infty} Q_{t,t+l}^{-1}\psi^l \left[P_{t+l}(j)(Y_{t+l}P_{t+i}(j)^{-\phi}) - r_{t+l}K_{t-1+l}(j) - w_{t+l}H_{t+l}(j)\right]$$
$$\text{subject to } \; Y_{t+l}P_{t+l}(j)^{-\phi} = T_{t+l}K_{t-1+l}(j)^{\theta}H_{t+l}(j)^{1-\theta} \quad , \tag{4}$$

where $Q_{t,t+l}$ is a discount factor. By solving the cost minimization problem, one can obtain the following marginal cost function:

$$MC_{t+l} = \theta^{-\theta}(1-\theta)^{\theta-1}\frac{1}{T_{t+l}}r_{t+l}^{\theta}w_{t+l}^{1-\theta}. \tag{5}$$

It turns out that the marginal cost MC is the same across all price-setting intermediate goods firms. By solving the maximization problem, one can obtain the following equation for the present value of the firm's monopoly profit stream at time t, Π_t:

$$\Pi_t = \sum_{l=0}^{\infty} Q_{t,t+l}^{-1}\psi^l Y_{t+l}MC_{t+l}^{1-\phi}\left(\frac{\phi}{\phi-1}\right)^{-\phi}\left(\frac{\phi}{\phi-1}-1\right). \tag{6}$$

2.1.3 Intermediate goods firms' R&D decisions

Intermediate goods firms need a product blueprint (i.e., applied technology) to produce new intermediate goods. To innovate such a new product blueprint,

[8] The rate, g_G, is considered to be something like the growth rate of TFP in the pre-industrial period (I assume that $T_t = G_t$ in the pre-industrial period, i.e., no human capital externality in that period). Hansen and Prescott (2002) and Parente and Prescott (2004) calibrate this pre-industrial period growth rate and I, later, use their calibrated value to simulate the model.

intermediate goods firms borrow money from households and invest in R&D. I assume "time-to-innovate," i.e., a "time-to-build" type of structure for R&D. In other words, I assume that a firm needs to consecutively invest in R&D to create a new product blueprint.[9] An intermediate goods firm that starts innovating a new blueprint at time t needs, in total, ${}_tRDC$ units of final goods to produce the blueprint and the cost is assumed to be the same across firms. Firms take the cost as given. Denoting the number of periods needed to complete the R&D process by $\overline{\varphi}$, the total R&D cost of a firm that starts its R&D at time t is given by

$$ {}_tRDC = \sum_{\varphi=1}^{\overline{\varphi}} ({}_t\lambda_\varphi \, \eta_\varphi) \text{ where } \sum\nolimits_{\varphi=1}^{\overline{\varphi}} \eta_\varphi = 1. \tag{7} $$

The term ${}_t\lambda_\varphi \, \eta_\varphi$ represents the cost at each R&D stage for a firm that commences R&D at time t and needs φ further periods before completing the R&D process. The term η_φ measures the relative importance of each R&D stage (subscript φ indicates that there are φ further periods before the R&D process is complete). ${}_t\lambda_\varphi$ is assumed to take the following form:

$$ \begin{aligned} {}_t\lambda_\varphi &= d\frac{\left[(\kappa N_{t+(\overline{\varphi}-\varphi)})^{\beta}\right]^{\gamma_N} G_{t+(\overline{\varphi}-\varphi)}^{\gamma_G} F_t^{\alpha}}{V_{t-1+(\overline{\varphi}-\varphi)}^{\gamma_V} - A_{t-1+(\overline{\varphi}-\varphi)}^{\gamma_V}}, \\ 0 &< d, 0 < \gamma_V, 0 < \alpha < 1 \text{ and } \varphi = 1, 2, ..\overline{\varphi}, \end{aligned} \tag{8} $$

where d is a scaling parameter; F_t is the number of firms searching for a new product blueprint; A_t is the number of intermediate-goods blueprints in the lagging country; and V_t is the stock of diffused technologies (i.e., intermediate-goods blueprints) from the leading country. Denoting $A_{L,t}$ and $(1-\psi_L)$ as the number of the leading country's applied technologies (product blueprints) and the technology depreciation rate in the leading country, respectively, and

[9] Kydland and Prescott (1982) apply the "time-to-build" structure to model capital development. Okada (2018) introduces the "time-to-innovate" structure for R&D investment into a new Keynesian DSGE model and examines its effect on inflation dynamics.

denoting χ as the probability that an applied technology of the leading country diffuses to the lagging country in any given period, the stock of diffused technologies, V_t, can be expressed by (subscript L denotes the leading country)

$$V_t = \chi \left[A_{L,t} + \psi_L (1-\chi) A_{L,t-1} + \psi_L^2 (1-\chi)^2 A_{L,t-2} + ... + \psi_L^m (1-\chi)^m A_{L,t-m} \right], \tag{9}$$

where

$$0 < \chi < 1,\ 0 < \psi < 1 \text{ and } m = \infty.$$

Appendix A-1 gives the derivation of equation (9).[10] It is assumed that the lagging country does not fall behind or advance beyond the applied technology frontier in the long run (at the steady state) so that $A_t/A_{L,t}$ is constant at the steady state.[11]

Equation (8) shows several assumptions about R&D in the lagging country. The first and most important assumption is "*IBL from abroad.*" That is, intermediate goods firms in the lagging country learn from technologies created in the leading country (i.e., the lagging country firms benefit from backwardness) and create their own blueprints to produce goods suitable to their own country's environment. In other words, firms in the lagging country need to change, to a greater or a lesser extent, the leading country's product blueprints to make the goods fit the requirements of their domestic customers. Such differences arise because of differences in culture, institutions, and other structural factors. This effect of IBL from abroad is captured by the term $V^{\gamma_V} - A^{\gamma_V}$, which shows that the R&D cost (λ) increases with a decrease in the gap between the stock of diffused technologies and the lagging country's applied technology level. Because firms in the lagging country can learn only from the unlearnt subset of V and ideas that are easier to learn are learned first, the cost increases as the gap decreases.[12]

The next assumption about the lagging country's R&D is that the gen-

[10] $1/\chi$ measures the average time for the leading country's new applied technology to diffuse to the lagging country.

[11] Cordoba and Ripoll (2008) make a similar assumption in their calibration to study cross-country differences in income per worker.

[12] Barro and Sala-i-Martin (1997) make a similar assumption.

eral technology level ($T_t = (\kappa N_t)^{\beta} G_t$) has an effect on R&D costs. The term $\left[(\kappa N_{,t})^{\beta}\right]^{\gamma_N} G_t^{\gamma_G}$ captures this effect, which can be positive or negative. On one hand, general technologies (T_t) might help firms to create applied technologies (A_t), but on the other hand, more advanced general technologies might make it harder for firms to innovate a new applied technology because applied technologies are built on general technologies which get more complicated and sophisticated as they advance. I allow $(\kappa N_t)^{\beta}$ and G to have different effects on the cost, i.e., γ_N can be different from γ_G.

The third assumption is that the R&D cost depends on the number of firms searching for new ideas. When more firms engage in R&D, some of the ideas created by individual firms are less likely to be new to an economy. Thus, an increase in the number of R&D-conducting firms makes it harder for individual firms to find a new idea. I call this the congestion effect. The term F_t^{α} with $0 < \alpha < 1$ in equation (8) captures the effect.

As for the leading country, the stage-dependent R&D cost of innovation (counterpart of ${}_t\lambda_{\varphi}$) is assumed to take the following from:

$$ {}_t\lambda_{L,\varphi} = d_L \left[(\kappa_L \; N_{L,\; t+(\overline{\varphi}-\varphi)})^{\beta_L}\right]^{\gamma_{N_L}} G_{t+(\overline{\varphi}-\varphi)}^{\gamma_{G_L}} \; F_{L,t}^{\alpha_L}. \tag{10} $$

Apart from the absence of the IBL effect ($V^{\gamma_V} - A^{\gamma_V}$), the leading country's innovation cost takes the same form as that of the lagging country.

Once an intermediate goods firm creates a product blueprint for new goods, it obtains a monopoly right over the production of those goods. A constant success-probability of R&D is assumed and denoted by ϵ. Free entry into R&D is also assumed. That is, any firm can pay λ to secure the monopoly profit of $\epsilon\Pi$. In equilibrium, free entry into the blueprint production must thus guarantee

$$ \sum_{\varphi=1}^{\overline{\varphi}} \left(\prod_{j=0}^{\overline{\varphi}-1} (1 + q_{t+\overline{\varphi}-j}) \right) (\eta_{\varphi} \; {}_t\lambda_{\varphi}) = \epsilon \Pi_{t+\overline{\varphi}} \tag{11} $$

where q is the interest rate on loans, and Π_t is given by equation (6). Firms take q and ϵ as given.

Using equation (11) with equation (8) and denoting $\pi_t = \frac{\Pi_t}{N_t}$, in the case of the lagging country, one can then obtain

$$\epsilon\pi_{t+\overline{\varphi}} = \frac{d}{(1+n)^{\overline{\varphi}}}\kappa^{\beta\gamma_N}N_t^{\beta\gamma_N-1}G_t^{\gamma_G}F_t^{\alpha} \left[\sum_{\varphi=1}^{\overline{\varphi}}\left(\begin{array}{c}(1+n)^{(\overline{\varphi}-\varphi)\beta\gamma_N}(1+g_G)^{(\overline{\varphi}-\varphi)\gamma_G}\left(\prod_{j=0}^{\varphi-1}(1+q_{t+\overline{\varphi}-j})\right)\\ \eta_{\varphi}\frac{1}{V_{t-1+(\overline{\varphi}-\varphi)}^{\gamma_V}-A_{t-1+(\overline{\varphi}-\varphi)}^{\gamma_V}}\end{array}\right)\right] \tag{12}$$

Similarly, in the case of the leading country,

$$\epsilon_L\pi_{L,t+\overline{\varphi}} = \frac{d_L}{(1+n_L)^{\overline{\varphi}}}\kappa^{\beta_L\ \gamma_{N_L}}N_{L,t}^{\beta_L\ \gamma_{N_L}-1}G_t^{\gamma_{G_L}}F_{L,t}^{\alpha_L} \left[\sum_{\varphi=1}^{\overline{\varphi}}\left(\begin{array}{c}(1+n_L)^{(\overline{\varphi}-\varphi)\beta_L\ \gamma_{N_L}}(1+g_G)^{(\overline{\varphi}-\varphi)\gamma_{G_L}}\\ \left(\prod_{j=0}^{\varphi-1}(1+q_{L,t+\overline{\varphi}-j})\right)\eta_{L,\varphi}\end{array}\right)\right]. \tag{13}$$

2.2 Households

The economy has a continuum mass of homogeneous households indexed by $i \in [0,1]$ and household i maximizes

$$\sum_{t=0}^{\infty}\Gamma^t N_{i,t}\left[\ln\left(\frac{C_{i,t}}{N_{i,t}}\right)+D\frac{H_{i,t}}{N_{i,t}}\right], 0<\Omega<1,\ D<0$$

s.t.

$$C_{i,t}+K_{i,t}-(1-\delta)K_{i,t-1}+B_{i.t}\leq w_tH_{i,t}+r_tK_{i,t-1}+(1+q_t)B_{i,t-1}+\Xi_{i,t}$$

where Γ is a discount factor, N_i is the number of household i's members (growth rate is exogenously given by n), C_i is household i's consumption, H_i is household i's labor inputs, K_i is household i's capital stock, $B_{i,t}$ is household i's one-period loan to intermediate goods firms (the loan is made at time t and given back at time $t+1$), $\Xi_{i,t}$ is household i's gains or losses from holding shares of intermediate goods firms in period t, and $D \equiv \frac{\vartheta\ln(1-\overline{h}_i)}{\overline{h}_i}$ where $\vartheta(>0)$

is a preference parameter for leisure and $\overline{h}$ $(0 < \overline{h} < 1)$ is indivisible labor.[13] Note here that the economy's total R&D investment, i.e., the economy's total amount of loans, is given by $B_t = \int_0^1 B_{t,i}\, di = B_t$.

Loans to intermediate goods firms are rolled over until the firms complete R&D. When firms that have rolled over their loans during their R&D complete their R&D, they repay all of their rolled-over loans by issuing shares. This implies that as a buyer of shares, on the one hand, a household invests an amount equivalent to the loan payment, whereas as an owner of the firm, on the other hand, a household disinvests the same amount (i.e., loses the firm's assets due to the loan payment). Because these two transactions cancel each other out, they are not shown in the budget constraint above. Also, because firms need to consecutively invest in R&D, they borrow money at each stage of R&D, and their (one-period) loans are rolled over. B_t consists of both initial and rolled-over loans. Household i as an owner of an intermediate goods firm gains or loses due to changes in the firm' value over time.[14] These gains or losses are shown by $\Xi_{t,i}$.

Note here that because firms are owned by households, a discount factor used in the problem for an intermediate goods firm is obtained from the solution to the household problem and is given (see Model Appendix)

$$Q_{t,t+1} = \Gamma^{-1}\frac{c_{i,t+1}}{c_{i,t}} = 1 + q_{t+1} = 1 + r_{t+1} - \delta,$$

where $c_{i,t} = C_{i,t}/N_{i,t}$. This shows a one-period gross interest rate on financial assets.

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[13] Each member of household i signs a contract with a firm to provide $\overline{h}$ units of labor in period t with probability $\frac{H_{t,i}/N_{t,i}}{\overline{h}_i}$. They recieve the same wages whether or not they work. This kind of labor contract (unemployment insurance contract) leads to the household utility shown above. See Hansen's indivisible-labor model (1986) and McCandless (2008) for the details.

[14] A change in the value of an intermediate goods firm is $\Pi_{t+1} - \Pi_t$ if the firm still exists in the market at time $t + 1$ and is $-\Pi_t$ if it is driven out of the market due to product obsolescence.

2.2.1 Aggregate Dynamics

By combining the optimization conditions and constraints with the equilibrium conditions, one can obtain the following system of equations describing the dynamics of the aggregate economy for the lagging country (see Model Appendix for the derivation):

$$c_t = -\frac{w_t}{D} \ , \tag{14}$$

$$c_{t+1} = \Gamma(1 + r_{t+1} - \delta)c_t \ , \tag{15}$$

$$c_t + k_t + rd_t = y_t + \frac{1-\delta}{1+n}k_{t-1} \ , \tag{16}$$

$$\frac{1-\theta}{\theta}r_t k_{t-1} = (1+n)w_t h_t, \tag{17}$$

$$y_t = \left(\frac{1}{1+n}\right)^{\theta} A_{t-1}^{\frac{1}{\phi-1}} T_t k_{t-1}^{\theta} h_t^{1-\theta} \ , \tag{18}$$

$$A_{t-1}^{\frac{1}{\phi-1}} = \frac{\phi}{\phi-1}\theta^{-\theta}(1-\theta)^{\theta-1}\frac{1}{T_t}r_t^{\theta}w_t^{1-\theta} \ , \tag{19}$$

$$A_t = \epsilon F_{t-\overline{\varphi}+1} + \psi A_{t-1} \ , \tag{20}$$

$$\pi_{t+1} = \frac{1+r_{t+1}-\delta}{\psi(1+n)}\left[\pi_t - \frac{1+g_{A_1}}{\phi}y_t \ A_{t-1}^{-1}\right], \tag{21}$$

$$q_t = r_t - \delta, \tag{22}$$

$$V_t = \chi \left[\begin{array}{l} A_{L,t} + \psi_L(1-\chi)A_{L,t-1} + \psi_L^2(1-\chi)^2 A_{L,t-2} \\ +.. + \psi_L^m(1-\chi)^m A_{L,t-m} \end{array} \right] \tag{23}$$

$$rd_t = d\kappa^{\beta\gamma_N} N_t^{\beta\gamma_N - 1} G_t^{\gamma_G} \frac{1}{V_{t-1}^{\gamma_V} - A_{t-1}^{\gamma_V}} \left[\sum_{\varphi=1}^{\overline{\varphi}} \eta_\varphi F_{t-(\overline{\varphi}-\varphi)}^{1+\alpha}\right], \tag{24}$$

$$\begin{aligned} \epsilon\pi_{t+\overline{\varphi}} = {} & d\kappa^{\beta\gamma_N} N_t^{\beta\gamma_N-1} G_t^{\gamma_G} F_t^{\alpha} \\ & \left[\sum_{\varphi=1}^{\overline{\overline{\varphi}}} \left(\begin{array}{c} (1+n)^{(\overline{\varphi}-\varphi)\beta\gamma_N - \overline{\varphi}}(1+g_G)^{(\overline{\varphi}-\varphi)\gamma_G} \\ \left(\prod_{j=0}^{\varphi-1}(1+q_{t+\overline{\varphi}-j})\right) \eta_\varphi \frac{1}{V_{t-1+(\overline{\varphi}-\varphi)}^{\gamma_V} - A_{t-1+(\overline{\varphi}-\varphi)}^{\gamma_V}} \end{array} \right)\right], \end{aligned} \tag{25}$$

where $y_t \equiv Y_t/N$, $c_t \equiv C_t/N_t$, $k_t \equiv K_t/N_t$, $b_t \equiv B_t/N_t$, $h_t \equiv H_t/N_t$, and $rd_t \equiv RD_t/N_t$. For the leading country, $rd_{L,t}$ and $\pi_{L,t+\overline{\varphi}}$ are given by

$$rd_{L,t} = d_L \kappa_L^{\beta_L \ \gamma_{N_L}} N_{L,t}^{\beta_L \ \gamma_{N_L}-1} G_t^{\gamma_{G_L}} \left[\sum_{\varphi=1}^{\overline{\varphi}} \eta_{L,\varphi} F_{L,t-(\overline{\varphi}-\varphi)}^{1+\alpha_L} \right], \tag{26}$$

$$\begin{aligned} \epsilon_L \pi_{L,t+\overline{\varphi}} &= d_L \kappa_L^{\beta_L \ \gamma_{N_L}} N_{L,t}^{\beta_L \ \gamma_{N_L}-1} G_t^{\gamma_{G_L}} F_{L,t}^{\alpha_L} \\ & \left[\sum_{\varphi=1}^{\overline{\varphi}} \left(\begin{array}{c} (1+n_L)^{(\overline{\varphi}-\varphi)\beta_L \ \gamma_{N\_L}-\overline{\varphi}} \ (1+g_G)^{(\overline{\varphi}-\varphi)\gamma_{G\_L}} \\ \left(\prod_{j=0}^{\varphi-1} (1+q_{L,t+\overline{\varphi}-j}) \right) \eta_{L,\varphi} \end{array} \right) \right]. \end{aligned} \tag{27}$$

The condition for the lagging country to have a BGP is given by

$$\gamma_G = \frac{1}{1-\theta}, \ \gamma_N = \frac{\beta+1-\theta}{\beta(1-\theta)} \text{ and } \ \gamma_V = \frac{(1+\alpha)(\phi-1)(1-\theta)-1}{(\phi-1)(1-\theta)}.$$

Note that these equations show the condition for the existence of a BGP for any exogenous growth rates of $g_{A_L^*}$, n and g_G, and they are used to pin down parameter values for the later simulation. Using conventional values of α, ϕ and θ, the condition ensures the assumption of $\gamma_v > 0$. In addition, the BGP condition for the leading country is given by

$$(1+g_{A_L^*}) = (1+g_G)^{\frac{(\phi_L-1)\left[\gamma_{G_L}(1-\theta_L) \ - \ 1\right]}{1-(\phi_L-1)(1-\theta_L)(1+\alpha_L)}} (1+n_L)^{\frac{(\phi_L-1)\left[(1-\theta_L)(\beta_L \ \gamma_{N_L}-1)-\beta_L\right]}{1-(\phi_L-1)(1-\theta_L)(1+\alpha_L)}}, \tag{28}$$

where $g_{A_L^*}$ is the BGP growth rate of $A_{L,t}$. As in Jones (1995), the BGP growth rate depends on population (workers) growth rate. The derivation of the above BGP conditions for the lagging and leading countries are shown in Appendix B-1.

2.2.2 Detrending

The system of aggregate equations is detrended. The detrending (i.e., stationalizing) is needed in order to solve the model. It also helps to calibrate the model because I later use the parameterized steady-state values of the detrended variables to pin down several of the model's parameters.

First, define the leading country's applied technology level at time t, $A_{L,t}$, as

$$A_{L,t} \equiv A^*_{L,t} \, \widetilde{A}_{L,t}, \tag{29}$$

where $A^*_{L,t}$ is the BGP value of $A_{L,t}$ and $\widetilde{A}_{L,t}$ is the cyclical component of $A_{L,t}$. The BGP value of $A_{L,t}$ is given by $A^*_{L,t+1} = (1 + g_{A^*_L}) \, A^*_{L,t}$ where $g_{A^*_L}$ is constant and semi-endogenously determined by equation (28). The mean of $\widetilde{A}_{L,t}$ is 1. Next, define the deviation of A_t (the lagging country's A) from $A^*_{L,t}$, $\widetilde{A}_t$, as

$$\widetilde{A}_t \equiv \frac{A_t}{A^*_{L,t}}. \tag{30}$$

It is assumed that even in the long run A_t does not catch up with $A_{L,t}$ (note that $\widetilde{A}_t (= A_t/A^*_{L,t})$ is constant at the steady state because it has been assumed that the lagging country does not fall behind or advance beyond the applied technology frontier at the steady state). That is, $A_t/A_{L,t} < 1$. This implies $\widetilde{A}_t < \widetilde{A}_{L,t}$. Finally, define a trend variable Z_t as

$$Z_t \equiv A^{*\frac{1}{(\phi-1)(1-\theta)}}_{L,t} T_t^{\frac{1}{1-\theta}}. \tag{31}$$

Because $T_t = (\kappa N_t)^{\beta} G_t$ (see equation 3) and $A^*_{L,t+1} = (1 + g_{A^*_L}) \, A^*_{L,t}$, from equation (31) the gross growth rate of Z_t is given by

$$1 + g_Z = (1 + g_{A^*_L})^{\frac{1}{(\phi-1)(1-\theta)}} (1 + g_G)^{\frac{1}{1-\theta}} (1 + n)^{\frac{\beta}{1-\theta}}.$$

The detrended variables are then defined by

$$\begin{aligned}\widetilde{w}_t &\equiv \frac{w_t}{Z_t},\ \widetilde{c}_t \equiv \frac{c_t}{Z_t},\ \widetilde{y}_t \equiv \frac{y_t}{Z_t},\ \widetilde{k}_t \equiv \frac{k_t}{Z_t},\ \widetilde{rd}_t \equiv \frac{rd_t}{Z_t},\\ \widetilde{\pi}_t &\equiv \frac{\pi_t A^*_{L,t}}{Z_t}, \widetilde{F}_t \equiv \frac{F_t}{A^*_{L,t}},\ \widetilde{V}_t \equiv \frac{V_t}{A^*_{L,t}},\ \text{ and } \widetilde{A}_t \equiv \frac{A_t}{A^*_{L,t}}.\end{aligned}$$

Similarly, the detrended variables for the leading country are defined by

$$\begin{aligned}\widetilde{w}_{L,t} &\equiv \frac{w_{L,t}}{Z_{L,t}},\ \widetilde{c}_{L,t} \equiv \frac{c_{L,t}}{Z_{L,t}},\ \widetilde{y}_{L,t} \equiv \frac{y_{L,t}}{Z_{L,t}},\ \widetilde{k}_{L,t} \equiv \frac{k_{L,t}}{Z_{L,t}},\ \widetilde{rd}_{L,t} \equiv \frac{rd_{L,t}}{Z_{L,t}},\\ \widetilde{\pi}_{L,t} &\equiv \frac{\pi_{L,t} A^*_{L,t}}{Z_{L,t}},\ \widetilde{F}_{L,t} \equiv \frac{F_{L,t}}{A^*_{L,t}} \text{ and } \widetilde{A}_{L,t} \equiv \frac{A_{L,t}}{A^*_{L,t}}\end{aligned}$$

where

$$Z_{L,t} \equiv A_{L,t}^{*\frac{1}{(\phi_L-1)(1-\theta_L)}} T_{L,t}^{\frac{1}{1-\theta_L}} = A_{L,t}^{*\frac{1}{(\phi_L-1)(1-\theta_L)}} \left[(\kappa_L N_{L,t})^{\beta_L} G_t\right]^{\frac{1}{1-\theta_L}} .$$

and

$$1+g_{Z_L} = (1+g_{A^*_L})^{\frac{1}{(\phi_L-1)(1-\theta_L)}} (1+g_G)^{\frac{1}{1-\theta_L}} (1+n_L)^{\frac{\beta_L}{1-\theta_L}}.$$

Using the above expressions, the detrended system is given by the following equations:

$$\widetilde{c}_t = -\frac{\widetilde{w}_t}{D}, \tag{32}$$

$$(1+g_Z)\widetilde{c}_{t+1} = \Gamma(1+r_{t+1}-\delta)\widetilde{c}_t\ , \tag{33}$$

$$\widetilde{c}_t + \widetilde{k}_t + \widetilde{rd}_t = \widetilde{y}_t + \frac{1-\delta}{(1+n)(1+g_Z)}\widetilde{k}_{t-1}\ , \tag{34}$$

$$\frac{1-\theta}{\theta}\frac{1}{1+g_Z} r_t \widetilde{k}_{t-1} = (1+n)\widetilde{w}_t h_t, \tag{35}$$

$$\widetilde{y}_t = \left[\frac{1}{(1+n)(1+g_Z)}\right]^{\theta} (1+g_{A^*_1})^{\frac{-1}{\phi-1}} \widetilde{A}_{t-1}^{\frac{1}{\phi-1}} \widetilde{k}_{t-1}^{\theta} h_t^{1-\theta}, \tag{36}$$

$$\widetilde{A}_{t-1}^{\frac{1}{\phi-1}} = \left(\frac{1}{1+g_{A^*_L}}\right)^{\frac{-1}{\phi-1}} \frac{\phi}{\phi-1}\theta^{-\theta}(1-\theta)^{\theta-1} r_t^{\theta} \widetilde{w}_t^{1-\theta}, \tag{37}$$

$$\widetilde{A}_t = \frac{\epsilon}{(1+g_{A_L^*})^3}\widetilde{F}_{t-\overline{\varphi}+1} + \frac{\psi}{(1+g_{A_L^*})}\widetilde{A}_{t-1}, \tag{38}$$

$$\widetilde{\pi}_{t+1} = \frac{1+r_{t+1}-\delta}{\psi(1+n)}\frac{1+g_{A_L^*}}{1+g_Z}\left[\widetilde{\pi}_t - \frac{1+g_{A_L^*}}{\phi}\frac{\widetilde{y}_t}{\widetilde{A}_{t-1}}\right], \tag{39}$$

$$q_t = r_t - \delta, \tag{40}$$

$$\widetilde{V}_t = \chi[\widetilde{A}_{L,t} + \frac{\psi(1-\chi)}{1+g_{A_L^*}}\widetilde{A}_{L,t-1} + \frac{\psi^2(1-\chi)^2}{(1+g_{A_L^*})^2}\widetilde{A}_{L,t-2} + ... + \frac{\psi^m(1-\chi)^m}{(1+g_{A_L^*})^m}\widetilde{A}_{L,t-m}], \tag{41}$$

$$\widetilde{rd}_t = \frac{\overline{d}(1+g_{A_L^*})^{\gamma_V}}{\left[\widetilde{V}_{t-1}\right]^{\gamma_V} - \left[\widetilde{A}_{t-1}\right]^{\gamma_V}}\left[\sum_{\varphi=1}^{\overline{\varphi}}\left(\eta_\varphi(1+g_{A_L^*})^{-(\overline{\varphi}-\varphi)(1+\alpha)}\widetilde{F}^{1+\alpha}_{t-(\overline{\varphi}-\varphi)}\right)\right], \tag{42}$$

$$\epsilon\widetilde{\pi}_{t+\overline{\varphi}} = \overline{d}(1+g_Z)^{-\overline{\varphi}}(1+g_{A_L^*})^{\overline{\varphi}}(1+n)^{-\overline{\varphi}}\left(\widetilde{F}_t\right)^\alpha$$
$$\left[\sum_{\varphi=1}^{\overline{\varphi}}\left\{\begin{array}{c}(1+g_{A_L^*})^{-(\overline{\varphi}-\varphi-1)\ \gamma_V}(1+n)^{(\overline{\varphi}-\varphi)\beta\gamma_N}(1+g_G)^{(\overline{\varphi}-\varphi)\gamma_G} \\ \left(\prod_{j=0}^{\varphi-1}(1+q_{t+\overline{\varphi}-j})\right)\eta_\varphi\frac{1}{\widetilde{V}^{\gamma_V}_{t-1+(\overline{\varphi}-\varphi)}-\widetilde{A}^{\gamma_V}_{t-1+(\overline{\varphi}-\varphi)}}\end{array}\right\}\right], \tag{43}$$

$$\widetilde{rd}_{L,t} = \overline{d}_L\left[\sum_{\varphi=1}^{\overline{\varphi}}\eta_{L,\varphi}(1+g_{A_L^*})^{-(\overline{\varphi}-\varphi)(1+\alpha_L)}\widetilde{F}^{1+\alpha_L}_{L,t-(\overline{\varphi}-\varphi)}\right], \tag{44}$$

$$\begin{aligned}\epsilon_L\widetilde{\pi}_{L,t+\overline{\varphi}} \;=\;& \overline{d}_L(1+g_{A_L^*})^{\overline{\varphi}\left(\frac{-1+(\phi_L-1)(1-\theta_L)}{(\phi_L-1)(1-\theta_L)}\right)} \\ & (1+n_L)^{\overline{\varphi}\left(\frac{-\beta_L}{1-\theta_L}-1\right)}(1+g_G)^{\overline{\varphi}\left(\frac{-1}{1-\theta_L}\right)} \\ & \widetilde{F}^{\alpha_L}_{L,t}\left[\sum_{\varphi=1}^{\overline{\varphi}}\left\{\begin{array}{c}(1+n_L)^{(\overline{\varphi}-\varphi)\beta_L\ \gamma_{N_L}}(1+g_G)^{(\overline{\varphi}-\varphi)\gamma_{G_L}} \\ \left(\prod_{j=0}^{\varphi-1}(1+q_{L,t+\overline{\varphi}-j})\right)\eta_{L,\varphi}\end{array}\right\}\right],\end{aligned} \tag{45}$$

where

$$\overline{d} = \kappa d,$$
$$\overline{d}_L = d_L(\kappa_L)^{\beta_L\ \gamma_{N_L}-\frac{\beta_L}{1-\theta_L}}\Delta,$$
$$1+g_Z = (1+g_{A_L^*})^{\frac{1}{(\phi-1)(1-\theta)}}(1+g_G)^{\frac{1}{1-\theta}}(1+n)^{\frac{\beta}{1-\theta}}, \tag{46}$$

$$1+g_{Z_L}=(1+g_{A_L^*})^{\frac{1}{(\phi_L-1)(1-\theta_L)}}(1+g_G)^{\frac{1}{1-\theta_L}}(1+n_L)^{\frac{\beta_L}{1-\theta_L}},$$

$$\gamma_G=\frac{1}{1-\theta}, \tag{47}$$

$$\gamma_N=\frac{\beta+1-\theta}{\beta(1-\theta)}, \tag{48}$$

$$\gamma_V=\frac{(1+\alpha)(\phi-1)(1-\theta)-1}{(\phi-1)(1-\theta)}, \tag{49}$$

$$\Delta\equiv\left(A_{L,t}^*\right)^{\frac{(\phi_L-1)(1-\theta_L)(1+\alpha_L)-1}{(\phi_L-1)(1-\theta_L)}}N_{L,t}^{\frac{(1-\theta_L)(\beta_L\ \gamma_{N_L}-1)-\beta_L}{1-\theta_L}}G_t^{\frac{(1-\theta_L)\gamma_{G_L}-1}{1-\theta_L}}. \tag{50}$$

Equations (47)-(49) together show the BGP condition, which has already been shown. Δ in equation (50) is constant when the leading country's BGP condition (28) is met. Equation (41) is derived in Appendix 1.

As shown by equations (32)-(45), the lagging country's economy is described by a set of detrended endogenous variables, $\widetilde{c}_t$, $\widetilde{w}_t$, r_t, $\widetilde{k}_t$, $\widetilde{y}_t$, $\widetilde{rd}_t$, h_t, $\widetilde{\pi}_t$, $\widetilde{A}_t, \widetilde{F}_t, \widetilde{V}_t$, and $\widetilde{A}_{L,t}$. To be importantly noted, $\widetilde{A}_{L,t}$ is totally independent from any economic activity in the lagging country and is determined by the *leading country's* R&D process. The equation for $\widetilde{A}_{L,t}$ is shown later in the context of simulation.

3 Calibration and Computation

This section first shows the calibration strategy, which uses the parameterized steady state values to pin down several parameters. The simulation results are then presented. The quantitative analysis below assumes that the U.S. is a technologically leading country and Japan is a technologically lagging country. The time frequency is annual rather than quarterly because reliable data on Japanese R&D are only available annually and the present paper aims to study fluctuations over a longer time horizon than those analyzed by conventional business cycle studies. The data sample period is 1963-2010.[15] The data are described in Appendix A-2. The required time for R&D process is assumed

[15] The sample period for U.S. R&D and applied technology level ($\widetilde{A}_{L,t}$) starts from 1960.

to take 4 years (i.e., $\overline{\varphi} = 4$). This is consistent with the finding of Griffin (2002). Based on survey data from 116 U.S. firms, Griffin (2002) finds that industrial firms, on average, require approximately 53 months to develop a new-to-the-world product.

3.1 Leading country: $\widetilde{A}_{L,t}$ and $\widetilde{V}_t$

To simulate the model for the lagging country, $\widetilde{V}_t$ is needed. To obtain $\widetilde{V}_t$, $\widetilde{A}_{L,t}$ is, in turn, needed as equation (41) shows.[16] To get $\widetilde{A}_{L,t}$, I calibrate parameters for the leading country model and estimate (calculate) $\widetilde{A}_{L,t}$ using U.S. R&D data (as already noted, the exogenous variable in the following simulation is U.S. R&D). $\widetilde{V}_t$ is then obtained using the best fitted $\widetilde{A}_{L,t}$ to the data on $\widetilde{A}_{L,t}$.

The procedure to obtain best fitted $\widetilde{A}_{L,t}$ is shown as follows. As shown in Appendix 6, the dynamics of $\widetilde{A}_{L,t}$ are given by

$$\widetilde{A}_{L,t+3} - \frac{\psi_L}{(1+g_{A_L^*})}\widetilde{A}_{L,t+2}$$
$$= \left[\begin{array}{c} (\overline{d}_L)^{-1}\ \varepsilon_L^{(1+\alpha_L)}(1+g_{A_L^*})^{-3(1+\alpha_L)}\rho_L^{-3}\eta_{L,1}^{-1} \\ \left\{\widetilde{rd}_{L,t} - (1+g_{A_L^*})^{-(1+\alpha_L)}\rho_L^{-1}\widetilde{rd}_{L,t-1}\right\} \\ +(1+g_{A_L^*})^{-4(1+\alpha_L)}\rho_L^{-4}\left\{\widetilde{A}_{L,t-1} - \frac{\psi_L}{(1+g_{A_L^*})}\widetilde{A}_{L,t-2}\right\}^{1+\alpha_L} \end{array}\right]^{\frac{1}{1+\alpha L}} . \quad (51)$$

By feeding data on $\widetilde{rd}_{L,t}$ into this equation, $\widetilde{A}_{L,t}$ is simulated. More specifically, to simulate $\widetilde{A}_L$ starting from *time t onwards*, I use actual data on $\widetilde{A}_{L,t-5}$, $\widetilde{A}_{L,t-4}$, $\widetilde{A}_{L,t-3}$, $\widetilde{A}_{L,t-2}$ and $\widetilde{A}_{L,t-1}$, and data on $\widetilde{rd}_L$ ($\widetilde{rd}_{L,t-4}, \widetilde{rd}_{L,t-3}, \ldots.\widetilde{rd}_{L,T}$). Note here that *constrained* regressions are used to obtain the detrended data variables, $\widetilde{A}_{L,t-5}$, $\widetilde{A}_{L,t-4}$, $\widetilde{A}_{L,t-3}$, $\widetilde{A}_{L,t-2}$, $\widetilde{A}_{L,t-1}$ and $\widetilde{rd}_L$ ($\widetilde{rd}_{L,t-4}, \widetilde{rd}_{L,t-3}, \ldots.\widetilde{rd}_{L,T}$). That is, I regress the log of the $A_{L,t}$ ($rd_{L,t}$) data on a log-linear time trend with a growth rate of $g_{A_L^*}$ (g_{z_L}) and estimate a constant term ($g_{A_L^*}$ and g_{z_L} are calibrated as shown later).[17] I then use the estimated constant term with $g_{A_L^*}$

[16] I obtain $\widetilde{V}_t$ by setting m in equation (41) to 3 because only a limited amount of data on $rd_{L,t}$ is available. To calculate $\widetilde{V}^*$, m is set to $1,000$.

[17] I use $TFP_{L,t} = A_{L,t-1}^{\frac{1}{\phi-1}}(\kappa_L N_{L,t})^{\beta} G_t$ to obtain $A_{L,t}$ data. That is, with β , ϕ, g_G and

(g_{z_L}) in order to obtain $A^*_{L,t}$ ($Z_{L,t}$), which I use to construct data on $\widetilde{A}_{L,t-5}$, $\widetilde{A}_{L,t-4}$, $\widetilde{A}_{L,t-3}$, $\widetilde{A}_{L,t-2}$ and $\widetilde{A}_{L,t-1}$ ($\widetilde{rd}_{L,t}$).

Next, the method used to assign values to the parameters in equation (51) is shown. For simplicity, assume

$$\eta_{L,\phi} = \eta_{L,1}\rho_L^{\varphi-1}, \ 0 < \rho_1. \tag{52}$$

This assumption implies that $\eta_{L,4} < \eta_{L,3} < \eta_{L,2} < \eta_{L,1}$ when $\rho_L < 1$ and $\eta_{L,4} > \eta_{L,2} > \eta_{L,3} > \eta_{L,1}$ when $\rho_L > 1$ (remember that $\overline{\varphi} = 4$ is assumed). By analyzing the steady state (see Appendix B-2), one can then obtain the following equations

$$\frac{1}{D_L}\Theta_{L,5} = \epsilon_L^{-1-\alpha_L}\overline{d}_L \left(\begin{array}{c} \phi_L \frac{\Theta_{L,1}\Theta_{L,2}}{\Theta_{L,3}} \Theta_{L,4}^{\alpha_L}\Theta_{L,5}^{1-\theta_L}\Theta_{L,6}\Theta_{L,8} \\ -\Theta_{L,4}^{\alpha_L}\left(\phi_L \frac{\Theta_{L,1}\Theta_{L,2}}{\Theta_{L,3}} - \Theta_{L,4}\Theta_{L,7}\right) \end{array} \right) \tag{53}$$

$$\eta_{L,1}(1+\rho_L+\rho_L^2+\rho_L^3) = 1, \tag{54}$$

$$(1+g_{A_L^*}) = (1+g_G)^{\frac{(\phi_L-1)\left[\gamma_{G_L}(1-\theta_L)\ -\ 1\right]}{1-(\phi_L-1)(1-\theta_L)(1+\alpha_L)}}(1+n_L)^{\frac{(\phi_L-1)\left[(1-\theta_L)(\beta_L\gamma_{N_L}-1)-\beta_L\right]}{1-(\phi_L-1)(1-\theta_L)(1+\alpha_L)}}, \tag{55}$$

where

$$\Theta_{L,1} \equiv (1+g_{A_L^*})^{4\left(\frac{-1+(\phi_L-1)(1-\theta_L)}{(\phi_L-1)(1-\theta_L)}\right)}(1+n_L)^{\frac{-4\beta_L}{1-\theta_L}-4}(1+g_G)^{\frac{-4}{1-\theta_L}},$$

$$\Theta_{L,2} \equiv \eta_{L,1}\left[\begin{array}{c} (1+n_L)^{3\beta_L\ \gamma_{N_L}}(1+g_G)^{3\gamma_{G_L}}(1+r_L^*-\delta_L) \\ +(1+n_L)^{2\beta_L\ \gamma_{N_L}}(1+g_G)^{2\gamma_{G_L}}(1+r_L^*-\delta_1)^2\rho_L \\ +(1+n_L)^{\beta_L\ \gamma_{N_L}}(1+g_G)^{\gamma_{G_L}}(1+r_L^*-\delta_L)^3\rho_L^2 \\ +(1+r_L^*-\delta_L)^4\rho_L^3\ , \end{array} \right],$$

$$\Theta_{L,3} \equiv \frac{(1+r_L^*-\delta_L)(1+g_{A_L^*})^2}{(1+r_L^*-\delta_L)(1+g_{A_L^*})-\psi_L(1+n_L)(1+g_{Z_L})},\ \Theta_{L,4} \equiv \left[(1+g_{A_L^*})^2(1+g_{A_L^*}-\psi_L)\right],$$

data on TFP and workers, one can calculate $xA_{L,t}$ where x is a constant. The term x does not cause any problem in the simulation analysis because the aim is to measure deviations of $A_{L,t}$ form a trend, i.e., $\widetilde{A}_{L,t}$. Later, I will show how values are assigned to β, ϕ, and g_G.

$$\Theta_{L,5} \equiv (1+g_{A_L^*})^{\frac{-1}{(\phi_L-1)(1-\theta_L)}} \left[\frac{\phi_L}{\phi_L-1}\theta_L^{-\theta_L}(1-\theta_L)^{\theta_L-1}(r_L^*)^{\theta_L}\right]^{\frac{-1}{(1-\theta L)}},$$

$$\Theta_{L,6} \equiv (1+g_{A_L^*})^{\frac{1}{\phi_L-1}} \left[\frac{\theta_L}{1-\theta_L}\frac{1}{r_L^*}\right]^{-\theta_L}, \ \Theta_{L,7} \equiv \eta_{L,1} \begin{bmatrix} (1+g_{A_L^*})^{-3(1+\alpha_L)} \\ +\rho_L(1+g_{A_L^*})^{-2(1+\alpha_L)} \\ +\rho_L^2(1+g_{A_L^*})^{-(1+\alpha_L)} \\ +\rho_L^3 \end{bmatrix},$$

$$\Theta_{L,8} \equiv \left[1-\frac{1-\delta_L}{(1+n_L)(1+g_{Z_L})}\right](1+n_L)(1+g_{Z_L})\frac{\theta_L}{1-\theta_L}\frac{1}{r_L^*},$$

$$1+g_{Z_L} = (1+g_{A_L^*})^{\frac{1}{(\phi_L-1)(1-\theta_L)}}(1+g_G)^{\frac{1}{1-\theta_L}}(1+n_L)^{\frac{\beta_L}{1-\theta_L}}, \ r_L^* = \frac{1+g_{Z_L}}{\Gamma_L}-1+\delta_L.$$

Equations (53)-(55) are used to assign values to $\eta_{L,1}$, ρ_L, $\overline{d}_L$, γ_{G_L}, and γ_{N_L} where $\overline{d}_L \equiv d_L \kappa_L^{\frac{(1-\theta_L)\beta_L \ \gamma_{N_L}-\beta_L}{1-\theta_L}} \Delta$. As shown later, the other parameter values are set to be consistent with previous studies and the U.S. data.

The expressions above show that there are only three equations to determined the five unknown parameters of η_L, ρ_L, $\overline{d}_L$, γ_{G_L}, and γ_{N_L}. However, this does not cause a problem. The reasoning is as follows. First, as long as the steady state restriction, i.e., equation (55), holds, the choice of values for γ_{G_L}, and γ_{N_L} does not affect the simulation exercise. This is because with given values of η_L, ρ_L, any pair of γ_{G_L} and γ_{N_L} that satisfies equation (55) gives the same value of $\overline{d}_L$ (see $\Theta_{L,2}$ and equation b12 in Appendix B-1). Next, as shown by equation (51), what is really needed for the simulation exercise (i.e., the simulation of $\widetilde{A}_{L,t}$) is the values of $\eta_{L,1}$, ρ_L, and $\overline{d}_L$. Thus, ultimately, three unknowns, namely $\eta_{L,1}$, ρ_L, and $\overline{d}_L$, must be determined with the two equations, namely, equations (53) and (54). One more restriction is still needed to determine $\eta_{L,1}$, ρ_L, and $\overline{d}_L$. I thus use a data matching restriction shown below.

The data matching restriction procedure to determine $\eta_{L,1}$, ρ_L, and $\overline{d}_L$ is as follows. First, with an arbitrarily chosen pair (γ_{G_L}, γ_{N_L}) that satisfies equation (55), values of $\eta_{L,1}$ are chosen in the range of $(0,1)$ and values of ρ_L, and $\overline{d}_L$ corresponding to each of the chosen value of $\eta_{L,1}$ are calculated using equations (53) and (54). Next, using equation (51), $\widetilde{A}_{L,t}$ is simulated for each of the chosen values of $\eta_{L,1}$ with the corresponding values of ρ_L and $\overline{d}_L$. Then, the combination of $\eta_{L,1}$, ρ_L and $\overline{d}_L$ that gives the best fit to the data on $\widetilde{A}_{L,t}$ is chosen. The fit is judged by root-mean-square error (RMSE) with the data. Finally, based on equation (41), $\widetilde{V}_t$ is constructed using the best fitted $\widetilde{A}_{L,t}$.

3.2 Lagging country: $\widetilde{A}_t$

As with the case of the leading country, some parameters are determined by best fitting the model's $\widetilde{A}_t$ values to the data on $\widetilde{A}_t$ using the following procedure.

Analyzing the steady state for the lagging country yields the following equations (see Appendix B-3).

$$\begin{aligned}
&\frac{1}{D}\left(\Theta_5+\frac{\Theta_3\Theta_4}{\Theta_1\Theta_2}\Theta_6\Theta_7-\Theta_7\right)^{-1} \\
&= \left[\begin{array}{c} \epsilon^{-(1+\alpha)}\overline{d}\Theta_1\Theta_2\Theta_3^{-1}\Theta_7^{-1}\Theta_8\Theta_9^{\frac{1}{(\phi-1)(1-\theta)}} \\ \frac{1}{\widetilde{V}^{*\gamma_V}-\widetilde{A}^{*\gamma_V}}\widetilde{A}^{*\frac{-1}{(\phi-1)(1-\theta)}+(1+\alpha)} \end{array}\right],
\end{aligned} \tag{56}$$

$$\eta_1(1+\rho+\rho^2+\rho^3)=1, \tag{57}$$

where

$$\Theta_1 \equiv (1+g_Z)^{-4}(1+g_{A_L^*})^4(1+n)^{-4}\ ,$$

$$\Theta_2 \equiv \eta_1\left[\begin{array}{c} (1+g_{A_L^*})^{-2\gamma_V}(1+n)^{3\beta\gamma_N}(1+g_G)^{3\gamma_G}(1+r^*-\delta) \\ +(1+g_{A_L^*})^{-\gamma_V}(1+n)^{2\beta\gamma_N}(1+g_G)^{2\gamma_G}(1+r^*-\delta)^2\rho \\ +(1+n)^{\beta\gamma_N}(1+g_G)^{\gamma_G}(1+r^*-\delta)^3\rho^2 \\ +(1+g_{A_L^*})^{\gamma_V}(1+r^*-\delta)^4\rho^3\ , \end{array}\right]$$

$$\Theta_3 \equiv \frac{(1+r^*-\delta)(1+g_{A_1^*})^2}{(1+r^*-\delta)(1+g_{A_L^*})-\psi(1+n)(1+g_Z)}$$

$$\Theta_4 \equiv (1+g_{A_L^*})^{\gamma_V}\eta_1\left[\begin{array}{c} (1+g_{A_L^*})^{-3(1+\alpha)}+\rho(1+g_{A_L^*})^{-2(1+\alpha)} \\ +\rho^2(1+g_{A_L^*})^{-(1+\alpha)}+\rho^3 \end{array}\right],$$

$$\Theta_5 \equiv \left[1 - \frac{1-\delta}{(1+n)(1+g_Z)}\right](1+n)(1+g_Z)\frac{\theta}{1-\theta}\frac{1}{r^*},$$

$$\Theta_6 \equiv \frac{1}{\phi}(1+g_{A_L^*})^2(1+g_{A_L^*}-\psi)\ ,\ \Theta_7 \equiv \frac{\phi}{\phi-1}\theta^{-\theta}(1-\theta)^{\theta-1}\left(\frac{\theta}{1-\theta}\right)^{\theta},$$

$$\Theta_8 \equiv \phi\left[(1+g_{A_L^*})^2(1+g_{A_L^*}-\psi)\right]^{\alpha},\ \Theta_9 \equiv (1+g_{A_L^*})\left[\frac{\phi}{\phi-1}\theta^{-\theta}(1-\theta)^{\theta-1}(r^*)^{\theta}\right]^{\phi-1},$$

$$1+g_Z = (1+g_{A_L^*})^{\frac{1}{(\phi-1)(1-\theta)}}(1+g_G)^{\frac{1}{1-\theta}}(1+n)^{\frac{\beta}{1-\theta}},$$

$$r^* = \frac{1+g_Z}{\Gamma} - 1 + \delta,\ \widetilde{V}^* = \chi\left(\frac{1-\left(\frac{\psi(1-\chi)}{1+g_{A_L^*}}\right)^m}{1-\left(\frac{\psi(1-\chi)}{1+g_{A_L^*}}\right)}\right),$$

$$\gamma_G = \frac{1}{1-\theta},\ \gamma_N = \frac{\beta+1-\theta}{\beta(1-\theta)},\ \gamma_V = \frac{(1+\alpha)(\phi-1)(1-\theta)-1}{(\phi-1)(1-\theta)}.$$

Equations (56) and (57) are used to assign values to η_1, ρ and $\overline{d}$. As shown later, I assign values to the remaining parameters such that the values are consistent with previous studies and the Japanese data.

The expressions above show that three unknown parameters, η_1, ρ, and $\overline{d}$ are present with only two equations. To determine η_1, ρ, and $\overline{d}$, I use a procedure similar to that used in the case of the leading country. First, values of η_1 are taken in the range of $(0,1)$. Then, values of ρ, and $\overline{d}$ corresponding to each of the chosen values of η_1 are calculated using equations (56) and (57). Next, $\widetilde{A}_t$ is simulated by feeding U.S. R&D data into the model (i.e., feeding U.S. R&D data into equation (51) to obtain $\widetilde{V}_t$ (as shown above) and then feeding $\widetilde{V}_t$ into the lagging country model). I then choose the combination of η_1 , ρ, and $\overline{d}$ that gives the model's $\widetilde{A}_t$ the best fit to the data on $\widetilde{A}_t$.[18] As earlier, the fit is measured by RMSE.

[18] Both actual and simulated data are filtered.

3.3 Parameters

The parameters other than those discussed above are assigned values as follows and are summarized in Table 1.

$g_G, n, n_L, \phi, \phi_L, g_{A_L^*}, \theta$, and θ_L

A value of $g_G = 0.0009$ is used. This value is the one calibrated by Hansen and Prescott (2002) and Parente and Prescott (2004) for the technology growth rate of the pre-industrial period. For n_L and n, linear trend growth rates of U.S. and Japanese labor are used. I set $\phi = 4.33$ to match the gross (value-added) markup rate of 1.3.[19] In the literature on markups in value added data, estimates range from 1.2 and 1.4 (see, for example, Basu and Fernald 1997). Here, an intermediate value is chosen. In addition, ϕ is assumed to be the same in each of the two countries. For $g_{A_L^*}$, a value of 0.025 is chosen, which is close to the average growth rate of the stock of knowledge in the U.S. as calculated by Bottazzi and Peri (2007), who calculated the stock of knowledge for several countries using the number of patent applications. θ_L and θ are set to match average capital shares in U.S. and Japanese GDP, respectively, over the sample period.

$\alpha, \alpha_L, \beta, \beta_L, \psi$, and ψ_L

α, β and ψ are assumed to be the same in each of the two countries. From equations (3) and (18) one can obtain the equation $g^*_{TFP_L} = \frac{1}{\phi_L - 1} g_{A_L^*} + \beta_L n_L + g_G$ where the subscript TFP denotes total factor productivity (TFP, i.e., Solow residuals) and $g^*_{TFP_L}$ is the growth rate of TFP in the BGP. Using the parameters specified above and the equation for $g^*_{TFP_L}$, I calculate β ($= \beta_L$). To obtain β in this way, I use a trend growth rate of TFP for $g^*_{TFP_L}$; specifically, to obtain TFP, equation (18) is used. For α ($= \alpha_L$), a value of 0.25 is chosen. Because $1/(1+\alpha)$ measures elasticity of innovation (i.e., new applied technology creation) to R&D in the BGP (see Appendix B-4), the chosen value of α implies that the elasticity equals 0.8. Branstetter (2001) uses U.S. firm-level data and finds that the elasticity is 0.81. Bottazzi and

[19] The markup in the present model is a value-added markup because firms that set their markups use only capital and labor for production. Jaimovich (2007, 2008) also uses the gross markup rate of 1.3 for his analyses.

Perri (2007) use OECD macro data and find it to be 0.79. Accordingly, an intermediate value is taken. For ψ, a value of $\psi = 0.8$ $(= \psi_L)$ is chosen. With $\psi = 0.8$, the implied rate of applied technology obsolescence is 0.2, which is consistent with the literature. For example, Mansfield, Schwartz and Wagner (1981) find that the rate is 0.2 (i.e., $\psi = 0.8$), Pakes and Schankerman (1984) estimate that the rate is 0.25 (i.e., $\psi = 0.75$), and Caballero and Jaffe (1993) find a mean rate of technology obsolescence between 0.1 (i.e., $\psi = 0.9$) and 0.12 (i.e., $\psi = 0.88$). Accordingly, an intermediate value is taken.

$\underline{\delta, \delta_L, \Gamma, \Gamma_L, D \text{ and } D_L}$

The depreciation rates of capital, δ and δ_L, are calibrated from Japanese and U.S. data (depreciation rate of the capital stock, "delta" in Penn World Table 8.0). I use average values over the data sample period. A discount factor, Γ $(= \Gamma_L)$, is set to 0.96 (both countries are assumed to have the same discount factor). The parameters related to leisure preference, D and D_L, are calibrated using equation (14) with U.S. and Japanese data on consumption and wages (I solve equation (14) and take an average of the calculated values over the sample period).

$\underline{\epsilon, \epsilon_L, \chi \text{ and } \widetilde{A}^*}$

Here, ϵ_L is set to 0.1, following Comin and Gertler (2006), and ϵ is set to 0.2. [20] The higher value is chosen for ϵ(Japan) than ϵ_L(the U.S.) because IBL from abroad is assumed to be easier than "pure" innovation. The value of χ is set to 0.35. This value of χ with $\psi = \psi_L = 0.8$ and $\epsilon = 0.2$ implies that the average time that a new U.S. innovation affects new Japanese innovations is about 22 years, i.e., $1/(\epsilon\psi_L\chi) + 4$ (where 4 (years) is the required time to complete the R&D process): it is taken 28 % $(\psi_L\chi)$ of newly innovated technologies in the U.S. diffuse to Japan (become available for learning in Japan) within the next year, and then by learning those technologies, Japanese firms succeed in innovating their own new technologies with a success probability 2% (ϵ) in 4 years' time.[21] The 22-year mean lag is close to Eaton and Kortum (1999)'s

[20] Comin and Gertler (2006) set a probability of 0.1 that firms adopt domestically-invented new ideas. This success probability of domestic adoption corresponds to a success probability of R&D in our model.

[21] In other words, there is a probability of 5.6 % $(= \epsilon\psi_L\chi)$ that a newly innovated tech-

estimate that the mean lag between innovation of an idea in one country and its arrival in (i.e., its impact on) another country is about 21 years. Next, $\widetilde{A}^*(= A^*/A_L^*)$ is set to 0.42 ($\widetilde{A}^*$is needed to find $\overline{d}$: see equation 56). It is estimated as follows. Using international patent data, Bottazzi and Peri (2007) estimate A_t (stock of knowledge in Japan) and $A_{L,t}$ (stock of knowledge in the U.S.). According to their estimate, $A_t/A_{L,t}$ is approximately 0.34 in 1999 (the end of their sample period is the year 1999).[22]. Using their estimate of $A_t/A_{L,t}$ in 1999, Japanese and U.S. TFP data in 1999, employment data in 1999 and the calibrated values of β ($=\beta_L$) and ϕ ($= \phi_L$), κ_{JP}/κ_{US} is obtained based on $TFP_{JP}/TFP_{US} = (\frac{A_{JP}}{A_{US}})^{\frac{1}{\phi-1}} \left(\frac{\kappa_{JP}}{\kappa_{US}}\right) (\frac{N_{JP}}{N_{US}})^{\beta}$.[23] Using the obtained value of κ_{JP}/κ_{US}, $A_t/A_{L,t}$ is then calculated. I use the highest value of $A_t/A_{L,t}$ after 1999 for $\widetilde{A}^*$(2007 is found to have the highest value, and it is assumed that Japan and the U.S. were both close to their BGPs in 2007).

3.4 Assessing the impact of U.S. R&D on the Japanese economy

3.4.1 Model performance

I now assess the role played by U.S. R&D in Japanese medium-run fluctuations. To do this, as already described, first, U.S. R&D data are fed into the leading country (the U.S.) model to obtain U.S. applied technology ($\widetilde{A}_{L,t}$). Then, this simulated $\widetilde{A}_{L,t}$ (the model $\widetilde{A}_{L,t}$) is, through $\widetilde{V}_t$, fed into the lagging country (Japan) model. Figure 2 shows the model $\widetilde{A}_{L,t}$ with the data on $\widetilde{A}_{L,t}$ (i.e., U.S. detrended applied technology data). For some periods, the model $\widetilde{A}_{L,t}$ is quite far away from the $\widetilde{A}_{L,t}$ data. This is probably because the model $\widetilde{A}_{L,t}$ only reflects U.S. R&D outcomes but the $\widetilde{A}_{L,t}$ data include other factors like factor utilization. The $\widetilde{A}_{L,t}$ data are constructed from U.S. TFP data unadjusted for factor utilization (see Appendix A-2 for the method used in construction of the U.S. TFP data).

nology in the U.S. will have an effect on a new Japanese innovation in 4 years' time.

[22] See Figure 5 in their paper for the estimate.

[23] κ is defined in equation (3) and G is assumed to be the same between the two countries.

As in Comin and Gertler (2006), the simulated variables are filtered to obtain medium-term cycles by using Christiano and Fitzgerald (2003)'s optimal band-pass filter.[24] To remove a reasonably smooth nonlinear trend by the band-pass filter, I applied two cutoffs for the trend: a 45-year cutoff and a 35-year cutoff. For the 45-year cutoff, all fluctuations with duration of more than 45 years (roughly the sample size) are removed, and for the 35-year cutoff, those with duration of more than 35 years are removed. Use of the shorter 35-year cutoff period could be a better choice than the 45-year cutoff period if the data have a less smooth trend caused by factors such as institutional or demographic changes. In addition, if the data have a very smooth and nearly monotonic trend, the choice of the shorter 35-year cutoff period gives almost an identical detrended series to that obtained when using the 45-year cutoff. In what follows, only the results for the 35-year cutoff are shown (the results for the 45-year cutoff are quite similar to those for the 35-year cutoff and are available upon request). As suggested by Christiano and Fitzgerald (2003), I drop 2 years of data (for both the actual data series and model series) from the beginning and end of the filtered series because these data are relatively poorly estimated. The resulting detrended simulated series consists of data covering the 1965-2008 period.

The model has three predetermined variables ($\widetilde{k}$, $\tilde{A}$ and $\widetilde{F}$).[25] To solve the model, one thus needs $\widetilde{k}(0)$, $\widetilde{A}(0)$, $\widetilde{F}(0)$, $\widetilde{F}(-1)$ and $\widetilde{F}(-2)$.[26] In finding these values, the economy is assumed to be around the steady state at the end of the data sample period. Appendix A-3 shows that with the steady-state value of $\tilde{A}^*$ one can estimate $\widetilde{k}(0)$, $\widetilde{A}(0), \widetilde{F}(0)$, $\widetilde{F}(-1)$, and $\widetilde{F}(-2)$ by using the actual values of k, N and TFP both at the beginning and end of the data

[24]Christiano and Fitzgerald (2003) develop a band-pass filter under the assumption that the data used are generated by a pure random walk. Although this assumption is most likely false, they find that getting the exactly-correct representation of the time series is not crucial at least for U.S. macroeconomic data and that their approach results in a nearly optimal filter.

[25]In Dynare, the timing of each variable depends on when that variable is decided. In the case of the present model, $\widetilde{k}$, $\tilde{A}$ and $\widetilde{F}$ are decided *yesterday*.

[26]For the initial values of the remaining variables, i.e, jump variables, arbitrary values are chosen. Obviously, this does not cause any problem because they are jump variables.

sample period. The model is solved and simulated using Dynare 4.5. After 2010 (the end of the data sample period), $\widetilde{V}_t$ is set to its steady-state value.[27] I stress here that the following simulation results show the Japanese economy's responses to changes in U.S. R&D and that $\widetilde{A}_{L,t}$ is not a Solow residual, which is usually used as a shock variable in conventional studies.

Figure 3 depicts plots of the model predictions of detrended (medium-term cycle filtered) technology (applied technology), output, labor (total hours worked), R&D, consumption, and investment, as well as the corresponding Japanese detrended data.[28] The figure shows that generally, the model does a good job of matching the Japanese macroeconomic data and captures the principal movements in the data well.

Tables 2 reports variabilities (standard deviations) and contemporaneous correlations between model predicted values and actual data values of each medium-term cycle filtered series. The columns under the "model" label provide the model simulation results (the results labeled "fixed $\widetilde{V}$," "fixed IBL," and "SS" are discussed later). Overall, the model does a good job in reproducing the variabilities of the data although model consumption is more volatile than consumption in the data and model technology is less volatile than technology in the data (this is also seen in Figure 3). The greater variability of technology in the data can arise from the fact that apart from technology variation, variations in the degree of factor utilization (e.g., variation in labor effort, capital workweek and so on) cause fluctuations in the measured TFP, especially in the short run. The tables also show that the correlations between the model and data time-series are quite high. The correlations for technology and output are especially high (above 0.8).

Next, because a key argument of the model is based upon the connection between Japanese technology and U.S. R&D, it is important to check whether

[27] Dynare solves a deterministic model by using a Newton method. The number of periods is set to 200 (200 years) to solve the model.

[28] The data for labor are total hours worked ("total number of employed persons" multiplied by "average annual hours worked by persons employed"). Japanese TFP data are used for Japanese technology (applied technology). This does not, however, cause any problem, because all of the variables are detrended.

the model can reproduce this relationship between Japanese technology and U.S. R&D in the data. Figure 4 reports the model and data cross-correlation function of Japanese technology and U.S. R&D as well as that of Japanese technology and U.S. technology and that of Japanese R&D and U.S. R&D.[29] The horizontal axis indicates the number of lags (years) in the U.S. variable. For example, the data cross-correlation function in the top-left corner of Figure 4 shows that the peak cross-correlation (0.82) occurs when current period Japanese technology is correlated with period t-5 U.S. R&D. Although the model misses the peak correlations of the data to some extent, the shapes of the model's cross-correlation functions fit well with those of the data. These findings support the key argument of the model that U.S. R&D affects U.S. technology, which diffuses to Japan and affects Japanese R&D, which in turn affect Japanese technology and other variables.

3.4.2 Counterfactual exercises and role of R&D spillovers and IBL from abroad

To assess the effect of R&D spillovers and IBL from abroad (i.e., technology diffusion effect from the U.S. to Japan) in more detail, I perform several counterfactual exercises, the results of which are shown in Figure 5. The blue lines in the figures depict plots of a counterfactual simulation in which the stock of diffused technologies, $\widetilde{V}_t$, is fixed at the initial level over the 1963-2010 period (i.e., the data sample period). Beyond 2010, $\widetilde{V}_t$ is set to its steady-state level. The blue lines with rhombus marks report plots of another counterfactual simulation in which both $\widetilde{V}_t$ and $(\widetilde{V}_t^{\gamma_V} - \widetilde{A}_t^{\gamma_V})$ are fixed over the 1963-2010 period. That is, $\widetilde{V}_t$ and $\widetilde{A}_t$ in equations (41), (42), and (43) are both fixed over the period. This implies that variations in the effect of IBL from abroad are totally shut down. In doing this, $\widetilde{A}_t$ in equations (42) and (43) is chosen to be equal to the steady-state level of $\widetilde{A}$ over the period ($\widetilde{V}_t$ is fixed as before). Note here that although the effect of IBL from abroad, $\widetilde{V}_t^{\gamma_V} - \widetilde{A}_t^{\gamma_V}$, is fixed, $\widetilde{A}_t$

[29] Note that the model cross-correlation functions with U.S. R&D are based on cross-correlations between the simulated series and the actual U.S. R&D data because U.S. R&D is an exogenous variable in the model.

itself is not fixed, i.e., $\widetilde{A}_t$ in equations (37) and (38) can vary even though the IBL effect is fixed. The red lines report plots of the model without fixing $\widetilde{V}_t$ and the effect of IBL from abroad (these lines are the same as those shown in Figure 3), and the black lines plot the data. According to the figures, it is very clear that fixing $\widetilde{V}_t$ and the IBL effect largely deteriorate the ability to capture principal movements in the data variables, except consumption, particularly for the case of technology.

The findings in Figure 5 suggests that the effect of diffusion of U.S. R&D outcomes to Japan accounts for substantial fractions of medium-run fluctuations in Japanese R&D and technology movements and thus other Japanese aggregate variables such as output and labor. This is also confirmed by **T**able 2, which reports standard deviations and contemporaneous correlations between predicted values of the two counterfactual simulations and actual data values of each medium-term cycle filtered time series (results are labeled "fixed $\widetilde{V}$" and "fixed IBL"). The tables show that fixing $\widetilde{V}_t$ and the effect of IBL from abroad reduce the standard deviations and correlations of most variables. Especially in the case of technology, the standard deviation and correlation both decrease substantially. For example (see Table 2), the standard deviation (correlation) decreases from 0.075 (0.824) to 0.034 (0.621) when $\widetilde{V}_t$ is fixed and it further decreases to 0.016 (-0.592) when the IBL effect is fixed. Relative RMSEs of the counterfactual models (relative RMSE is defined as the counterfactual model's RMSE over the model's RMSE) reported in Table 3 also confirm the findings in Figure 5. The table shows that in both cases of fixing $\widetilde{V}_t$ and fixing the IBL effect, the relative RMSEs are generally greater than 1 except for consumption (i.e., RMSEs of the counterfactual models generally increase relative to those of the model). In particular, the relative RMSEs for technology are much higher than 1.

The importance of the effect of technology diffusion is also shown by examining the relationship between Japanese technology and U.S. R&D in counterfactual simulations. Similar to Figure 4, Figure 6 reports the cross-correlation function of Japanese technology and U.S. R&D, that of Japanese technology and U.S. technology, and that of Japanese R&D and U.S. R&D for the counterfactual models and the data. The figure clearly shows that fixing $\widetilde{V}_t$ and the IBL effect reduce the fit. This reduction in fit is very large when the IBL effect is fixed.

The above simulation results suggest that a change in U.S. R&D, which causes a change in $\widetilde{V}_t$ (the stock of diffused U.S. technology to Japan), is an important driver of Japanese medium-run fluctuations. However, due to diminishing returns to capital and technology catching up, the low initial levels of Japanese capital stock and technology are also highly likely to play a role in Japanese medium-run fluctuations, especially in terms of the rapid (upward) movements of the Japanese economy seen in the early sample period. To address this point, another counterfactual simulation is undertaken. To control the effects of the low initial levels of Japanese capital stock and technology in order to isolate and focus on the effect of a change in U.S. R&D, I simulate the model starting from the steady state (SS). That is, I feed $\widetilde{V}_t$ into the model by setting the initial levels of the predetermined variables ($\widetilde{k}$, $\tilde{A}$ and $\widetilde{F}$) to their steady-state levels. Figure 7 presents plots of the "SS starting" model's predictions of output, technology, labor, R&D, consumption and investment, as well as the corresponding detrended data and "model" predictions (the model without the SS-starting manipulation). The figures show that "SS starting" explains significant fractions of principal movements of Japanese technology, output, R&D, labor and investment data even in the first half of the sample period. Although "SS starting" explains less of the technology movements, compared with the "model," the figures suggest that U.S. R&D can *solely* account for quite a large part of Japanese medium-run technology movements. This finding is also indicated in the standard deviations and correlations of Table 2 as well as the model and data cross-correlation functions of Figure 8.

4 Conclusion

This paper examines how international technology diffusion and "IBL from abroad" affect medium-run macroeconomic fluctuations. The paper finds that the diffusion of U.S.-originated technologies to Japan had a large impact on

Japan's medium-run fluctuations during the postwar period. The model succeeds in reproducing well the patterns of medium-run Japanese fluctuations in GDP, TFP, R&D, labor, consumption, and investment. The results show that changes in U.S. R&D can largely account for Japan's fluctuations. This finding can be explained by U.S. innovations generated by U.S. R&D diffusing to Japan, where they make a large impact on the process of technology creation in Japan.

The paper emphasizes the role of a technologically lagging country's R&D in facilitating technology learning from abroad. The model indicates that in a technologically leading country, R&D spending stimulates "pure" innovation; in contrast, in a technologically lagging country, R&D spending induces IBL from the technological leader. Several recent studies have shown the important role of R&D in enhancing technology transfers. Griffith, Redding and Reenen (2004) find that R&D greatly contributes to technology imitation. The present paper argues that especially in developed economies, R&D processes can be well characterized by IBL from abroad.

Finally, a few remarks will be made regarding some points that are beyond the scope of this paper. The present paper ignores trade, despite its potentially important role in spreading new ideas throughout the world. Standard international business cycle models have trouble replicating the degree of empirical correlation between trade and business cycle comovement, see Kose and Yi (2006). Extending the present model to include trade may help to solve this "trade-comovement puzzle." Another issue for future work would be the introduction of uncertainty, which would be especially important for an analysis of short-run fluctuations. Extending the model to incorporate some types of friction (e.g., price rigidity and financial friction) with uncertainty might lead to some new findings regarding short-run fluctuations.

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Appendix A-1: Stock of diffused technologies

Denoting by $1-\psi_L$ the depreciation rate of technologies (product blueprints), the amount of newly created technologies in the leading country at time t is

$$A_{L,t} - \psi_L A_{L,t-1},$$

where $A_{L,t}$ is the number of technologies in the leading country at time t. Assuming that the technologies created by the leading country gradually diffuse to the lagging country at a constant rate of χ, the amount of $\chi(A_{L,t} - \psi_L A_{L,t-1})$ diffuses to the lagging country at time t.

For technologies created by the leading country at time $t-1$, the amount of $\chi(A_{L,t-1} - \psi_L A_{L,t-2}) + \chi(1-\chi)(A_{L,t-1} - \psi_L A_{L,t-2})$ diffuses to the lagging country by time t. Because the ideas depreciate at the rate $1-\psi_L$, in net terms, the actual diffused amount is

$$\begin{aligned} & \psi_L \left[\chi(A_{L,t-1} - \psi_L A_{L,t-2}) + \chi(1-\chi)(A_{L,t-1} - \psi_L A_{L,t-2})\right] \\ = \quad & \chi\psi_L \frac{1-(1-\chi)^2}{1-(1-\chi)}(A_{L,t-1} - \psi_L A_{L,t-2}). \end{aligned}$$

Similarly, for technologies created by the leading country at time $t-2$, the net amount of diffused technologies by time t is given by

$$\begin{aligned} \psi_L^2 & \left[\begin{array}{c} \chi(A_{L,t-2} - \psi_L A_{L,t-3}) + \chi(1-\chi)(A_{L,t-2} - \psi_L A_{L,t-3}) \\ +\chi^2(1-\chi)^2(A_{L,t-2} - \psi_L A_{L,t-3}) \end{array} \right] \\ & = \chi\psi_L^2 \frac{1-(1-\chi)^3}{1-(1-\chi)}(A_{L,t-2} - \psi_L A_{L,t-3}). \end{aligned}$$

The total stock of diffused technologies to the lagging country at time t, denoted by V_t, is thus given by

$$\begin{aligned} V_t = \chi(A_{L,t} - \psi_L A_{L,t-1}) + \chi\psi_L \frac{1-(1-\chi)^2}{1-(1-\chi)}(A_{L,t-1} - \psi_L A_{L,t-2}) \\ +\chi\psi_L^2 \frac{1-(1-\chi)^3}{1-(1-\chi)}(A_{L,t-2} - \psi_L A_{L,t-3}) + ... \\ +\chi\psi_L^m \frac{1-(1-\chi)^{m+1}}{1-(1-\chi)}(A_{L,t-m} - \psi_L A_{L,t-m-1}), \quad m = \infty. \end{aligned}$$

This can be rewritten as

$$V_t = \chi \left[A_{L,t} + \psi_L(1-\chi)A_{L,t-1} + \psi_L^2(1-\chi)^2 A_{L,t-2} + ... + \psi_L^m(1-\chi)^m A_{L,t-m}\right].$$

From equation (9), $\widetilde{A}_{L,t} = A_{L,t}/A^*_{L,t}$, and $A^*_{L,t} = (1+g_{A^*_L})A^*_{L,t-1}$ (where $A^*_{L,t}$ is the trend of $A_{L,t}$ and $g_{A^*_L}$ is the growth rate of $A^*_{L,t}$ which is the growth rate of $A_{L,t}$ on the balanced growth path), one can obtain

$$V_t = A^*_{L,t}\chi \left[\begin{array}{c} \widetilde{A}_{L,t} + \frac{\psi_L(1-\chi)}{(1+g_{A^*_L})}\widetilde{A}_{L,t-1} + \frac{\psi_L^2(1-\chi)^2}{(1+g_{A^*_L})^2}\widetilde{A}_{L,t-2} + ... \\ + \frac{\psi_L^m(1-\chi)^m}{(1+g_{A^*_L})^m}\widetilde{A}_{L,t-m} \end{array} \right].$$

Defining $\widetilde{V}_t = V_t/A^*_{L,t}$, one can obtain

$$\widetilde{V}_t = \chi \left[\widetilde{A}_{L,t} + \frac{\psi_L(1-\chi)}{(1+g_{A^*_L})}\widetilde{A}_{L,t-1} + \frac{\psi_L^2(1-\chi)^2}{(1+g_{A^*_L})^2}\widetilde{A}_{L,t-2} + ... + \frac{\psi_L^m(1-\chi)^m}{(1+g_{A^*_L})^m}\widetilde{A}_{L,t-m} \right]$$

Appendix A-2: Data

The primary data sources for the Japanese data set are Penn World Table and Economic and Social Research Institute, Cabinet Office, "SNA (National Accounts of Japan)." The primary data sources for the U.S. data set are Penn World Table and FRED Economic Data. The data sample period is from 1963 to 2010 (U.S. R&D data are from 1960). The details of the data are as follows.

- Y (output): Real GDP PPP adjusted. The data are constructed as follows. Real GDP at chained PPP (US$) in 2005 ("rgdpo" in Penn World Table 8.0) is extended from 2005 data to onward and backward by annual percentage changes in constant 2005 national price GDP ("rgdpna" in Penn World Table 8.0)

- K (capital): (ck/cgdp)×Y where "ck" is capital stock at current PPPs from Penn World Table 9.0 and "cgdp" is real GDP at current PPPs from Penn World Table 9.0.

- N (employment): Number of persons engaged, "emp" in Penn World Table 8.0.

- h (hours): Average annual hours worked by persons engaged, "avh" in Penn World Table 8.0.

- WI (wage income): Real wage income, labsh$\times$Y where "labsh" is the share of labor compensation in GDP at current national prices from Penn World Table 8.0. WI data are used to calibrate D and D_L, the parameters related to leisure preference (see the "Parameters" subsection).

- δ (deprecation rate of capital): Average depreciation rate of the capital stock, "delta" in Penn World Table 8.0.

Japanese data

- C (consumption): Real consumption, (ncjp/nyjp)$\times$Y where "ncjp" is nominal private consumption in Japanese Yen and "ngdpjp" is nominal GDP in Japanese Yen. Both "ncjp" and "ncjp" data are obtained from 2008SNA and 68SNA. Because the 2008SNA data are available only from 1994 onwards and the 68SNA data are available only from 1958 to 1998, the 2008SNA data are extended from 1993 data to backward using annual percentage changes of the 68 SNA data.

- IV (investment): Real investment, (nivjp/ngdpjp)$\times$Y where "nivjp" is nominal private investment in Japanese Yen. "nivjp" data are obtained from 2008SNA and 68SNA.

- RD (R&D): Real R&D expenditure, (nrdjp/ngdpjp)$\times$Y where "nrdvjp" is nominal privately-funded R&D expenditure in Japanese Yen. The "nrdjp" data are obtained from the Japanese Ministry of Internal Affairs and Communications, "Survey of Research and Development." Because the surveyed category changed in 1996, 2001 and 2002, the series is extended by annual changes from 1995 data to onward.

- TFP (total factor productivity): TFP=Y/(K$^\theta\times$(N$\times$h)$^{1-\theta}$) where θ is Japanese capital share value. The capital share is calculated as 1 minus the average value of "labsh" data over the sample period (for "ladsh", see the data description for WI above).

U.S. data

- RD: Real R&D expenditure, (nrdus/ngdpus)×Y where "nrdus" is nominal total R&D expenditures in U.S. dollars and "ngdpus" is nominal GDP in U.S. dollar. The "nrdus" data are obtained from National Science Foundation, "National Patterns of R&D Resources" and "ngdpus"data are from FRED Economic Data.

- TFP: TFP=Y/(K$^{\theta}$×(N×h)$^{1\text{-}\theta}$) where θ is U.S. capital share value. The capital share is calculated in the same way as in the case of Japan.

- $\widetilde{A}_L$: $A_{L,t}/A^*_{L,t}$. The data on $\widetilde{A}_L$ are constructed as follows. Using $TFP_{L,t}$ $= A_{L,t-1}^{\frac{1}{\phi-1}}(\kappa_L N_{L,t})^{\beta} G_t$ and setting the value of κ_L and the initial levels of $N_{L,t}$ and G_t as one, "$A_{L,t}$ data" is constructed from data on $TFP_{L,t}$ and $N_{L,t}$ (see the "Parameters" subsection for the values of ϕ, β and g_G). Construct a series with the constant growth rate of $g_{A_L^*}$ (the growth rate of $A^*_{L,t}$) by setting the initial level as 1, and define this series as $A^{*pre}_{L,t}$. Next, regress the log of "$A_{L,t}$ data" on the log of $A^{*pre}_{L,t}$ and obtain the constant term. Then, using $g_{A_L^*}$ and the estimated constant term as the initial level, "$A^*_{L,t}$ data" are constructed. Finally, divide "$A_{L,t}$ data" by "$A^*_{L,t}$ data" to construct data on $\widetilde{A}_L$. Note that in this data construction method, arbitrarily setting κ_L and initial levels of N_t and G_t as 1 in constructing "$A_{L,t}$ data" does not cause any problem because $\widetilde{A}_L = A_{L,t}/A^*_{L,t}$.

Appendix A-3: Estimating $\widetilde{k}(0)$, $\widetilde{A}(0)$, $\widetilde{F}(0)$, $\widetilde{F}(-1)$, and $\widetilde{F}(-2)$

This appendix shows how to estimate $\widetilde{k}(0)$, $\widetilde{A}(0)$, $\widetilde{F}(0)$, $\widetilde{F}(-1)$, and $\widetilde{F}(-2)$. In the followings the economy is assumed to be at the steady state at time t. First, consider $\widetilde{k}(0)$. The following equations can be obtained.

$$\widetilde{k}(0) = \frac{k(0)}{Z(0)}, \tag{a1}$$

$$\widetilde{k}^* = \frac{k_t^*}{Z_t} = \frac{k_t^*}{Z(0)(1+g_Z)^t}, \tag{a2}$$

$$Z_t = A_{L,t}^{*\frac{1}{(\phi-1)(1-\theta)}} \left((\kappa N_t)^\beta G_t\right)^{\frac{1}{1-\theta}} . \tag{a3}$$

By substituting equation (a2) into equation (a1) for $Z(0)$, $\widetilde{k}(0)$ can be expressed by

$$\widetilde{k}(0) = k(0)(1+g_Z)^t \frac{\widetilde{k}^*}{k_t^*}.$$

Substituting equation (46) into this equation for $1+g_Z$ gives

$$\widetilde{k}(0) = k(0)\frac{\widetilde{k}^*}{k_t^*}\left[(1+g_{A_L^*})^{\frac{t}{(\phi-1)(1-\theta)}}(1+g_G)^{\frac{t}{1-\theta}}(1+n)^{\frac{\beta t}{1-\theta}}\right] \tag{a4}$$

As shown later, $\widetilde{k}(0)$ is estimated using equations (a4).

Next, consider $\widetilde{A}(0)$. From equations (18) and (3)

$$\begin{aligned} Y_t &= W_t K_{t-1}^\theta (N_t h_t)^{1-\theta} \\ W_t &= A_{t-1}^{\frac{1}{\phi-1}} (\kappa N_t)^\beta G_t. \end{aligned} \tag{a5}$$

Using equation (a5) yields (assuming $\beta = \beta_L$ and $\phi = \phi_L$)

$$\frac{A_t(0)}{A_L(0)} = \left[\left(\frac{W(0)}{W_L(0)}\right)\left(\frac{\kappa}{\kappa_L}\right)^{-\beta}\left(\frac{N(0)}{N_L(0)}\right)^{-\beta}\right]^{\phi-1}. \tag{a6}$$

Using equation (a5) also yields

$$\left(\frac{\kappa}{\kappa_L}\right)^\beta = \left[\left(\frac{W_t^*}{W_{L,t}^*}\right)\left(\frac{A_t^*}{A_{L,t}^*}\right)^{\frac{-1}{(\phi-1)}}\left(\frac{N_t}{N_{L,t}}\right)^{-\beta}\right].$$

Substituting this into equation (a6) gives

$$\frac{A_t(0)}{A_L(0)} = \widetilde{A}^* \left[\left(\frac{W(0)}{W_L(0)}\right)\left(\frac{W_t^*}{W_{L,t}^*}\right)^{-1}\left(\frac{N(0)}{N_L(0)}\right)^{-\beta}\left(\frac{N_t}{N_{L,t}}\right)^{\beta}\right]^{\phi-1}$$

This is used as an estimate of $\frac{A_t(0)}{A_L^*(0)}$ as follows:

$$\widetilde{A}(0) \approx \widetilde{A}^* \left[\left(\frac{W(0)}{W_L(0)} \right) \left(\frac{W_t^*}{W_{L,t}^*} \right)^{-1} \left(\frac{N(0)}{N_L(0)} \right)^{-\beta} \left(\frac{N_t}{N_{L,t}} \right)^{\beta} \right]^{\phi-1} \tag{a7}$$

As shown below, $\widetilde{A}(0)$ can be estimated using equation (a7).

Using equations (a4) and (a7), one can obtain $\widetilde{k}(0)$ and $\widetilde{A}(0)$ as follows. The steady state values $\widetilde{k}^*$ and $\widetilde{A}^*$can be found once values are assigned to the parameters. Per labor capital values of $k(0)$ and k_t^* can be obtained from data. $TFP(0)$ and TFP_t^* can be easily obtained. Plugging this information into equations (a4) and (a7), one can then obtain $\widetilde{k}(0)$ and $\widetilde{A}(0)$.

Lastly, consider $\widetilde{F}(-2)$, $\widetilde{F}(-1)$, and $\widetilde{F}(0)$. From equation (38),

$$\widetilde{A}_t = \frac{\epsilon}{(1+g_{A_L^*})^3} \widetilde{F}_{t-3} + \frac{\psi}{(1+g_{A_L^*})} \widetilde{A}_{t-1}.$$

One can then obtain

$$\widetilde{F}_{t-3} = \frac{(1+g_{A_L^*})^3}{\epsilon} \left[\widetilde{A}_t - \frac{\psi}{(1+g_{A_L^*})} \widetilde{A}_{t-1} \right].$$

Thus,

$$\widetilde{F}(-2) = \frac{(1+g_{A_L^*})^3}{\epsilon} \left[\widetilde{A}(1) - \frac{\psi}{(1+g_{A_L^*})} \widetilde{A}(0) \right] \tag{a8}$$

Using equation (a8), one can estimate the value of $\widetilde{F}(-2)$. The values of $\widetilde{F}(-1)$ and $\widetilde{F}(0)$ can be similarly estimated.

Appendix B-1: Conditions for the existence of a steady state (not for publication)

Equation (20) gives

$$\frac{A_t}{A_{t-1}} = \epsilon \frac{F_{t-\overline{\varphi}+1}}{A_{t-1}} + \psi. \tag{b1}$$

Thus, one can obtain

$$1 + g_{A^*} = (1 + g_{F^*}), \tag{b2}$$

where * denotes a steady state. Because $RD_t = \sum_{\varphi=1}^{\overline{\varphi}} \eta_\varphi ({}_{t-\overline{\varphi}+\varphi}\lambda_\varphi) F_{t-\overline{\varphi}+\varphi}$, (see Model Appendix), the following steady state restriction is obtained.

$$1 + g_{RD^*} = (1 + g_{\lambda^*})(1 + g_{F^*}), \tag{b3}$$

where

$$1 + g_{\lambda^*} = \frac{{}_t\lambda_1^*}{{}_{t-1}\lambda_1^*} = \frac{{}_t\lambda_2^*}{{}_{t-1}\lambda_2^*} = \frac{{}_t\lambda_3^*}{{}_{t-1}\lambda_3^*} = \frac{{}_t\lambda_4^*}{{}_{t-1}\lambda_4^*}$$

Substituting equation (b2) into (b3) gives

$$1 + g_{\lambda^*} = \frac{1 + g_{RD^*}}{1 + g_{A^*}}$$

Using $(1 + g_{y^*})(1 + n) = 1 + g_{RD^*}$, the above equation can be rewritten as

$$1 + g_{\lambda^*} = \frac{(1 + g_{y^*})(1 + n)}{(1 + g_{A^*})}. \tag{b4}$$

Because $y_t = A_{t-1}^{\frac{1}{\phi-1}} (\frac{1}{1+n})^\theta T_t k_{t-1}^\theta h_t^{1-\theta}$, the following relationship must hold at the steady state.

$$1 + g_{y^*} = (1 + g_{A^*})^{\frac{1}{\phi-1}} (1 + g_T)(1 + g_{k^*})^\theta,$$

where $g_T = \frac{T_t}{T_{t-1}} - 1$. Because $g_{y^*} = g_{k^*}$, this can be rewritten as

$$1 + g_{y^*} = (1 + g_{A^*})^{\frac{1}{(\phi-1)(1-\theta)}} (1 + g_T)^{\frac{1}{1-\theta}}. \tag{b5}$$

Substituting this equation into equation (b4) for $(1 + g_y)$ and using the fact that $1 + g_T = (1 + g_G)(1 + n)^\beta$ from equation (3) gives

$$1 + g_{\lambda^*} = (1 + g_{A^*})^{\frac{1}{(\phi-1)(1-\theta)} - 1} (1 + g_G)^{\frac{1}{1-\theta}} (1 + n)^{\frac{\beta}{1-\theta} + 1}. \tag{b6}$$

The lagging country Next, because $\left(\frac{V_t^{\gamma_V}-A_t^{\gamma_V}}{V_{t-1}^{\gamma_V}-A_{t-1}^{\gamma_V}}\right)^* = (1+g_{A_L^*})^{\gamma_V}$, using equation (8), one can show that at the steady state, the following relationship must hold for the lagging country

$$1+g_{\lambda^*} = \frac{\left[(1+n)^{\beta}\right]^{\gamma_N}(1+g_G)^{\gamma_G}(1+g_{F^*})^{\alpha}}{(1+g_{A_L^*})^{\gamma_V}}.$$

Substituting equation (b2) into this for g_{F^*} gives

$$1+g_{\lambda^*} = (1+g_{A^*})^{\alpha}(1+g_{A_L^*})^{-\gamma_V}\ (1+g_G)^{\gamma_G}(1+n)^{\beta\gamma_N} \tag{b7}$$

Considering equations (b6) and (b7) and $g_{A^*} = g_{A_L^*}$, a steady state exists for any exogenous growth rates of $g_{A_L^*}$, n and g_G only if the following relationship holds

$$(1+g_{A_L^*})^{\frac{1}{(\phi-1)(1-\theta)}-1}(1+g_G)^{\frac{1}{1-\theta}}(1+n)^{\frac{\beta}{1-\theta}+1} = (1+g_{A_L^*})^{\alpha-\gamma_V}\ (1+g_G)^{\gamma_G}(1+n)^{\beta\gamma_N}.$$

Thus, the following parameter restrictions are obtained.

$$\gamma_G = \frac{1}{1-\theta}, \tag{b8}$$

$$\gamma_N = \frac{\beta+1-\theta}{\beta(1-\theta)}, \tag{b9}$$

$$\gamma_V = \frac{(1+\alpha)(\phi-1)(1-\theta)-1}{(\phi-1)(1-\theta)}. \tag{b10}$$

These restrictions indicate that with conventional values of α, ϕ and θ, the assumption of $\gamma_v > 0$ is ensured to hold.

The leading country Next, similarly, using equations (10), one can show that at the steady state, the following relationship must hold for the leading country.

$$1+g_{\lambda_L^*} = (1+n_L)^{\beta_L\ \gamma_{N_L}}(1+g_G)^{\gamma_{G_L}}\left(1+g_{F_L^*}\right)^{\alpha_L}.$$

Substituting equation (b2) into this for g_{F^*} gives

$$1 + g_{\lambda_L^*} = (1 + g_{A^*})^{\alpha_L}\ (1 + g_G)^{\gamma_{G_L}}(1 + n_L)^{\beta_L\ \gamma_{N_L}} \tag{b11}$$

Considering equations (b6) and (b11), at the steady state the following relationship must hold

$$\begin{aligned} & (1 + g_{A_L^*})^{\frac{1}{(\phi_L - 1)(1-\theta_L)} - 1}(1 + g_G)^{\frac{1}{1-\theta_L}}(1 + n)^{\frac{\beta_L}{1-\theta_L} + 1} \\ = \ & (1 + g_{A_L^*})^{\alpha_L}\ (1 + g_G)^{\gamma_{G_L}}(1 + n_L)^{\beta_L\ \gamma_{N_L}}. \end{aligned} \tag{b12}$$

This leads to

$$\begin{aligned} (1 + g_{A_L^*}) = \ & (1 + g_G)^{\frac{\gamma_{G_L}(1-\theta_L)\ -\ 1}{1-\theta_L}\frac{(\phi_L-1)(1-\theta_L)}{1-(\phi_L-1)(1-\theta_L)(1+\alpha_L)}} \\ & (1 + n_L)^{\frac{(1-\theta_L)(\beta_L\ \gamma_{N_L}-1)-\beta_L}{1-\theta_L}\frac{(\phi_L-1)(1-\theta_L)}{1-(\phi_L-1)(1-\theta_L)(1+\alpha_L)}}. \end{aligned}$$

We thus have

$$(1+g_{A_L^*}) = (1+g_G)^{\frac{(\phi_L-1)\left[\gamma_{G_L}(1-\theta_L)\ -\ 1\right]}{1-(\phi_L-1)(1-\theta_L)(1+\alpha_L)}}(1+n_L)^{\frac{(\phi_L-1)\left[(1-\theta_L)(\beta_L\ \gamma_{N_L}-1)-\beta_L\right]}{1-(\phi_L-1)(1-\theta_L)(1+\alpha_L)}}. \tag{b13}$$

This equation shows that the growth rate of output per labor at the steady state depends on the population growth rate as in Jones (1995). If the steady state exists, the growth rate of A at the steady state is given by equation (b13). Note that the parameter values are not constrained as in the case of the lagging country because such restriction method leads to $g_{A_L^*} = 0$.

Appendix B-2: Steady-state parameterization of the leading country (not for publication)

An equation used to pin down $\eta_{L,1}$, ρ_L and $\overline{d}_L$ is derived. In the following, the notation L, which denotes the leading country, is omitted. From Model

Appendix,

$$\widetilde{c}^{*} = -\frac{\widetilde{w}^{*}}{D}, \tag{b14}$$

$$r^{*} = \frac{1+g_Z}{\Gamma} - 1 + \delta \ , \tag{b15}$$

$$\widetilde{y}^{*} = \widetilde{c}^{*} + \left[1 - \frac{1-\delta}{(1+n)(1+g_Z)}\right]\widetilde{k}^{*} + \widetilde{rd}^{*} \ , \tag{b16}$$

$$\widetilde{k}^{*} = (1+n)(1+g_Z)\frac{\theta}{1-\theta}\frac{\widetilde{w}^{*}h^{*}}{r^{*}}, \tag{b17}$$

$$\widetilde{y}^{*} = \left[\frac{1}{(1+n)(1+g_Z)}\right]^{\theta}(1+g_{A^*})^{\frac{-1}{\phi-1}}\left(\widetilde{k}^{*}\right)^{\theta}(h^{*})^{1-\theta} \ , \tag{b18}$$

$$1 = (1+g_{A^*})\left[\frac{\phi}{\phi-1}\theta^{-\theta}(1-\theta)^{\theta-1}(r^{*})^{\theta}(\widetilde{w}^{*})^{1-\theta}\right]^{\phi-1} \ , \tag{b19}$$

$$\widetilde{F}^{*} = \frac{(1+g_A)^2}{\epsilon}(1+g_{A^*}-\psi) \tag{b20}$$

$$\widetilde{rd}^{*} = \begin{array}{c} d(\kappa)^{\frac{-\beta}{1-\theta}+\beta\gamma_N}\ \Delta\widetilde{F}^{*\ 1+\alpha}\cdot \\ \left[\begin{array}{c} \eta_1(1+g_{A^*})^{-3(1+\alpha)} \\ +\eta_2(1+g_{A^*})^{-2(1+\alpha)} \\ +\eta_3(1+g_{A^*})^{-(1+\alpha)} \\ +\eta_4 \end{array}\right] \end{array} \ , \tag{b21}$$

$$\widetilde{\pi}^{*} = \begin{array}{c} \frac{1}{\epsilon}d\kappa^{\frac{(1-\theta)\beta\gamma_N-\beta}{1-\theta}}\ (1+g_{A_1^*})^{4\left(\frac{-1+(\phi-1)(1-\theta)}{(\phi-1)(1-\theta)}\right)}(1+n)^{\frac{-4\beta}{1-\theta}-4}(1+g_G)^{\frac{-4}{1-\theta}}\Delta\cdot \\ \widetilde{F}^{*\ \alpha}\left[\begin{array}{c} (1+n)^{3\beta\gamma_N}(1+g_G)^{3\gamma_G}(1+q^{*})\eta_1 \\ +(1+n)^{2\beta\gamma_N}(1+g_G)^{2\gamma_G}(1+q^{*})^2\eta_2 \\ +(1+n)^{\beta\gamma_N}(1+g_G)^{\gamma_G}(1+q^{*})^3\eta_3 \\ +(1+q^{*})^4\eta_4 \ , \end{array}\right] \end{array} , \tag{b22}$$

$$\widetilde{\pi}^{*} = \frac{1}{\phi}\frac{(1+r^{*}-\delta)(1+g_{A^*})^2}{(1+r^{*}-\delta)(1+g_{A^*})-\psi(1+n)(1+g_Z)}\widetilde{y}^{*} \ , \tag{b23}$$

$$q^{*} = r^{*} - \delta.$$

From equations (b22) and (b23),

$$\frac{d}{\epsilon}\kappa^{\frac{(1-\theta)\beta\gamma_N-\beta}{1-\theta}}\ \Delta\widetilde{F}^{*\alpha}\Theta_1\Theta_2=\frac{1}{\phi}\Theta_3\widetilde{y}^*, \tag{b24}$$

where

$$\Theta_1=(1+g_{A^*})^{4\left(\frac{-1+(\phi-1)(1-\theta)}{(\phi-1)(1-\theta)}\right)}(1+n)^{\frac{-4\beta}{1-\theta}-4}(1+g_G)^{\frac{-4}{1-\theta}},$$

$$\Theta_2=\left[\begin{array}{c}(1+n)^{3\beta\gamma_N}(1+g_G)^{3\gamma_G}(1+q^*)\eta_1\\+(1+n)^{2\beta\gamma_N}(1+g_G)^{2\gamma_G}(1+q^*)^2\eta_2\\+(1+n)^{\beta\gamma_N}(1+g_G)^{\gamma_G}(1+q^*)^3\eta_3\\+(1+q^*)^4\eta_4\ ,\end{array}\right],$$

$$\Theta_3=\frac{(1+r^*-\delta)(1+g_{A^*})^2}{(1+r^*-\delta)(1+g_{A^*})-\psi(1+n)(1+g_Z)}.$$

Substituting equation (b20) into equation (b24) for $\widetilde{F}^*$ can give

$$\widetilde{y}^*=\epsilon^{-1-\alpha}d\kappa^{\frac{(1-\theta)\beta\gamma_N-\beta}{1-\theta}}\ \Delta\phi\frac{\Theta_1\Theta_2}{\Theta_3}\Theta_4^{\alpha}. \tag{b25}$$

where

$$\Theta_4=\left[(1+g_A)^2(1+g_{A^*}-\psi)\right].$$

From equation (b19)

$$1=(1+g_{A^*})\left[\frac{\phi}{\phi-1}\theta^{-\theta}(1-\theta)^{\theta-1}(r^*)^{\theta}(\widetilde{w}^*)^{1-\theta}\right]^{\phi-1}.$$

Then,

$$\widetilde{w}^*=\Theta_5, \tag{b26}$$

where

$$\Theta_5=(1+g_{A^*})^{\frac{-1}{(\phi-1)(1-\theta)}}\left[\frac{\phi}{\phi-1}\theta^{-\theta}(1-\theta)^{\theta-1}(r^*)^{\theta}\right]^{\frac{-1}{(1-\theta)}}.$$

From equations (b16) and (b17), one can obtain

$$\epsilon^{-1-\alpha} d\kappa^{\frac{(1-\theta)\beta\gamma_N-\beta}{1-\theta}} \ \Delta\phi \frac{\Theta_1\Theta_2}{\Theta_3}\Theta_4^{\alpha} = \frac{\left[\frac{1}{(1+n)(1+g_Z)}\right]^{\theta}(1+g_{A^*})^{\frac{-1}{\phi-1}}}{\left[(1+n)(1+g_Z)\frac{\theta}{1-\theta}\frac{\widetilde{w}^*h^*}{r^*}\right]^{\theta}(h^*)^{1-\theta}}.$$

Substituting equation (b26) into this equation for $\widetilde{w}^*$ can give

$$h^* = \epsilon^{-1-\alpha} d\kappa^{\frac{(1-\theta)\beta\gamma_N-\beta}{1-\theta}} \ \Delta\phi \frac{\Theta_1\Theta_2}{\Theta_3}\Theta_4^{\alpha}\Theta_5^{-\theta}\Theta_6, \tag{b27}$$

where

$$\Theta_6 = (1+g_{A^*})^{\frac{1}{\phi-1}}\left(\frac{\theta}{1-\theta}\frac{1}{r^*}\right)^{-\theta}.$$

Substituting equation (b20) into equation (b21) for $\widetilde{F}^*$ gives

$$\widetilde{rd}^* = \epsilon^{-1-\alpha} d(\kappa)^{\frac{-\beta}{1-\theta}+\beta\gamma_N} \ \Delta\Theta_4^{1+\alpha}\Theta_7 \ , \tag{b28}$$

where

$$\Theta_7 = \begin{bmatrix} \eta_1(1+g_{A^*})^{-3(1+\alpha)} \\ +\eta_2(1+g_{A^*})^{-2(1+\alpha)} \\ +\eta_3(1+g_{A^*})^{-(1+\alpha)} \\ +\eta_4 \end{bmatrix}.$$

Substituting equations (b14), (b17), (b25), and (b28) into equation (b16) gives

$$\epsilon^{-1-\alpha} d\kappa^{\frac{(1-\theta)\beta\gamma_N-\beta}{1-\theta}} \ \Delta\Theta_4^{\alpha}\left(\phi\frac{\Theta_1\Theta_2}{\Theta_3} - \Theta_4\Theta_7\right) = \widetilde{w}^*\left\{\begin{array}{c} \left[1-\frac{1-\delta}{(1+n)(1+g_Z)}\right] \\ (1+n)(1+g_Z)\frac{\theta}{1-\theta}\frac{h^*}{r^*} - \frac{1}{D}\end{array}\right\}.$$

Then, substituting equations (b27) and (b26) into this gives

$$\epsilon^{-1-\alpha} d\kappa^{\frac{(1-\theta)\beta\gamma_N-\beta}{1-\theta}} \ \Delta\Theta_4^{\alpha}\left(\phi\frac{\Theta_1\Theta_2}{\Theta_3} - \Theta_4\Theta_7\right) = \Theta_5\left\{\Theta_8\left(\begin{array}{c}\epsilon^{-1-\alpha} d\kappa^{\frac{(1-\theta)\beta\gamma_N-\beta}{1-\theta}} \ \Delta \\ \phi\frac{\Theta_1\Theta_2}{\Theta_3}\Theta_4^{\alpha}\Theta_5^{-\theta}\Theta_6\end{array}\right) - \frac{1}{D}\right\}$$

where

$$\Theta_8 = \left[1-\frac{1-\delta}{(1+n)(1+g_Z)}\right](1+n)(1+g_Z)\frac{\theta}{1-\theta}\frac{1}{r^*}.$$

The following equation can finally be obtained

$$\frac{1}{D}\Theta_5 = \epsilon^{-1-\alpha} d\kappa^{\frac{(1-\theta)\beta\gamma_N-\beta}{1-\theta}} \Delta \left\{ \phi \frac{\Theta_1\Theta_2}{\Theta_3}\Theta_4^{\alpha}\Theta_5^{1-\theta}\Theta_6\Theta_8 - \Theta_4^{\alpha}\left(\phi\frac{\Theta_1\Theta_2}{\Theta_3} - \Theta_4\Theta_7\right)\right\}.$$

Appendix B-3: Steady-state parameterization of the lagging country (not for publication)

First consider h^*. In the following, the case of $\overline{\varphi} = 4$ is considered. From Model Appendix,

$$\widetilde{c}^* = -\frac{\widetilde{w}^*}{D}, \tag{b29}$$

$$r^* = \frac{1+g_Z}{\Gamma} - 1 + \delta \ , \tag{b30}$$

$$\widetilde{y}^* = \widetilde{c}^* + \left[1 - \frac{1-\delta}{(1+n)(1+g_Z)}\right]\widetilde{k}^* + \widetilde{rd}^* \ , \tag{b31}$$

$$\widetilde{k}^* = (1+n)(1+g_Z)\frac{\theta}{1-\theta}\frac{\widetilde{w}^* h^*}{r^*}, \tag{b32}$$

$$\widetilde{y}^* = \left[\frac{1}{(1+n)(1+g_Z)}\right]^{\theta}(1+g_{A_L^*})^{\frac{-1}{\phi-1}}\left(\widetilde{A}^*\right)^{\frac{1}{\phi-1}}\left(\widetilde{k}^*\right)^{\theta}(h^*)^{1-\theta} \ , \tag{b33}$$

$$\widetilde{A}^* = (1+g_{A_L^*})\left[\frac{\phi}{\phi-1}\theta^{-\theta}(1-\theta)^{\theta-1}(r^*)^{\theta}(\widetilde{w}^*)^{1-\theta}\right]^{\phi-1} \ , \tag{b34}$$

$$\widetilde{F}^* = \frac{(1+g_{A_L^*})^2}{\epsilon}(1+g_{A_L^*}-\psi)\widetilde{A}^*, \tag{b35}$$

$$\widetilde{rd}^* = \left[\begin{array}{c} d\kappa\ (1+g_{A_L^*})^{\gamma_V}\frac{\widetilde{N}_F^{*\ 1+\alpha}}{\widetilde{V}^{*\ \gamma_V}-\widetilde{A}^{*\ \gamma_V}}\cdot \\ \eta_1(1+g_{A_L^*})^{-3(1+\alpha)} + \eta(1+g_{A_L^*})^{-2(1+\alpha)} \\ +\eta_3(1+g_{A_L^*})^{-(1+\alpha)} + \eta_4 \end{array}\right] \ , \tag{b36}$$

$$\widetilde{\pi}^* = \left[\begin{array}{c} \frac{1}{\epsilon}d\kappa(1+g_Z)^{-4}(1+g_{A_L^*})^4(1+n)^{-4}\frac{\widetilde{F}^{*\ \alpha}}{\widetilde{V}^{*\ \gamma_V}-\widetilde{A}^{*\ \gamma_V}}\cdot \\ (1+g_{A_L^*})^{-2\gamma_V}(1+n)^{3\beta\gamma_N}(1+g_G)^{3\gamma_G}(1+q^*)\eta_1 \\ +(1+g_{A_L^*})^{-\gamma_V}(1+n)^{2\beta\gamma_N}(1+g_G)^{2\gamma_G}(1+q^*)^2\eta_2 \\ +(1+n)^{\beta\gamma_N}(1+g_G)^{\gamma_G}(1+q^*)^3\eta_3 \\ +(1+g_{A_L^*})^{\gamma_V}(1+q^*)^4\eta_4 \ , \end{array}\right] \tag{b37}$$

$$\widetilde{\pi}^* = \frac{1}{\phi}\frac{(1+r^*-\delta)(1+g_{A_L^*})^2}{(1+r^*-\delta)(1+g_{A_L^*})-\psi(1+n)(1+g_Z)}\frac{\widetilde{y}^*}{\widetilde{A}^*}\ , \tag{b38}$$

$$q^* = r^* - \delta$$

$$\widetilde{V}^* = \chi\left(\frac{1-\left(\frac{\psi(1-\chi)}{1+g_{A_L^*}}\right)^m}{1-\left(\frac{\psi(1-\chi)}{1+g_{A_L^*}}\right)}\right). \tag{b39}$$

Using equations (b37) and (b38)

$$\frac{1}{\widetilde{V}^{*\gamma_V}-\widetilde{A}^{*\gamma_V}}\frac{d\kappa}{\epsilon}\widetilde{F}^{*\alpha}\Theta_1\Theta_2 = \frac{1}{\phi}\Theta_3\frac{\widetilde{y}^*}{\widetilde{A}^*}, \tag{b40}$$

where

$$\Theta_1 = (1+g_Z)^{-4}(1+g_{A_L^*})^4(1+n)^{-4}\ ,$$

$$\Theta_2 = \left[\begin{array}{c} (1+g_{A_L^*})^{-2\gamma_V}(1+n)^{3\beta\gamma_N}(1+g_G)^{3\gamma_G}(1+q^*)\eta_1 \\ +(1+g_{A_L^*})^{-\gamma_V}(1+n)^{2\beta\gamma_N}(1+g_G)^{2\gamma_G}(1+q^*)^2\eta_2 \\ +(1+n)^{\beta\gamma_N}(1+g_G)^{\gamma_G}(1+q^*)^3\eta_3 \\ +(1+g_{A_L^*})^{\gamma_V}(1+q^*)^4\eta_4\ , \end{array}\right],$$

$$\Theta_3 = \frac{(1+r^*-\delta)(1+g_{A_L^*})^2}{(1+r^*-\delta)(1+g_{A_L^*})-\psi(1+n)(1+g_Z)}.$$

Substituting equation (b40) into equation (b36) for $\frac{1}{\widetilde{V}^{*\gamma_V}-\widetilde{A}^{*\gamma_V}}$ and then substituting equation (b35) into the obtained equation for $\widetilde{F}^*$, one can obtain

$$\widetilde{rd}^* = \frac{1}{\phi}(1+g_{A_L^*})^2(1+g_{A_L^*}-\psi)\frac{\Theta_3\Theta_4}{\Theta_1\Theta_2}\widetilde{y}, \tag{b41}$$

where

$$\Theta_4 = (1+g_{A_L^*})^{\gamma_V}\left[\begin{array}{c} \eta_1(1+g_{A_L^*})^{-3(1+\alpha)}+\eta_2(1+g_{A_L^*})^{-2(1+\alpha)} \\ +\eta_3(1+g_{A_L^*})^{-(1+\alpha)}+\eta_4 \end{array}\right].$$

Substituting equations (b29), (b32), and (b41) into (b31) gives

$$\widetilde{y}^* = \begin{array}{c} -\frac{\widetilde{w}^*}{D} + \left[1 - \frac{1-\delta}{(1+n)(1+g_Z)}\right](1+n)(1+g_Z)\frac{\theta}{1-\theta}\frac{\widetilde{w}^* h^*}{r^*} \\ +\frac{1}{\phi}(1+g_{A_L^*})^2(1+g_{A_L^*}-\psi)\frac{\Theta_3\Theta_4}{\Theta_1\Theta_2}\widetilde{y}^* \end{array}. \tag{b42}$$

Substituting equations (b32) and (b34) into equation (b33) gives

$$\widetilde{y}^* = \frac{\phi}{\phi-1}\theta^{-\theta}(1-\theta)^{\theta-1}\left(\frac{\theta}{1-\theta}\right)^{\theta} h^*(\widetilde{w}^*)$$

Substituting this into equation (b42) and solving for h^* gives

$$\frac{1}{h^* D} = \left\{ \begin{array}{c} \left[1 - \frac{1-\delta}{(1+n)(1+g_Z)}\right](1+n)(1+g_Z)\frac{\theta}{1-\theta}\frac{1}{r^*} \\ +\frac{1}{\phi}(1+g_{A_L^*})^2(1+g_{A_L^*}-\psi)\frac{\Theta_3\Theta_4}{\Theta_1\Theta_2}\frac{\phi}{\phi-1}\theta^{-\theta}(1-\theta)^{\theta-1}\left(\frac{\theta}{1-\theta}\right)^{\theta} \\ -\frac{\phi}{\phi-1}\theta^{-\theta}(1-\theta)^{\theta-1}\left(\frac{\theta}{1-\theta}\right)^{\theta} \end{array} \right\}.$$

Defining

$$\begin{aligned} \Theta_5 &= \left[1 - \frac{1-\delta}{(1+n)(1+g_Z)}\right](1+n)(1+g_Z)\frac{\theta}{1-\theta}\frac{1}{r^*}, \\ \Theta_6 &= \frac{1}{\phi}(1+g_{A_L^*})^2(1+g_{A_L^*}-\psi), \\ \Theta_7 &= \frac{\phi}{\phi-1}\theta^{-\theta}(1-\theta)^{\theta-1}\left(\frac{\theta}{1-\theta}\right)^{\theta}, \end{aligned}$$

one can finally obtain

$$h^* = \frac{1}{D}\left(\Theta_5 + \frac{\Theta_3\Theta_4}{\Theta_1\Theta_2}\Theta_6\Theta_7 - \Theta_7\right)^{-1}. \tag{b43}$$

Next, with r^*, $\widetilde{V}^*$ and h^*given by equations (b30), (b39) and (b43), $\widetilde{w}^*$ can be parameterized as follows. From equation (b40)

$$\widetilde{y}^* = \frac{1}{\widetilde{V}^{*\gamma_V} - \widetilde{A}^{*\gamma_V}}\frac{d\kappa}{\epsilon}\widetilde{F}^{*\alpha}\Theta_1\Theta_2\widetilde{A}^*\phi\Theta_3^{-1}.$$

Substituting equation (b35) into this gives

$$\widetilde{y}^* = \frac{d\kappa}{\epsilon}\phi\left[\frac{(1+g_{A_L^*})^2}{\epsilon}(1+g_{A_L^*}-\psi)\right]^\alpha \Theta_1\Theta_2\Theta_3^{-1}\frac{1}{\widetilde{V}^{*\gamma_V}-\widetilde{A}^{*\gamma_V}}\widetilde{A}^{*\ 1+\alpha}.$$

Substituting this equation into equation (b33) and substituting equations (b32) and (b34) into the resulting equation, one can then obtain

$$\epsilon^{-(1+\alpha)}d\kappa\Theta_1\Theta_2\Theta_3^{-1}\Theta_8\Theta_9^{1+\alpha}\frac{(\widetilde{w}^*)^{(1-\theta)(\phi-1)(1+\alpha)}}{\widetilde{V}^{*\gamma_V}-\Theta_9^{\gamma_V}(\widetilde{w}^*)^{(1-\theta)(\phi-1)\gamma_V}} = \Theta_7(\widetilde{w}^*)h^* \qquad \text{(b44)}$$

where

$$\Theta_8 = \phi\left[(1+g_{A_L^*})^2(1+g_{A_1^*}-\psi)\right]^\alpha,$$

$$\Theta_9 = (1+g_{A_L^*})\left[\frac{\phi}{\phi-1}\theta^{-\theta}(1-\theta)^{\theta-1}(r^*)^\theta\right]^{\phi-1}.$$

$\widetilde{w}^*$ is then given by solving equation (b44) for $\widetilde{w}^*$.

Finally, an equation that can be used to pin down η_1, ρ, and $\overline{d}$ is derived. From equation (b34),

$$\widetilde{A}^{*\frac{1}{\phi-1}} = (1+g_{A_L^*})^{\frac{1}{\phi-1}}\left[\frac{\phi}{\phi-1}\theta^{-\theta}(1-\theta)^{\theta-1}(r^*)^\theta(\widetilde{w}^*)^{1-\theta}\right].$$

This can be rewritten as

$$(\widetilde{w}^*)^{-(1-\theta)} = (1+g_{A_L^*})^{\frac{1}{\phi-1}}\left[\frac{\phi}{\phi-1}\theta^{-\theta}(1-\theta)^{\theta-1}(r^*)^\theta\right]\widetilde{A}^{*\frac{-1}{\phi-1}}.$$

Then,

$$\widetilde{w}^* = \Theta_9^{\frac{-1}{(\phi-1)(1-\theta)}}\widetilde{A}^{*\frac{1}{(\phi-1)(1-\theta)}}.$$

By substituting this into equation (b44) for $\widetilde{w}^*$, one can then obtain

$$h^* = \epsilon^{-(1+\alpha)}d\kappa\Theta_1\Theta_2\Theta_3^{-1}\Theta_7^{-1}\Theta_8\Theta_9^{\frac{1}{(\phi-1)(1-\theta)}}\frac{1}{\widetilde{V}^{*\gamma_V}-\widetilde{A}^{*\gamma_V}}\widetilde{A}^{*\frac{-1}{(\phi-1)(1-\theta)}+(1+\alpha)}.$$

Using this with equation (b43) yields

$$\frac{1}{D}\left(\Theta_5+\frac{\Theta_3\Theta_4}{\Theta_1\Theta_2}\Theta_6\Theta_7-\Theta_7\right)^{-1}$$
$$=\left[\begin{array}{c}\epsilon^{-(1+\alpha)}\overline{d}\Theta_1\Theta_2\Theta_3^{-1}\Theta_7^{-1}\Theta_8\Theta_9^{\frac{1}{(\phi-1)(1-\theta)}}\\ \frac{1}{\widetilde{V}^{*\gamma_V}-\widetilde{A}^{*\gamma_V}}\widetilde{A}^{*\frac{-1}{(\phi-1)(1-\theta)}+(1+\alpha)}\end{array}\right].$$

Appendix B-4: $\widetilde{A}_L$ and $\widetilde{rd}_L$ (not for publication)

From equations (38) and (44), one can obtain

$$\widetilde{F}_{L,t-3}=\frac{(1+g_{A_L^*})^3}{\epsilon_L}\left(\widetilde{A}_{L,t}-\frac{\psi_L}{(1+g_{A_L^*})}\widetilde{A}_{L,t-1}\right), \tag{b45}$$

$$\widetilde{rd}_{L,t}=\overline{d}_L\left[\begin{array}{c}\eta_{L,1}(1+g_{A_L^*})^{-3(1+\alpha_L)}\widetilde{F}_{L,t-3}^{1+\alpha_L}\\ +\eta_{L,2}(1+g_{A_L^*})^{-2(1+\alpha_L)}\widetilde{F}_{L,t-2}^{1+\alpha_L}\\ +\eta_{L,3}(1+g_{A_L^*})^{-(1+\alpha_L)}\widetilde{F}_{L,t-1}^{1+\alpha_L}\\ +\eta_{L,4}\widetilde{F}_{L,t}^{1+\alpha_L}\end{array}\right] \tag{b46}$$

Substituting equation (b45) into equation (b46) for $\widetilde{F}_L$ and using $\eta_{L,\varphi}=\rho_L^{\varphi_L-1}\eta_{L,1}$, one can obtain the following equations:

$$\widetilde{rd}_{L,t}=\begin{array}{c}\overline{d}_L\ \varepsilon_L^{-(1+\alpha_L)}(1+g_{A_L^*})^{3(1+\alpha_L)}\eta_{L,1}\\ \left[\begin{array}{c}(1+g_{A_L^*})^{-3(1+\alpha_L)}\left\{\left(\widetilde{A}_{L,t}-\frac{\psi_L}{(1+g_{A_L^*})}\widetilde{A}_{L,t-1}\right)\right\}^{1+\alpha_L}\\ +\rho_L(1+g_{A_L^*})^{-2(1+\alpha_L)}\left\{\left(\widetilde{A}_{L,t+1}-\frac{\psi_L}{(1+g_{A_L^*})}\widetilde{A}_{L,t}\right)\right\}^{1+\alpha_L}\\ +\rho_L^2(1+g_{A_L^*})^{-(1+\alpha_L)}\left\{\left(\widetilde{A}_{L,t+2}-\frac{\psi_L}{(1+g_{A_L^*})}\widetilde{A}_{L,t+1}\right)\right\}^{1+\alpha_L}\\ +\rho_L^3\left\{\left(\widetilde{A}_{L,t+3}-\frac{\psi_L}{(1+g_{A_L^*})}\widetilde{A}_{L,t+2}\right)\right\}^{1+\alpha_L}\end{array}\right]\end{array}, \tag{b47}$$

Using this, one can then obtain the following equation:

$$\begin{array}{c}\overline{d}_L\ \varepsilon_L^{-(1+\alpha_L)}(1+g_{A_L^*})^{3(1+\alpha_L)}\eta_{L,1} \\ \left[\begin{array}{c}(1+g_{A_L^*})^{-3(1+\alpha_L)}\left\{\left(\widetilde{A}_{L,t}-\frac{\psi_L}{(1+g_{A_L^*})}\widetilde{A}_{L,t-1}\right)\right\}^{1+\alpha_L} \\ +\rho_L(1+g_{A_L^*})^{-2(1+\alpha_L)}\left\{\left(\widetilde{A}_{L,t+1}-\frac{\psi_L}{(1+g_{A_L^*})}\widetilde{A}_{L,t}\right)\right\}^{1+\alpha_L} \\ +\rho_L^2(1+g_{A_L^*})^{-(1+\alpha_L)}\left\{\left(\widetilde{A}_{L,t+2}-\frac{\psi_L}{(1+g_{A_L^*})}\widetilde{A}_{L,t+1}\right)\right\}^{1+\alpha_L}\end{array}\right] \\ = \begin{array}{c}\frac{1}{\rho_L}\widetilde{rd}_{L,t-1}(1+g_{A_L^*})^{-(1+\alpha_L)}-\overline{d}_L\ \varepsilon_L^{-(1+\alpha_L)}(1+g_{A_L^*})^{3(1+\alpha_L)}\eta_{L,1} \\ \left[\frac{1}{\rho_L}(1+g_{A_L^*})^{-4(1+\alpha_L)}\left(\widetilde{A}_{L,t-1}-\frac{\psi_L}{(1+g_{A_L^*})}\widetilde{A}_{L,t-2}\right)^{1+\alpha_L}\right]\end{array}.\end{array}$$

By substituting this into equation (b47), one can finally obtain

$$\begin{array}{l}\widetilde{A}_{L,t+3}-\dfrac{\psi_L}{(1+g_{A_L^*})}\widetilde{A}_{L,t+2} \\ = \left[\begin{array}{c}(\overline{d}_L)^{-1}\ \varepsilon_L^{(1+\alpha_L)}(1+g_{A_L^*})^{-3(1+\alpha_L)}\rho_L^{-3}\eta_{L,1}^{-1}\left\{\widetilde{rd}_{L,t}-(1+g_{A_L^*})^{-(1+\alpha_L)}\rho_L^{-1}\widetilde{rd}_{L,t-1}\right\} \\ +(1+g_{A_L^*})^{-4(1+\alpha_L)}\ \rho_L^{-4}\left(\widetilde{A}_{L,t-1}-\frac{\psi_L}{(1+g_{A_L^*})}\widetilde{A}_{L,t-2}\right)^{1+\alpha_L}\end{array}\right]^{\frac{1}{1+\alpha_L}}.\end{array}$$

Note that by using equations (b20) and (b21), it can be shown that at the steady state the following equation holds:

$$(1+g_{A_L^*}-\psi_L)=\frac{\epsilon_L}{(1+g_{A_L^*})^2}(\overline{d}_L)^{-\frac{1}{1+\alpha_L}}\left[\begin{array}{c}\eta_{L,1}(1+g_{A_L^*})^{-3(1+\alpha_L)} \\ +\eta_{L,2}(1+g_{A_L^*})^{-2(1+\alpha_L)} \\ +\eta_{L,3}(1+g_{A_L^*})^{-(1+\alpha_L)} \\ +\eta_{L,4}\end{array}\right]^{-\frac{1}{1+\alpha_L}}\widetilde{rd}_L^{*\frac{1}{1+\alpha_L}}.$$

Thus, $\frac{1}{1+\alpha_L}$ measures elasticity of innovation (new applied technologies) to R&D at the steady state.

Table 1: Model calibration

Parameters	Values	Description
α_L, α	0.25	R&D steady state elasticity (= 1/(1+α))
β_L, β	0.11	parameter related to general technology
θ_L, θ	0.35, 0.43	capital share
ϕ_L, ϕ	4.33	gross markup (=ϕ/(ϕ-1))
ψ_L, ψ	0.8	product survival rate
n_L, n	0.016, 0.0064	population growth rate
δ_L, δ	0.038, 0.048	capital depreciation rate
Γ_L, Γ	0.96	discount factor
D_L, D	-0.00057, -0.00054	preference parameter for leisure
ε_L, ε	0.1, 0.2	R&D success probability
χ	0.35	technology diffusion rate
$\tilde{A}^*$	0.42	steady state relative applied technology level (=A^*/A_L^*)
g_G	0.0009	growth rate of general technology
$g_{A_L^*}$	0.025	steady sate growth rate of applied technology in the leading country
η_{L1}, η_1	0.20, 0.59	parameter related to a relative importance of each statge of R&D
ρ_L, ρ	1.14, 0.43	parameter related to a relative importance of each statge of R&D
$\bar{d}_L$, $\bar{d}$	176, 243	scaling parameter in the R&D cost function
γ_G	1.54	parameter in the R&D cost function
γ_N	10.85	parameter in the R&D cost function
γ_V	0.24	parameter in the R&D cost function

Table 2: Volatility and cross-correlations (35-year cutoff for detrending by the band-pass filter)

	Data	Model	Fixed $\tilde{V}$	Fixed IBL	SS
		Standard deviation			
Technology (A)	0.118	0.075	0.034	0.016	0.040
Output	0.055	0.047	0.031	0.014	0.018
Labor	0.018	0.013	0.007	0.017	0.014
R&D	0.092	0.069	0.052	0.048	0.046
Consumption	0.030	0.042	0.036	0.030	0.008
Investment	0.157	0.088	0.051	0.028	0.084
		Correlation with data			
Technology (A)	-	0.824	0.610	-0.592	0.783
Output	-	0.889	0.797	0.663	0.702
Labor	-	0.578	0.240	-0.400	0.221
R&D	-	0.385	0.030	0.249	0.495
Consumption	-	0.840	0.805	0.712	0.500
Investment	-	0.552	0.467	-0.317	0.241

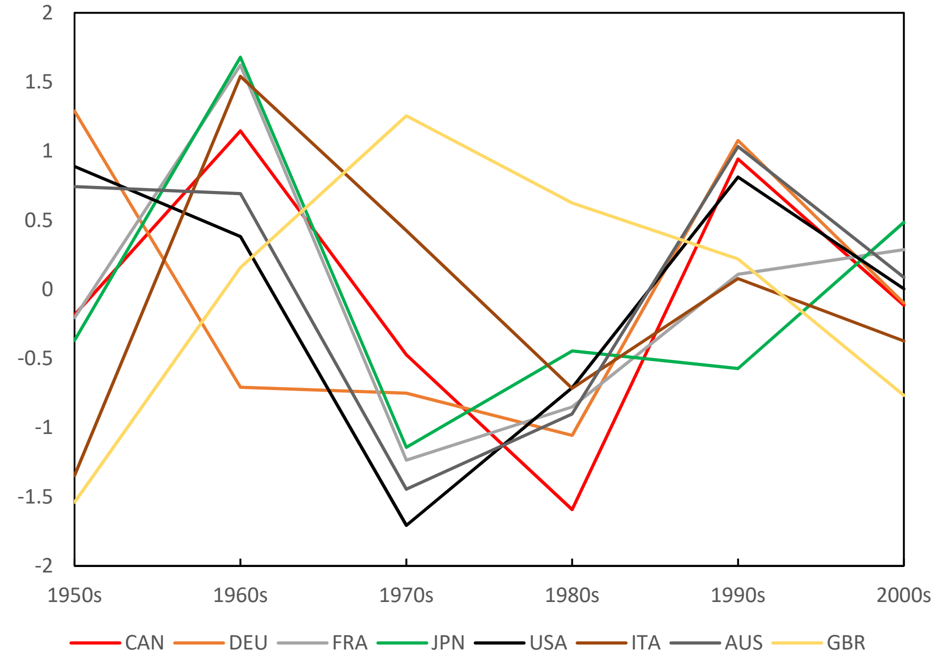
Notes: Columns under the label of "data" show the results of the data and those under the label "model" show the results of the model simulation. The columns under the label "fixed Ṽ" show the results of a counterfactual simulation in which no variation in Ṽ is allowed. Those under the label "fixed IBL" show the results of a counterfactual simulation in which the effect of IBL from abroad is fixed over the sample period (i.e.,$\tilde{V}^{Y} - \tilde{A}^{Y}$ is fixed). Those under the label "SS" show the results of a counterfactual simulation in which the initial levels of the predetermined variables are set to their steady state levels. All of the variables are medium-term cycle filtered and all fluctuations with a duration of longer than 35 years are removed.

Table 3: Root mean square of counterfactuals' forecasting errors relative to the model (35-year cutoff for detrending by the band-pass filter)

	RMSE: model	Relative RMSE: fixed Ṽ	Relative RMSE: fixed IBL
Technology	0.070	1.43	1.83
Output	0.025	1.40	1.86
Labor	0.015	1.19	1.99
R&D	0.091	1.14	1.02
Cosumption	0.024	0.90	0.97
Investment	0.129	1.07	1.29

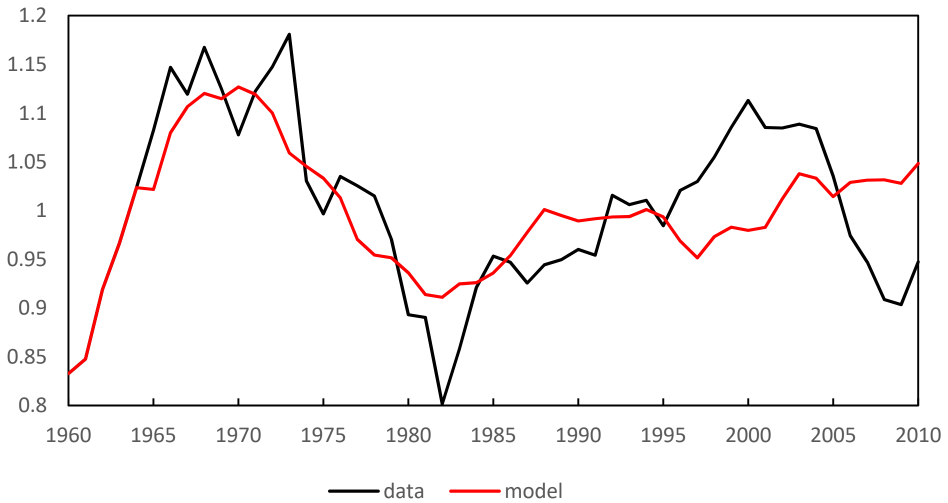
Notes: Columns labeled "RMSE: model" report root-mean-suqare errors (RMSEs) of the model. The columns labeled "Relative RMSE: fixed Ṽ " report ratios of the RMSE of "fixed Ṽ to the RMSE of the model. The columns labeled "Relative RMSE: fixed IBL" report ratios of the RMSE of "fixed IBL to the RMSE of the model.

Figure 1: International TFP growth



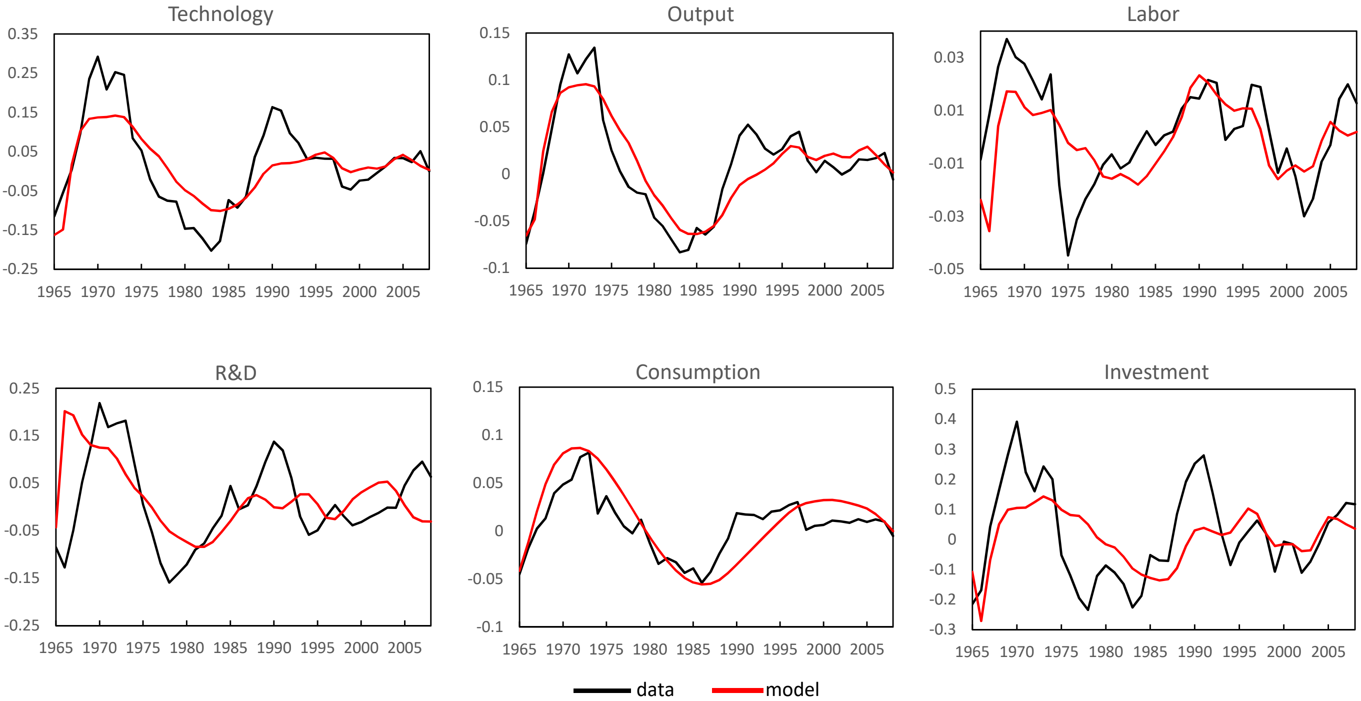
Notes: Growth rates of TFP are linearly detrended and then the detrended growth rates are averaged over 10-year intervals. These averaged growth rates are divided by their respective standard deviations. The data on TFP are from Penn World Table ("rtfpna" in Penn World Table 9.0). CAN: Canada, DEU: Germany, FRA: France, JPN: Japan, USA: U.S.A, ITA: Italy, AUS: Australia, GBR: U.K.

Figure 2: U.S. technology ($\tilde{A}_L$) : Model and data



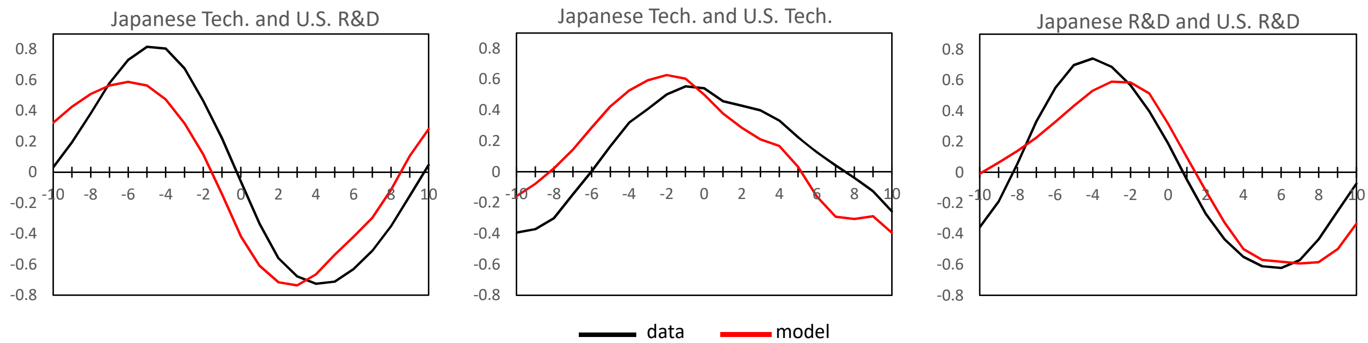
Notes: The data series is a detrended series constructed from U.S. TFP data.

Figure 3: Model predictions and data (35-year cutoff for detrending by the band-pass filter)



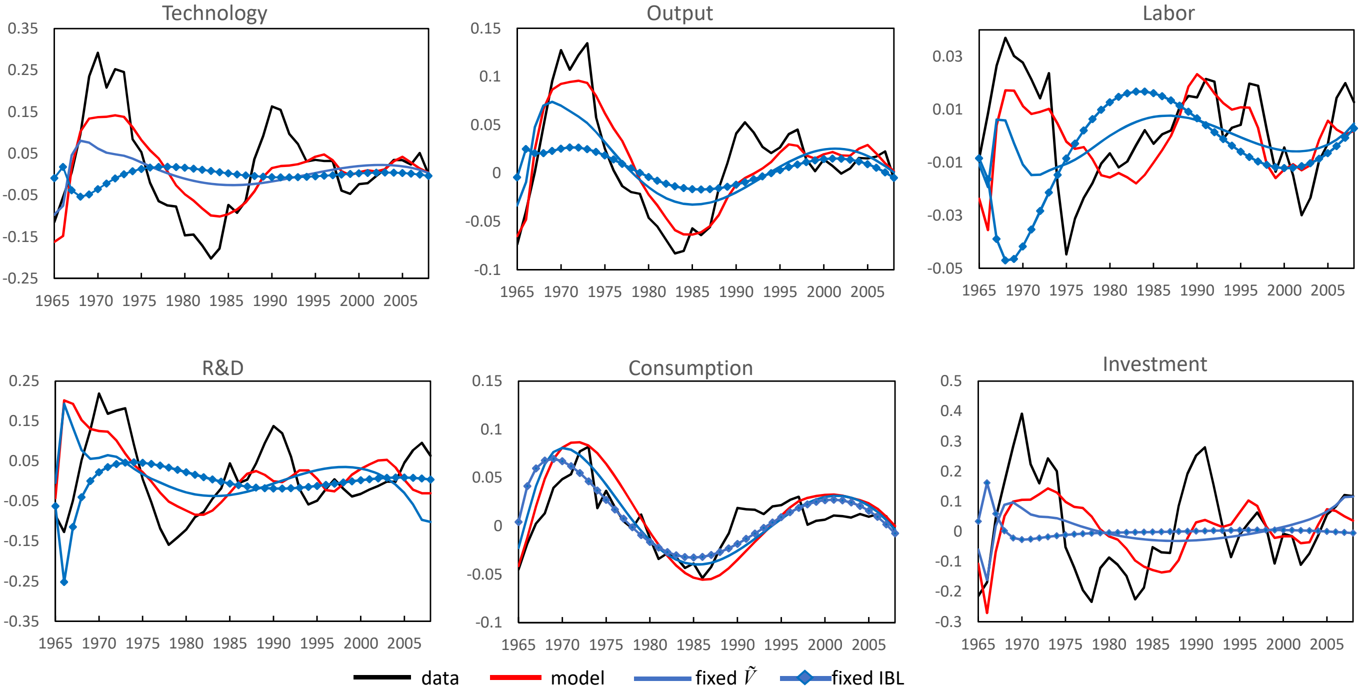
Notes: Variables are medium-term cycle filtered using Christiano and Fitzgerald (2003) optimal band-pass filter. All of the variables are medium-term cycle filtered and all fluctuations with a duration of longer than 35 years are removed.

Figure 4: Cross-correlation functions: Japanese technology and R&D with U.S. technology and R&D (35-year cutoff for detrending by the band-pass filter)



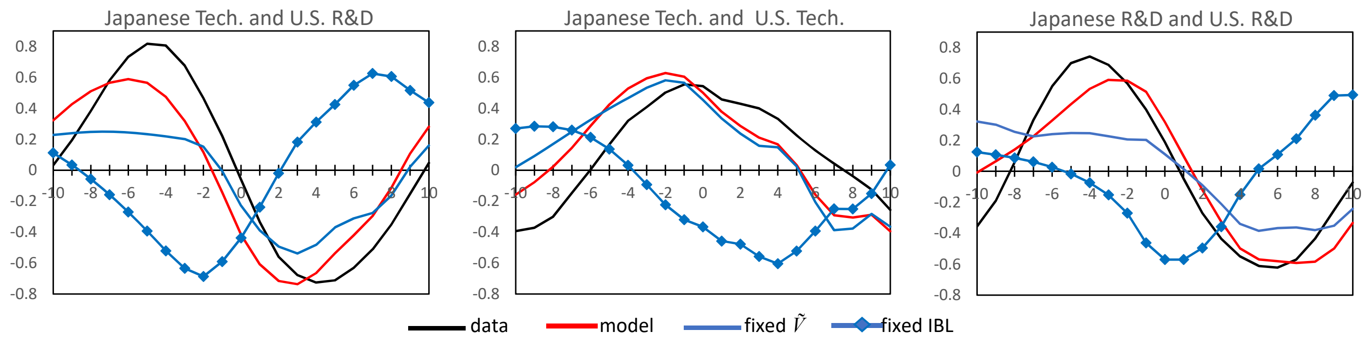
Notes: The horizontal axis indicates the number of lags (years) of the U.S. variable.

Figure 5: Counterfactual simulations (35-year cutoff for detrending by the band-pass filter)



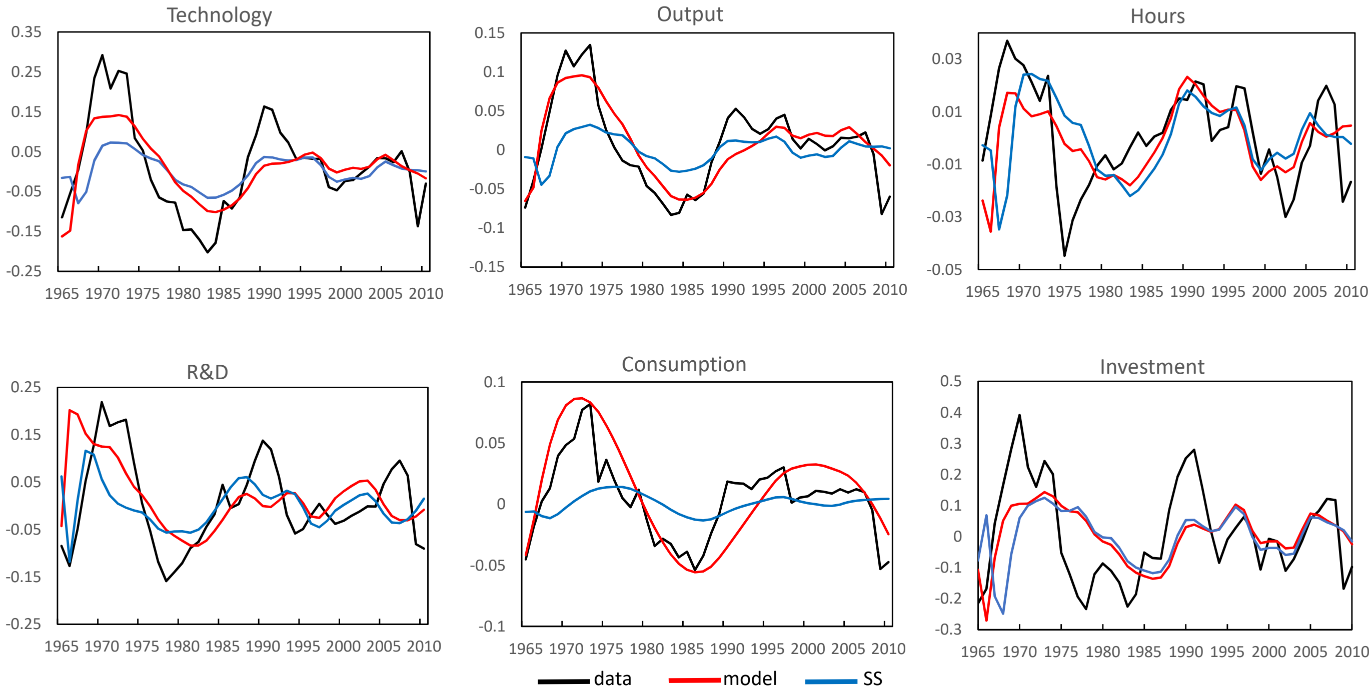
Notes: Variables are medium-term cycle filtered using Christiano and Fitzgerald (2003) optimal band-pass filter. All fluctuations with a duration of longer than 35 years are removed.

Figure 6: Cross-correlation functions of counterfactual simulations: Japanese technology and R&D with U.S. technology and R&D (35-year cutoff for detrending by the band-pass filter)



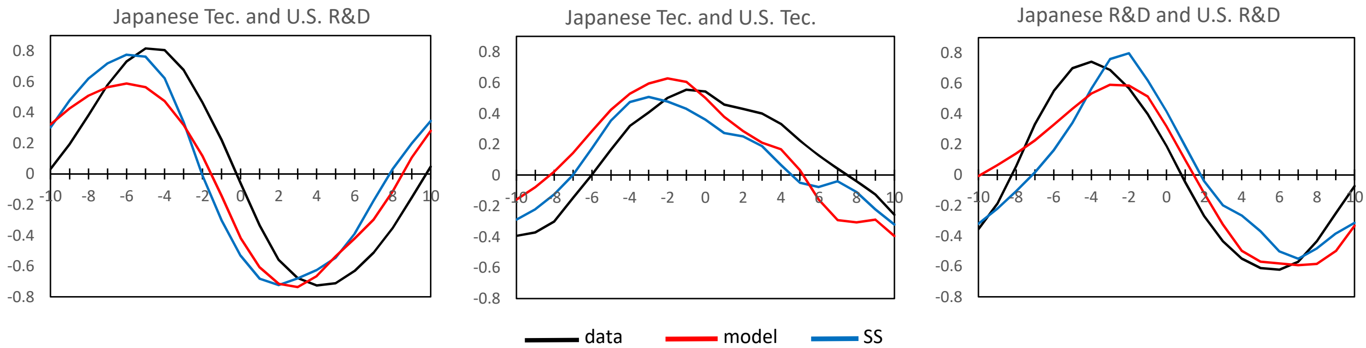
Notes: The horizontal axis indicates the number of lags (years) of the U.S. variable.

Figure 7: Counterfactual simulation of the "steady-state starting" model (35-year cutoff for detrending by the band-pass filter)



Notes: Variables are medium-term cycle filtered by using Christiano and Fitzgerald (2003) optimal band-pass filter. All fluctuations with a duration of longer than 35 years are removed. "SS" shows the results of a counterfactual simulation in which the initial levels of the predetermined variables are set to their steady state levels. The results of "data" and "model" are the same as those of figure 3.

Figure 8: Cross-correlation functions of counterfactual simulation of the "steady-state starting" model: Japanese technology and R&D with U.S. technology and R&D (35-year cutoff for detrending by the band-pass filter)



Notes: The horizontal axis indicates the number of lags (years) of the U.S. variable.