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Analysis of Merger Control in A Network Products Market

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Abstract

Using a horizontally differentiated three-firm model, we consider horizontal merger and antitrust policy in a network products market, where we observe network externalities and compatibilities (interconnectivities) between products and services. In particular, if the degree of network compatibilities in the case of a merger is sufficiently larger than that of product substitutability, consumer surplus is larger than in the premerger case. Thus, the proposed merger is allowed by antitrust authorities based on the positive effect on consumer surplus. In this case, the merger is Pareto improving.

Keywords: horizontal merger; antitrust policy; network externality; compatibility; consumer surplus standard; horizontally differentiated Cournot competition

JEL Classification: D43; K21; L12; L15; L41

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1. Introduction

Since the 1990s, waves of domestic and global mergers and acquisitions (M&A) have been observed in various industries, including telecommunications, internet businesses, banking, airlines, and railways.\textsuperscript{1} These industries are commonly characterized as network product markets where we observe network externalities and compatibilities (interconnectivities) between products and services. In a seminal paper, Economides and White (1994) demonstrate the parallels between the concepts of compatibility and networks and those of complementarity, and then apply the analysis to antitrust policy. In this paper, focusing on the role of network compatibilities as merger efficiencies, we consider horizontal mergers and associated policy.

There is a consensus that market concentration through M&A or collusive agreements reduces social welfare and thus should be forbidden. That is, horizontal mergers are likely to reduce consumer surplus and thus social welfare, even though they increase the merged firms’ profits. This is not allowed from an antitrust and competition policy perspective. In other words, from an antitrust perspective, the worrying cases are

\textsuperscript{1} For example, Breinlich, et al. (2017) surveyed international aspects of merger policy. Furthermore, di Giovanni (2005) empirically analyzed the relationship between cross-border M&A activity and financial deepening.
issues and welfare-reducing mergers.\(^2\)

However, it has been known in the field of industrial organization that market concentration by horizontal mergers does not necessarily worsen market performance. That is, mergers can increase market power and also create efficiencies.\(^3\) As shown in the “Williamson trade-off” (Williamson, 1968), a merger assessment requires to the trade-off of welfare-reducing price effects against welfare-increasing gains in productive efficiency. In particular, Farrell and Shapiro (1990) consider the welfare effects of horizontal mergers, assuming scale economies, synergies, and supply-side externalities (e.g., cost-saving) as merger efficiencies. For example, by combining their assets, the merging firms can operate at a lower unit cost. Thus, if the efficiency gains from the merger are sufficiently large, it is welfare improving.

As mentioned above, to consider horizontal mergers and their effects in this paper, we assume the impacts of merger-related synergies on the demand-side in a network products market, i.e., the effects of network externalities and compatibilities (interconnectivities). For example, Farrell and Shapiro (2001) use improved interoperability and network configuration as hypothetical examples of mergers with synergies. Researches have considered horizontal mergers and associated policy in

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\(^2\) However, Salant et al. (1983) demonstrate that horizontal mergers decrease the profits of participating firms compared with the premerger situation, i.e., merger paradox.

\(^3\) See Perry and Porter (1985) and Farrell and Shapiro (1990).
airline networks (e.g., Brueckner & Spiller, 1991; Encaouna et al., 1996; Bilptkach & Hüschelrath, 2012; Hüschelrath & Müller, 2014).

Furthermore, it is important to investigate the policy implications of merger control in a network products market. As explained by Shapiro and Varian (1999, pp. 304–305), the US Department of Justice (DOJ) conducted an extensive review of the proposed WorldCom–MCI merger in Internet and long-distance telephone services. Furthermore, both the DOJ and the Federal Trade Commission (FTC) are unlikely to challenge mergers in high-technology industries because of low entry barriers in the rapidly changing environments. However, they do not allow proposed mergers in the software industry. They recognize that entry may be difficult in this industry where there are high consumer switching costs and intellectual property rights of the incumbents. Related to this issue, following Faulhaber (2002) and Gandal (2002), we examine the external effect of a merger on the outsider (nonparticipant) firm as an application of our model.

As will be shown below, we appreciate that introducing a common standard to make products and services compatible (interoperable) is an important consideration with network externalities; therefore, with respect to collusive behavior of merging firms, we make the specific assumption that the merging firms upgrade the level of compatibility

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4 See also Spulber (2002, pp. 502–505).
(interoperability) compared with the case of noncooperative competition. Consequently, we demonstrate that a welfare-improving merger arises in the market if the net degree of network compatibilities in the merger case is sufficiently large.

In the next section, we first develop a horizontally differentiated three-firm model with network externalities and compatibilities. Second, we show the noncooperative Cournot equilibrium as the premerger case and then the equilibrium in the merger case. In Section 3, we first show the equilibrium outcomes, i.e., quantities, consumer surplus, and profits, in the merger case compared with the premerger case. Second, we consider horizontal mergers and associated policy from a consumer surplus perspective. Third, as an application of merger control, we examine the American Online (AOL)–Warner case. Finally, in Section 4, we summarize our main results and present some remaining problems.

2. The Horizontally Differentiated Three-Firm Model with Network Externality

2.1 Preliminary

We develop a three-firm (product) \( \{i, j, k\} \) model in a network industry where each
firm provides a single horizontally differentiated product with a network externality. Applying the frameworks of Economides (1996) and Häckner (2000), we assume a linear inverse demand function of firm $i$’s product as follows:

$$ p_i = A - q_i - \gamma Q_{-i} + N(S^e_i), $$

where $Q_{-i} = q_j + q_k$ is the sum of the rival firms’ output, $A$ is the intrinsic market size, $q_i$ is the output of firm $i$, and $\gamma \in (0,1)$ represents the degree of product substitutability. In other words, $1 - \gamma$ represents the degree of product differentiation.

Given equation (1), $N(S^e_i)$ is a network externality function of $S^e_i$, which represents the expected network size of firm $i$’s product. We assume a linear network externality function; $N(S^e_i) = nS^e_i$, where $n \in [0,1)$ represents the degree of network externality. Furthermore, the expected network size of product $i$ is given by:

$$ S^e_i \equiv q^e_i + \phi_h Q^e_{-i}, \quad h = C, M, $$

where $Q^e_{-i} = q^e_j + q^e_k$ is the sum of the rival firms’ expected outputs, $\phi_h \in [0,1]$ is the degree of product $i$’s compatibility (interconnectivity) with the other firms’ product $-i$, subscript $C$ denotes the case of noncooperative Cournot competition, i.e., premerger, and subscript $M$ denotes the case of a merger.

In considering the concept of a fulfilled expectation, following Katz and Shapiro (1985) and Economides (1996), we assume that consumers form expectations regarding network sizes before firms’ output decisions. Thus, when deciding the outputs, the
expected network sizes are given for the firms.

For the following analysis, we make some important assumptions:

Assumptions

(i) \(1 \geq \Delta > 0\) where \(\Delta \equiv \phi_M - \phi_C\).

(ii) \(n > \gamma\).

(iii) \(1 > n + \gamma\).

Assumption (i) implies that the degree of compatibility between insider firms’ products in the merger is larger than that in the premerger. This implies efficiency gains from merger-related synergies for network compatibilities.\(^5\) Assumption (ii) implies a stronger network externality. Assumption (iii) implies that the own-price effect exceeds the cross-price effect at the fulfilled expectation equilibrium.\(^6\) In other words, because Assumption (iii) can be rewritten as \(1 - \gamma > n\), the left-hand side measures the degree of product differentiation. Thus, the degree of network externality is lower than that of product differentiation.

\(^5\) We assume nil or negligible costs to increase the level of compatibility among the merging firms.

\(^6\) \(\left| \frac{\partial p_i}{\partial q_i} \right| > \left| \frac{\partial p_i}{\partial Q_{-i}} \right| \iff 1 - n > \gamma - n\phi_h > \gamma\), at \(q_i^e = q_i\), \(Q_{-i}^e = Q_{-i}\).
Furthermore, we assume that production costs are zero. For example, we readily observe low and even negligible marginal production costs in telecommunications and Internet businesses.

2.2 Premerger: Noncooperative Cournot competition

We consider the initial situation (premerger) where three firms compete on quantities à la Cournot in the market. Based on equation (1), the profit function of firm \( i \) is given by:

\[
\pi_i = \{A - q_i - \gamma Q_i - N(S_i^e)\}q_i.
\]  

(3)

The first-order condition (FOC) of profit-maximization is

\[
\frac{\partial \pi_i}{\partial q_i} = p_i - q_i = A - 2q_i - \gamma Q_i - N(S_i^e) = 0.
\]  

(4)

At a fulfilled expectation, i.e., \( q_i^e = q_i, \quad q_j^e = q_j, \) and \( q_k^e = q_k \), based on equations (2) and (4), we obtain:

\[
A - (2 - n)q_i - (\gamma - n\phi_C)Q_i = 0.
\]  

(5)

Assuming a symmetric equilibrium, i.e., \( q_i = q_j = q_k = q_C \), we derive the following fulfilled expectation Cournot equilibrium.

\[
q_C = \frac{A}{2 - n + 2(\gamma - n\phi_C)}.
\]  

(6)

Because it holds that \( p_C = q_C \), based on equation (4), the profit in the premerger case is expressed as:

\[
\pi_C = (q_C)^2.
\]
2.3 Horizontal merger

We consider the case of horizontal merger where a merger takes place between two firms in the market. Without loss of generality, we assume that there is an insider \( I \), providing two products \( \{i, j\} \) and an outsider \( O \), providing one product \( \{k\} \).

The aggregate profit of the insider under the merger \( M \) is expressed as:

\[
\Pi_M = \pi_i + \pi_j
\]

\[
= \left\{ A - q_i - \gamma Q_{-j} + N(S_i^e) \right\}q_i + \left\{ A - q_j - \gamma Q_{-j} + N(S_j^e) \right\}q_j.
\]

Furthermore, the profit of the outsider is given by:

\[
\pi_O = A - q_k - \gamma Q_{-k} + N(S_k^e)q_k.
\]

Based on equations (7) and (8), the FOCs for the insider and outsider are respectively given by:

\[
\frac{\partial \Pi_M}{\partial q_i} = p_i - q_i - \gamma q_j = A - 2q_i - 2\gamma q_j - \gamma q_k + N(S_i^e) = 0, \tag{9}
\]

\[
\frac{\partial \pi_O}{\partial q_k} = p_k - q_k = A - 2q_k - \gamma Q_{-k} + N(S_k^e) = 0, \tag{10}
\]

where we can obtain a similar FOC to equation (9) with respect to product \( j \).

At a fulfilled expectation, i.e., \( q_i^e = q_i, q_j^e = q_j, \) and \( q_k^e = q_k \), in view of equations (2), (9), and (10), we have the following equations.

\[
A - (2 - n)q_i - (2\gamma - n\phi_M)q_j - (\gamma - n\phi_C)q_k = 0, \tag{11}
\]
\[ A - (2-n)q_k - (\gamma - n\phi_C)Q_k = 0. \]  

Assuming a symmetric equilibrium, i.e., \( q_i = q_j = q_I \) and \( q_k = q_O \), equations (11) and (12) can be rewritten as:

\[
A - (2-n)(2-\gamma)q_I - (\gamma - n\phi_C)q_O = 0, \tag{13}
\]

\[
A - (2-n)q_O - 2(\gamma - n\phi_C)q_I = 0. \tag{14}
\]

Thus, we derive the following fulfilled expectation equilibrium at the merger.

\[
q_I = \frac{2-n-(\gamma - n\phi_C)}{D} A, \tag{15}
\]

\[
q_O = \frac{2-n-(\gamma - n\phi_C) + (\gamma - n\Delta)}{D} A, \tag{16}
\]

where \( D \equiv (2-n)[2-(2\gamma - n\phi_M)] - 2(\gamma - n\phi_C)^2 > 0. \)

Based on equations (15) and (16), we derive the following relationship:

\[
q_I - q_O = \frac{n\Delta - \gamma}{D} (\leftarrow 0 \Leftrightarrow n\Delta > (\leftarrow \gamma). \tag{17}
\]

where \( n\Delta \) denotes the net degree of network compatibilities. Equation (17) shows that if the net degree of network compatibilities is larger (smaller) than the degree of product substitutability, the output per insider firm is larger (smaller) than that of the outsider. In this case, if \( n\Delta > \gamma \), then the merger increases the outputs and prices of the insider compared with those of the outsider. Because it holds that \( p_I = (1+\gamma)q_I \) and \( p_O = q_O \), the profit per insider firm is larger than that of the outsider:

\[ \pi_I = (1+\gamma)(q_I)^2 > \pi_O = (q_O)^2. \] Otherwise, the merger reduces the outputs of the insider.
whereas it may or not increase the prices of the insider compared with the price of the outsider. Thus, the effect on the profits is ambiguous.

In general, we obtain the following relationship for the profits:

$$\pi_i > (\prec) \pi_o \iff \left( \sqrt{1 + \gamma} - 1 \right) (2 - n + (\gamma - n \phi_c)) + n \Delta - \gamma > (\prec) 0. \quad (18)$$

Given equation (18), even with $n \Delta \leq \gamma$, the profit per insider firm can be larger than that of the outsider.\(^7\)

3. The Analysis

3.1 The effects of horizontal merger

3.1.1 Outputs

From equations (6), (15), and (16), with respect to the outputs per firm in the merger

\(^7\) Equation (18) can be rewritten as follows:

$$\pi_i > (\prec) \pi_o \iff \Gamma_{IO}(\gamma) - (1 + \phi_c) \Phi_{IO}(\phi_M, \phi_C, \gamma)n > (\prec) 0,$$

where $\Gamma_{IO}(\gamma) = \sqrt{1 + \gamma} (2 + \gamma) - 2(1 + \gamma) > 0$ and $\Phi_{IO}(\phi_M, \phi_C, \gamma) = \sqrt{1 + \gamma} - \frac{1 + \phi_M}{1 + \phi_C}$.

Thus, if $\Phi_{IO}(\phi_M, \phi_C, \gamma) \leq 0$, e.g., $\phi_M = 1 > \phi_C = 0$, it holds that $\pi_i > \pi_o$. Conversely, if $\Phi_{IO}(\phi_M, \phi_C, \gamma) > 0$, e.g., $\phi_M \approx \phi_C$, we derive the following relationship:

$$\pi_i > (\prec) \pi_o \iff N_{IO}(\phi_M, \phi_C, \gamma) > (\prec) n,$$

where $N_{IO}(\phi_M, \phi_C, \gamma) = \frac{\Gamma_{IO}(\gamma)}{(1 + \phi_c) \Phi_{IO}(\phi_M, \phi_C, \gamma)} > 0$. 

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compared with those in the premerger, we obtain the following:

\[ q_I - q_C = \frac{(2 - n)(n\Delta - \gamma)A}{D[2 - n + 2(\gamma - n\phi_C)]}, \]

\[ q_O - q_C = \frac{(\gamma - n\phi_C)(\gamma - n\Delta)A}{D[2 - n + 2(\gamma - n\phi_C)]}, \]

Thus, we derive the following Lemma 1:

Lemma 1

(i) \( q_I > (\gamma)q_C \Leftrightarrow n\Delta > (\gamma). \)

(ii) \( q_O > (\gamma)q_C \Leftrightarrow (\gamma - n\phi_C)(\gamma - n\Delta) > (\gamma). \)

In view of equations (5) and (11), if \( n\phi_C > (\gamma). \) then the strategic relationship for the outsider (or competition firm \( k \)) is complementary (substitutionary). Similarly for the outsider firm \( k \), in view of equation (12), if \( n\phi_C > (\gamma). \) the strategic relationship for the insider firm \( i \) (and \( j \)) is complementary (substitutionary). Furthermore, if \( n\Delta > (\gamma). \) then the reaction curve of the insider firm \( i \) shifts upward (downward) compared with the premerger case. Thus, the insider firm has an incentive to increase (decrease) output.

If \( n\Delta > \gamma \) and \( n\phi_C > \gamma \), the merger increases the outputs of the insider and outsider firms compared with those in the premerger case. However, if \( n\phi_C > \gamma \), then
the merger increases the output of the insider firm while it decreases the output of the outsider firm compared with those in the premerger case.

Conversely, if $n\Delta < \gamma$ and $n\phi C < \gamma$, the merger increases the output of the outsider while it decreases that of the insider compared with those in the premerger case. This is the same well-known result found in previous studies on horizontal merger without network externalities. However, if $n\phi C > \gamma$, then the merger decreases the outputs of the insider and outsider firms compared with those in the premerger case.

3.1.2 Consumer surplus

Consumer surplus in the cases of a merger and a premerger are as follows:

$$CS_M = (1 + \gamma)(q_I)^2 + 2\gamma q_I q_O + \frac{1}{2}(q_O)^2$$ and $$CS_C = \frac{3(1 + 2\gamma)}{2}(q_C)^2.$$ Thus, with respect to the difference in consumer surplus between the cases of a merger and a premerger, i.e., the effect of a merger on consumer surplus, we derive the following relationships:

$$\Delta CS_M = CS_M - CS_C = (1 + \gamma)(q_I)^2 - (q_C)^2 + 2\gamma(q_I q_O - (q_C)^2) + \frac{1}{2}(q_O)^2 - (q_C)^2.$$

Using equations (19) and (20), we derive the following Lemma 2:

Lemma 2

If $n\Delta > (\gamma)$, it holds that $\Delta CS_M > (\gamma)0$. 

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Proof. See Appendix.

The condition for Lemma 2 can be rewritten as: 
\[ n\Delta > (\gamma) \iff n\phi_M > (\gamma)n\phi_C + \gamma. \]
That is, if the level of network compatibilities in the merger, i.e., \( n\phi_M \), is sufficiently large, then the merger increases consumer surplus compared with the premerger case. In this case, even if the output of the outsider decreases, because an increase in the outputs of the insider is sufficiently large, as a result, consumer surplus increases compared with the premerger case.\(^8\) Otherwise, the opposite case arises. For example, Hüschelrath and Müller (2014) show that mergers in airline networks increase consumer welfare.

3.1.3 Profits

To compare the profit a firm in the merger case to that in the premerger case, we define the difference in the profits between them as follows: 
\[ \Delta \Pi_M = 2(\pi_I - \pi_C) \]
and 
\[ \Delta \pi_O = \pi_O - \pi_C. \]
In this case, taking equations (6), (15), and (16), we have the following Lemma 3:

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\(^8\) With respect to total outputs in the merger case compared with the premerger case, using equations (19) and (20), we have the following relationship:
\[ Q_M > (\gamma)Q_C \iff 2(q_I - q_C) + (q_O - q_C) > (\gamma)0 \iff n\Delta > (\gamma). \]
Lemma 3

$$\Delta \Pi_m > (-)0 \iff \pi_i > (-)\pi_C$$

(i) $$\iff (\sqrt{1 + \gamma - 1}[2 - n + 2(\gamma - n\phi_C)](2 - n - (\gamma - n\phi_C)) + 2(n\Delta - \gamma) > (-)0.$$

(ii) $$\Delta \pi_o > (-)0 \iff \pi_o > (-)\pi_C \iff (n\phi_C - \gamma)(n\Delta - \gamma) > (-)0.$$

In view of Lemma 4 (i), if $n\Delta > \gamma$, i.e., the reaction curve of the insider shifts upward, then the merger increases the profit per insider firm compared with the premerger case. This is because, as in Lemma 1, the output and price of the insider increase compared with those in the premerger case.

Contrarily, if $n\Delta < \gamma$, although the output in the merger case decreases, i.e., $q_C > q_I$, the price may rise, compared with those in the premerger case. In particular, if $q_C > q_I > 0$, then it holds that $p_I > p_C$. Otherwise, the profit in the merger case is lower than the profit in the premerger case, i.e., an unprofitable merger. In other words, it is a necessary condition for the merger paradox that $n\Delta < \gamma$.

Furthermore, with respect to Lemma 4 (ii), if either $n\Delta > \gamma$ and $n\phi_C > \gamma$ or $n\Delta < \gamma$ and $n\phi_C < \gamma$, then the merger increases the profit of the outsider. The former case corresponds to efficiency from merger-related synergies for network

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9 Gugler and Szücs (2016) use datasets from the European Commission and find positive merger externalities.
compatibilities. The latter is the same as the case without network externalities.

However, if either $n\Delta > \gamma > n\phi_C$ or $n\Delta < \gamma < n\phi_C$, the profit of the outsider decreases compared with the premerger case. For the former case, because the level of network compatibility of the insider is sufficiently large while that of the outsider is small, the output and price of the outsider decrease with increased output of the insider because of strategic substitutes. For the latter case, the profit of the outsider decreases with decreasing output of the insider because of strategic complements. The AOL–Time Warner case is an example of the former case.

3.2 Antitrust policy and the evaluation of horizontal merger

As mentioned in the Introduction, i.e., “Williamson trade-off,” the evaluation of proposed horizontal mergers involves the following two factors: mergers may increase market concentration whereas they may create efficiencies. We know that there are various evaluation rules and merger controls used by antitrust authorities. In particular, there has been considerable debate concerning whether consumer surplus or social (total) surplus should be the criteria used by the antitrust authorities.\(^{10}\)

For example, based on the three-firm model in this paper, social surplus in the

\(^{10}\) See Farrell and Katz (2006) and Heyer (2006). Because this issue is beyond the scope of this paper, we do not consider it in detail.
merger case is given by: \[ W_M \equiv CS_M + \Pi_M + \pi_O, \] where \( \Pi_M = 2\pi_I \). Thus, the welfare change induced by a merger can be expressed as: \[ \Delta W_M = \Delta CS_M + \Delta \Pi_M + \Delta \pi_O, \] where \( \Delta CS_M = CS_M - CS_C \), \( \Delta \Pi_M = 2(\pi_I - \pi_C) \), and \( \Delta \pi_O = \pi_O - \pi_C \). Given the profitable merger, i.e., \( \Delta \Pi_M > 0 \), if the antitrust authorities use the social surplus standard, the merger will be allowed, i.e., \( \Delta W_M > 0 \). In this case, the condition \( \Delta W_M - \Delta \Pi_M = \Delta CS_M + \Delta \pi_O > 0 \) is not necessary if \( \Delta \Pi_M > 0 \) is sufficiently large. Furthermore, even if it holds that \( \Delta CS_M + \Delta \pi_O > 0 \), it is possible that \( \Delta \pi_O > 0 > \Delta CS_M \). This implies that social surplus increases through a merger if the increase in producer surplus is sufficiently large, i.e., \( \Delta \Pi_M > 0 \) and \( \Delta \pi_O > 0 \), even though consumer surplus decreases. In this case, if the antitrust authorities approve the proposed merger, based on the social surplus standard, this decision may not be preferable for consumers.

In this paper, following Neven and Röller (2005) and Nocke and Whinston (2010, 2013), we use the consumer surplus standard. For example, Neven and Röller (2005) consider three related parties: consumers, merging (insider) firms, and nonmerging

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11 The points plotted, i.e., B and C, lie in the northeast half-plane in Figure 1 in Farrell and Shapiro (1990, p. 117).

12 As mentioned by Nocke and Whinston (2010, 2013), antitrust authorities apply a consumer surplus standard to merger approval decisions in the US and to legal regimes in the EU, e.g., the Horizontal Merger Guidelines of the DOJ and FTC.
(outsider) firms. In particular, consumers do not lobby antitrust authorities because they may not be informed about the consequences of proposed mergers and because they may incur prohibitive costs in representing their interests. Furthermore, in Section 3.3, we consider a case in which a proposed merger may affect the interests of an outsider firm negatively.

Therefore, we assume that antitrust authorities will allow a proposed merger if and only if it holds that \( \Delta CS_M > 0 \). This criterion is more restrictive than that based on a social surplus standard, i.e., \( \Delta W_M > 0 \).

In view of Lemma 2 and 3 (i), if it holds that \( n\Delta > \gamma \), the proposed merger satisfies not only the consumer surplus criteria, but also the profitability of the merger. We note that, given equation (18), the merger is internally sustainable among the insider firms because they do not have an incentive to be an outsider.\(^{13}\) Furthermore, with respect to the external effect on the profits of the outsider, based on Lemma 3 (ii), if it holds that \( n\phi_c > \gamma \), then the merger increases the profit of the outsider compared with that in the

\(^{13}\) To confirm that the merger is externally sustainable, we should examine whether the outsider providing product \( \{k\} \) has an incentive to be merged into the existing merger providing products \( \{i, j\} \) and whether the profits of the existing insider firms do not decrease if the outsider is merged. Thus, if the outsider has an incentive to be merged and the profits of the insider firms do not fall, a multiproduct monopoly providing three products \( \{i, j, k\} \) arises in the market. However, this is an issue for future research.
premerger case, i.e., positive merger externalities. As a result, under these conditions, social surplus increases. Let us summarize the result as Proposition 1.

Proposition: The welfare-improving merger

If the degrees of network compatibilities are sufficiently larger than that of product substitutability, i.e., \( n\phi_M > n\phi_C + \gamma > n\phi_C > \gamma \), then the proposed horizontal merger increases consumer surplus, profits, and thus social surplus compared with the premerger case.

However, we assume that these products are incompatible in the premerger case, i.e., \( \phi_C = 0 \). In this case, because it holds that \( n\phi_C < \gamma \), the external effect on the profit of the outsider is negative, based on Lemma 3 (ii). Thus, the outsider firm has an incentive to bring a case to the court. In addition, considering the judgement of the case, the antitrust authorities may not allow the proposed merger. Although the proposed merger itself is preferable for consumers as well as the corresponding firms, it could be rejected by the antitrust authorities affected by the lobbying of the outsider firm. In the next section, as an application of our model, we consider the problem related to this case.

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14 See Gugler and Szücs (2016) for an empirical analysis of merger externalities.
3.3 Application: The AOL–Time Warner case

As mentioned above, related to the case where a merger reduces the profit of the outsider compared with the premerger case, we apply our model to the AOL–Time Warner case as an example. Although Faulhaber (2002) examines the AOL–Time Warner case in detail, in this paper, we cite the following description in Gandal (2002, p. 87) about how network effects affect merger policy:

“One of the main concerns of the regulatory agencies, i.e., FTC and the Federal Communications Commission (FCC), was interoperability or compatibility between AOL’s instant-messaging (IM) services and those of competitors. Although AOL offered a basic IM service before the proposed merger, merging IM services require broadband capabilities. AOL gained significant broadband capabilities with its acquisition of Time Warner. Hence, the FCC imposed the conditions that AOL must offer an interoperability with other providers of advanced IM services before it is allowed to offer such services itself.”

Using our model, we will try to interpret the case cited above. Assuming that the level of compatibility of the outsider can be expressed as $\phi_D$, we derive the following output of the outsider at the equilibrium under the merger:
Furthermore, with respect to the profit of the outsider compared with the premerger case, we obtain the following:

\[ \pi'_O > (<) \pi_C \iff (2 - n - \gamma + n\phi_O)\gamma (\phi_O - \phi_C) + (\gamma - n\phi_C) (\gamma - n(\phi_M - \phi_O)) > (<) 0. \]  

(22)

Initially, we assume that \( \phi_O = \phi_C \). In this case, Lemmas 3 (ii) holds. That is, equation (22) can be rewritten as follows:

\[ \pi'_O > (<) \pi_C \iff (\gamma - n\phi_C)(\gamma - n\Delta) > (<) 0. \]  

(23)

Now, based on the rule of the consumer surplus standard, i.e., \( \Delta CS_M > 0 \), the regulatory agencies allow the proposed merger if it satisfies the condition that the insider must offer interoperability with the outsider competitors (e.g., the other providers), as cited in italic above. In this situation, it holds that \( n\Delta > \gamma \). In addition, let us assume that if the insider firm does not offer interoperability with the outsider firm, its profit decreases, i.e., \( \pi'_O < \pi_C \). This implies that, in view of Lemma 3 (ii) and equation (23), it holds that \( \gamma > n\phi_C \). That is, as mentioned in Section 3.2, because it holds in this situation that \( n\Delta > \gamma \) and \( \gamma > n\phi_C \), negative merger externalities arise.

Following the condition imposed by the regulatory agencies, if the insider firm offers the same level of interoperability (compatibility) as that of the insider to the outsider, i.e., \( \phi_O = \phi_M \), based on equation (22), we obtain the following:
Therefore, because the profit of the outsider firm increases compared with the premerger case, the proposed merger can be allowed by the regulatory agencies.

4. Conclusions

Horizontal merger leads to monopolization and market concentration and various efficiencies are also created. In this paper, focusing on the role of network externalities and compatibilities, i.e., network compatibility (connectivity), we considered a horizontal merger and associated merger policy in a network products market. In particular, based on the model where participating firms cooperatively increase the level of compatibility (interconnectivity) between their products, we demonstrated that a social welfare-improving merger can arise in an oligopolistic industry, if the degree of network compatibilities of the products is sufficiently large.

Although we have used a consumer surplus standard as a criterion for evaluating a proposed merger, there are various criteria for judging planned mergers, e.g., Herfindahl–Hirschman Index and an upward price pressure Index. Relating to the latter
index, we have assumed a zero marginal cost of production in our model. That is, because we have focused on the efficiency gains on the demand-side such as a rising level of network compatibilities by mergers, we have not dealt with efficiency gains on the supply-side such as cost savings (decreasing marginal costs). Thus, in our model, horizontal mergers increase not only prices but also outputs. However, considering efficiency gains on both the supply-side and demand-side, we can demonstrate that horizontal mergers decrease prices but increase outputs. This is because horizontal mergers shift the demand curve upward whereas they shift the marginal cost curve downward simultaneously.

We appreciate that the results have some limitations because our model is based on specific assumptions, e.g., quantity competition, the linearity of functions, a three-firm case, and strong network externalities. In future, we intend to discuss more general cases by relaxing these assumptions and extending the model to oligopolistic competition. Furthermore, as mentioned above, we should examine the stability of the merger. To do so, we must consider the full merger case where the merger is composed of all firms existing in the market. These issues should be addressed in future research.
Appendix: Proof of Lemma 3

Equation (20) can be rewritten as follows.

\[
\Delta CS_M = \left(\frac{1}{2} - \gamma\right)\Delta CS_{M1} + 2\gamma\Delta CS_{M2} + \frac{1}{2}\Delta CS_{M3},
\]

(A.1)

where

\[
\Delta CS_{M1} \equiv (q_I)^2 - (q_C)^2,
\]

(A.1.1)

\[
\Delta CS_{M2} \equiv (q_I)^2 + q_I q_O - 2(q_C)^2,
\]

(A.1.2)

\[
\Delta CS_{M3} \equiv (q_I)^2 + (q_O)^2 - 2(q_C)^2.
\]

(A.1.3)

Regarding equation (A.1), based on assumptions (ii) and (iii), it holds that \(\frac{1}{2} > \gamma\).

Substituting equations (6), (15), and (16) into equations (A.1.1), (A.1.2), and (A.1.3), we derive the following:

\[
\Delta CS_{M1} = \frac{n\Delta - \gamma}{D} (q_C)^2 G_1, \quad \Delta CS_{M2} = \frac{n\Delta - \gamma}{D} (q_C)^2 G_2, \quad \text{and} \quad \Delta CS_{M3} = \frac{n\Delta - \gamma}{D} (q_C)^2 G_3,
\]

where

\[
G_1 \equiv 2(2 - n) + \frac{(2 - n)(n\Delta - \gamma)}{D} > 0,
\]

\[
G_2 \equiv 2(2 - n) + \{2 - n - 2(\gamma - n\phi_C)\} + \frac{(2 - n)(2 - n - 2(\gamma - n\phi_C))(n\Delta - \gamma)}{D} > 0,
\]

\[
G_3 \equiv 2[2 - n - 2(\gamma - n\phi_C)] + \left\{\frac{(2 - n)^2 + 4(\gamma - n\phi_C)^2}{D}(n\Delta - \gamma)\right\} > 0.
\]

In this case, equation (A.1) can be rewritten as follows:

\[
\Delta CS_M = \frac{n\Delta - \gamma}{D} (q_C)^2 \left\{\left(\frac{1}{2} - \gamma\right)G_1 + 2\gamma G_2 + \frac{1}{2} G_3\right\}.
\]

(A.2)

Therefore, we obtain that \(\Delta CS_M (>0) \Leftrightarrow n\Delta (>0)\gamma\).
References


