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Elderly Care Service in an Aging Society

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Elderly Care Service in an Aging Society[†]

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Abstract

An increase in life expectancy brings about an aging society, necessitating increasing demand for elderly care services. This paper presents an examination of how an aging society affects the demand for elderly care services and the labor market for elderly care services. Related reports of the literature describe that an aging society raises the share of labor dedicated to elderly care services. However, considering a closed economy in which saving affects the capital stock, an aging society does not always raise the share of labor allocated for elderly care services as derived by the related literature. This paper presents an examination of how the labor share and wage inequality between the final goods sector and elderly care sector are determined. In addition, this paper presents an examination of whether the subsidy for elderly care service increases demand for elderly care services, or not.

Keywords: Aging society, Elderly care service, Labor mobility, Two-sector model

JEL Classifications: H14, J21, H20

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1. Introduction

This paper presents an examination of how an aging society affects the elderly care services sector. In some OECD countries, including Japan and Germany, elderly care services are provided through long-term care insurance. Many reports of the related literature describe long-term care insurance that can be used if individuals need elderly care services. Pauly (1990) and Miyazawa, Moudoukoutas and Yagi (2000) have demonstrated that long-term care insurance should be managed by the government, not by private insurance, because of adverse selection and moral hazard problems. Because of long-term insurance, individuals might fail to make an effort to improve or maintain their health status. Thereby, inefficiencies arise. Some studies have examined optimal subsidies for elderly care and optimal long-term care insurance. Subsidies for elderly care services decrease the costs of using the long-term care insurance, but taxation is expected to be necessary to finance the subsidy. Income taxation decreases household disposable income. Then savings decrease. Consequently, capital accumulation is prevented, which slows income growth. Considering these positive and negative effects of subsidies, Lundholm and Ohlsson (1998), Tabata (2005), and Mizushima (2009) derive optimal subsidies for long-term care. Cremer and Pestiau (2014) examine public subsidies for elderly care services as a redistribution policy.

Long-term care of two types exists: formal care and informal care. Formal care is provided by the market service: if individuals need elderly care, they buy elderly care services from the market. However, the children provide elderly care with their time. They must reduce the labor supply time because of the time necessary for informal care. This point is examined by Byrne, Goeree, Hiedemann and Stern (2009), Courbage and Zweifiel (2011), Korn and Wrede (2013), and Mou and Winer (2015). Younger people must remain dedicated to informal elderly care. Therefore, they cannot obtain sufficient income to survive. They therefore descend into poverty.

In an aging society, the labor supply of the elderly care services sector increases with the progress of aging. This result is explained by Hashimoto and Tabata (2010). Both the ratio of older people among the population and the labor supplied to the elderly care services sector increase are presented in Fig. 1. However, a problem exists by which the wage rate of the elderly care services sector is lower than that of other industries, as presented in Fig. 2. These data are consistent with those presented by Cremer and Pestieau (2012) who explain that elderly care services will necessarily increase in OECD countries.

[Insert Fig. 1 around here.]

[Insert Fig. 2 around here.]

Aisa and Pueyo (2013) derive that an aging society can decrease the labor supplied to the elderly care services sector. An aging society has higher levels of household savings because households require savings for consumption. An increase in savings raises the capital stock and the labor productivity, which then increases the wage rate of the sector that uses the capital stock as an input. Some papers examine the effect of an aging society on the labor supplied to elderly care service sector in both small open and closed economies.

In contrast to those earlier efforts, this paper presents derivation of how an aging society affects the wage inequality between the final goods sector and elderly care sector. Neither Hashimoto and Tabata (2010) nor Aisa and Pueyo (2013) examine wage inequality. As presented in Fig. 2, this wage inequality is a problem by which workers in elderly care services work at low wages, which brings about a shortage of labor in the elderly care services sector. To consider wage inequality and labor mobility, this paper uses a model based on Caselli (1999), which considers the training cost necessary to work in a high-skill sector. By considering the training cost for labor mobility, wage inequality can be examined.

Results obtained through these analyses are presented as follows. An increase in life expectancy decreases the labor supply for the elderly care services sector. By considering not the small open economy but the closed economy, an aging society does not always increase its supply of labor for the elderly care services sector. Wage inequality between the final goods sector and the elderly care sector is diminished by an increase in life expectancy.

Moreover, this paper presents an examination of how the subsidy for elderly care service affects the labor share of elderly care services and wage inequality. Canta, Pestiau and Thibault (2016) describe consideration of how elderly care services affect capital accumulation via financing of subsidies for elderly care services. However, they do not examine wage inequality. The present paper has some new points and consideration of the importance of incorporating wage inequality.

Considering the results presented in this paper, an aging society reduces the elderly care sector workers and raises the wage rate of the elderly care sector as a pure effect. However, the subsidy for elderly care service raises the elderly care sector worker wages. Wage inequality between two sectors shrinks as shown by the results presented herein. The results are consistent with real data, as shown in Fig. 1 and Fig. 2.

The remainder of this paper consists of the following. Section 2 explains the model. Section 3 derives the equilibrium. Section 4 presents an examination of the effects of an aging society. Section 5 reports effects of subsidies on elderly care services. Section 6 examines the effect of an aging society and the subsidy for elderly care service on the labor market with numerical examples. The final section concludes this paper.

2. Model

There exist agents of three types: households, firms, and a government.

2.1 Household

This paper assumes that individuals in households exist in two periods: young and old. In each period, younger people and older people live simultaneously. This paper presents consideration of a two-period overlapping generations model assuming no population growth and assuming population size as unity over time. These analyses also assume a survival rate: some younger people cannot live to the old period.

Individuals obtain utility from consumption in young and old periods, and from elderly care services for themselves in old periods. The utility function u_t is assumed as

$$u_t = \alpha ln c_{1t} + p(1-\alpha)\beta ln c_{2t+1} + p(1-\alpha)(1-\beta) ln e_{t+1}, 0 < \alpha < 1, 0 < \beta < 1. \quad (1)$$

where c_{1t} and c_{2t+1} respectively denote consumption during young and old periods.[1] e_{t+1} denotes the demand for elderly care services. p denotes the survival rate and the share of p at the younger people in t period can live in the old period. The range of p is assumed as $0 < p < 1$.

During the young period, younger people work inelastically and obtain the wage income. Wage income is allocated into the consumption during the young period and savings for consumption during the old period. Then, the budget constraints in young and old periods are shown as

$$\bar{w}_t = c_{1t} + s_t, \quad (2)$$

$$R_{t+1} s_t = c_{2t+1} + z_{t+1} e_{t+1}, \quad (3)$$

where $\bar{w}_t$ and s_t respectively denote the wage rate and saving. In addition, R_{t+1} denotes the interest rate of annuity. The $1-p$ share of younger people cannot live to the old period. The savings and interest that the $1-p$ share of people had are distributed equally for the share p of people. The interest rate at that time is given as R_{t+1}. z_{t+1} denotes the price of elderly care services.

Allocations to maximize the utility function (1) subject to the budget constraint (2) and (3) are reduced to the following.[2]

$$c_{1t} = \frac{\alpha}{\alpha + p(1-\alpha)} \bar{w}_t, \quad (4)$$

$$c_{2t+1} = \frac{R_{t+1} p(1-\alpha)\beta}{\alpha + p(1-\alpha)} \bar{w}_t, \quad (5)$$

$$e_{2t+1} = \frac{R_{t+1}}{z_{t+1}} \frac{p(1-\alpha)(1-\beta)}{\alpha + p(1-\alpha)} \bar{w}_t. \quad (6)$$

2.2 Firms

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[1] Even if the heterogeneous household is assumed to consider heterogeneous demand for elderly care services, the amount presented in the paper is not changed because the heterogeneous demand for elderly care does not play a role in changing the result.

[2] Data show that the labor participation of elderly people rises in an aging society in developed countries such as Japan. On the other hand, this paper does not consider the labor participation of elderly people. However, even if the paper considers the labor participation of elderly people, the results presented herein do not change because eq. (30) always holds irrespective of the labor supply. (Data: Statistics Japan)

This model economy has firms of two types, respectively serving the final goods sector and elderly care sector. The production function in the final goods sector is assumed as

$$Y_t = K_t^{\theta} L_t^{1-\theta}, 0 < \theta < 1, \tag{7}$$

where K_t and L_t respectively denote capital stock and labor input. Also, Y_t denotes the final goods. Final goods are used as consumption goods and investment goods. With perfect competition, the wage rate in the final goods sector and interest rate are shown as

$$w_t = (1-\theta) K_t^{\theta} L_t^{-\theta}, \tag{8}$$

$$1 + r_t = \theta K_t^{\theta-1} L_t^{1-\theta}. \tag{9}$$

R_t is given as $R_t = \frac{1+r_t}{p}$. If younger people want to work in the final goods sector, then they must pay the training cost σ_t. σ_t is assumed to distribute uniformly between $[0, \bar{\sigma}]$. Then, younger people can obtain the net wage rate as $w_t - \sigma_t$.[3]

In this model economy, there exists an elderly care sector as another sector. The production function in the elderly care sector is assumed as

$$Y_t^c = \rho L_t^c, 0 < \rho, \tag{10}$$

where L_t^c denotes labor supply for elderly care. Considering profit function π_t as $\pi_t = z_t \rho L_t^c - w_t^c L_t^c$, the wage rate in elderly care sector in the case of perfect competition is the following.

$$w_t^c = z_t \rho, \tag{11}$$

3. Equilibrium

This section presents derivation of the equilibrium. Considering the equilibrium, occupational choice should be considered. An individual in a household has training cost σ_t to work in the final goods sector. Then, the following three types of work are shown.

$$w_t^c > w_t - \sigma_t, \text{ Working in the elderly care sector} \tag{12a}$$

$$w_t^c < w_t - \sigma_t, \text{ Working in the final goods sector} \tag{12b}$$

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[3] This setting is the same as that described by Caselli (1999), who considers the effect of the change on the labor mobility between two sectors. This paper can consider the effect of change $\bar{\sigma}$ on labor mobility. The wage rate of the final goods sector rises as shown by the wage rate (23) if $\bar{\sigma}$ rises. Then, there exists a positive effect on the labor supply in the final goods sector. As another form of training cost, $w_t \sigma_t$ can be considered. Then, an increase in the wage rate raises the training cost. This paper assumes the same training costs as those posited by Caselli (1999).

$$w_t^c = w_t - \sigma_t, \text{ Indifferent} \tag{12c}$$

Therefore, with small σ_t, individuals work in the final goods sector because the utility in the case in which that they work in final goods sector is greater than the case in which they work in the elderly care sector. Otherwise, they work in the elderly care sector.

If individuals work in final goods sector, then the following allocations are obtained because of $\bar{w}_t = w_t - \sigma_t$, (4)–(6):

$$c_{1t} = \frac{\alpha}{\alpha + p(1-\alpha)}(w_t - \sigma_t), \tag{13}$$

$$c_{2t+1} = \frac{R_{t+1}p(1-\alpha)\beta}{\alpha + p(1-\alpha)}(w_t - \sigma_t), \tag{14}$$

$$e_{2t+1} = \frac{R_{t+1}}{z_{t+1}}\frac{p(1-\alpha)(1-\beta)}{\alpha + p(1-\alpha)}(w_t - \sigma_t). \tag{15}$$

If the individuals work in the elderly care sector, then the following allocations are obtained because of $\bar{w}_t = w_t^c$, (4)–(6):

$$c_{1t} = \frac{\alpha}{\alpha + p(1-\alpha)}w_t^c, \tag{16}$$

$$c_{2t+1} = \frac{R_{t+1}p(1-\alpha)\beta}{\alpha + p(1-\alpha)}w_t^c, \tag{17}$$

$$e_{2t+1} = \frac{R_{t+1}}{z_{t+1}}\frac{p(1-\alpha)(1-\beta)}{\alpha + p(1-\alpha)}w_t^c. \tag{18}$$

The amounts of labor supplied to the final goods sector and in the elderly care sector are given respectively as follows.[4]

$$L_t = \frac{\sigma_t^*}{\bar{\sigma}}, \text{ Labor supply in final goods sector} \tag{19}$$

$$L_t^c = \frac{\bar{\sigma} - \sigma_t^*}{\bar{\sigma}}, \text{ Labor supply in the elderly care sector} \tag{20}$$

Considering the market clearing condition of the elderly care sector at the $t+1$ period, the price of elderly care service is shown as

4 $\frac{\sigma_t^*}{\bar{\sigma}}$ and $\frac{\bar{\sigma} - \sigma_t^*}{\bar{\sigma}}$ respectively denote the labor supply share of final goods sector and elderly care sector. Considering the unity of population size, the labor supply share is the same as the labor supply level. An increase in σ_t^* raises the labor share of final goods sector and reduces the labor share of the elderly care sector.

$$\rho\frac{\bar{\sigma}-\sigma_{t+1}^{*}}{\bar{\sigma}}=p\frac{R_{t+1}}{z_{t+1}}\frac{p(1-\alpha)(1-\beta)}{\alpha+p(1-\alpha)}\left(\int_{0}^{\sigma_t^*}(w_t-\sigma_t)\frac{1}{\bar{\sigma}}d\sigma+\int_{\sigma_t^*}^{\bar{\sigma}}w_t^c\frac{1}{\bar{\sigma}}d\sigma\right). \tag{21}$$

σ_t^* is given as

$$\sigma_t^*=w_t-w_t^c=w_t-\rho z_t. \tag{22}$$

Then, the wage rate (8) and interest rate (9) are given as

$$w_t=(1-\theta)K_t^{\theta}\left(\frac{\sigma^*}{\bar{\sigma}}\right)^{-\theta}, \tag{23}$$

$$1+r_t=\theta K_t^{\theta-1}\left(\frac{\sigma^*}{\bar{\sigma}}\right)^{1-\theta}. \tag{24}$$

The equilibrium of capital market is given as $K_{t+1}=\int_0^{\bar{\sigma}}s_t\frac{1}{\bar{\sigma}}d\sigma$:

$$K_{t+1}=\frac{p(1-\alpha)}{\alpha+p(1-\alpha)}\left(\int_{0}^{\sigma_t^*}(w_t-\sigma)\frac{1}{\bar{\sigma}}d\sigma+\int_{\sigma_t^*}^{\bar{\sigma}}w_t^c\frac{1}{\bar{\sigma}}d\sigma\right)=\frac{p(1-\alpha)}{\alpha+p(1-\alpha)}\left(w_t-\sigma_t^*+\frac{\sigma_t^{*2}}{2\bar{\sigma}}\right). \tag{25}$$

In addition, (21) is reduced to

$$(w_{t+1}-\sigma_{t+1}^*)\left(1-\frac{\sigma_{t+1}^*}{\bar{\sigma}}\right)=(1-\beta)(1+r_{t+1})K_{t+1}. \tag{26}$$

In addition, (26) holds for any t period. Therefore, the dynamics in this paper is specified by the capital stock K_t. Given K_t, the equilibrium $K_{t+1}, z_t, \sigma_t^*, w_t, 1+r_t$ are given respectively by (22)–(26). The equilibrium in the steady state is shown as

$$K=\frac{p(1-\alpha)}{\alpha+p(1-\alpha)}\left((1-\theta)K^{\theta}\left(\frac{\sigma^*}{\bar{\sigma}}\right)^{-\theta}-\sigma^*+\frac{\sigma^{*2}}{2\bar{\sigma}}\right), \tag{27}$$

$$(1-\theta)K^{\theta}\left(\frac{\sigma^*}{\bar{\sigma}}\right)^{-\theta}\left(1-\frac{\sigma^*}{\bar{\sigma}}\right)-\sigma^*+\frac{\sigma^{*2}}{\bar{\sigma}}=(1-\beta)\theta K^{\theta}\left(\frac{\sigma^*}{\bar{\sigma}}\right)^{1-\theta}. \tag{28}$$

Variables in the steady state K, σ^* are obtained. The price of elderly care z, wage rate w, and an interest rate $1+r$ are given respectively as (22)–(24).

The wage rate (23) can be shown by another equation. Because of constant returns to scale of the production function of the final goods sector, $(1+r_{t+1})K_{t+1}=\frac{\theta}{1-\theta}w_{t+1}L_{t+1}$ is obtainable. Substituting this equation into (26), (26) changes to the following equation.

$$\frac{w_t}{\sigma_t^*}=\frac{1-\frac{\sigma_t^*}{\bar{\sigma}}}{1-\left(1+\frac{\theta(1-\beta)}{1-\theta}\right)\frac{\sigma_t^*}{\bar{\sigma}}}. \tag{29}$$

Because of positive wage rate w_t, the following inequality must hold.

$$\frac{\sigma_t^*}{\bar{\sigma}} < \frac{(1-\theta)}{(1-\theta)+(1-\beta)\theta}. \tag{30}$$

4. Aging Society

This section presents an assessment of how an increase in the number of elderly people deriving from an increase in life expectancy affects the elderly care service market.

By total differentiation of (27) with respect to K, σ^*, p, the following equation is obtained as

$$(1-A)\frac{dK}{dp} - B\frac{d\sigma^*}{dp} = \frac{\alpha K}{\big(\alpha + p(1-\alpha)\big)p}, \tag{31}$$

where

$$A = \frac{p(1-\alpha)}{\alpha + p(1-\alpha)}\frac{\partial w}{\partial K} > 0,$$

$$B = \frac{p(1-\alpha)}{\alpha + p(1-\alpha)}\left(\frac{\partial w}{\partial \sigma^*} - \frac{\bar{\sigma}-\sigma^*}{\bar{\sigma}}\right) < 0.$$

By total differentiation of (28) with respect to K, σ^*, we obtain

$$C\frac{dK}{dp} - D\frac{d\sigma^*}{dp} = 0, \tag{32}$$

where[5]

$$C = \left(1 - \frac{\sigma^*}{\bar{\sigma}}\right)\frac{\partial w}{\partial K} - (1-\beta)\theta(1+r),$$

$$D = \frac{(1-\beta)\theta w}{\bar{\sigma}} - \frac{\partial w}{\partial \sigma^*}\left(1 - \frac{\sigma^*}{\bar{\sigma}}\right) + 1 + \frac{w}{\bar{\sigma}} - \frac{2\sigma^*}{\bar{\sigma}} > 0.$$

Therefore, the comparative static analyses reveal that

$$\begin{pmatrix} 1-A & -B \\ C & -D \end{pmatrix}\begin{pmatrix} \frac{dK}{dp} \\ \frac{d\sigma^*}{dp} \end{pmatrix} = \begin{pmatrix} \frac{\alpha K}{(\alpha + p(1-\alpha))p} \\ 0 \end{pmatrix}.$$

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5 D is positive because $\frac{\partial w}{\partial \sigma^*} < 0$ and $1 + \frac{w}{\bar{\sigma}} - \frac{2\sigma^*}{\bar{\sigma}} = 1 - \frac{\sigma^*}{\bar{\sigma}} + \frac{w - \sigma^*}{\bar{\sigma}} > 0$.

$\frac{d\sigma^*}{dp}$ is shown as

$$\frac{d\sigma^*}{dp} = -\frac{\alpha KC}{D\left(A + \frac{BC}{D} - 1\right)\left(\alpha + p(1-\alpha)\right)p}. \tag{33}$$

The sign of the denominator of (33) is negative.[6] Therefore, if $C > 0$, then the sign of $\frac{d\sigma^*}{dp}$ is positive in this model. However, $C > 0$ is obtained because (30) can always hold.

An increase in life expectancy raises the capital stock because

$$\frac{dK}{dp} = -\frac{\alpha K}{\left(A + \frac{BC}{D} - 1\right)\left(\alpha + p(1-\alpha)\right)p} > 0. \tag{34}$$

An increase in capital stock increases the wage rate in final goods market given by (23) because the marginal productivity of labor in final goods market increases for given σ^*. However, increased life expectancy raises demand for elderly care services. The wage rate of elderly care sector rises. Labor mobility occurs from the final goods sector to the elderly care service. This mobility is shown as the following figure.

[Insert Fig. 3 around here.]

However, the case of Fig. 3 is excluded because of the inequality (30). Therefore, in this paper, only the case of Fig. 4 can be shown.[7]

[Insert Fig. 4 around here.]

An increase in life expectancy raises the labor productivity of the final goods sector because of an increase in capital stock. Then, the wage rate of the final goods sector rises. Labor mobility from the elderly care sector to the final goods sector might occur.

An aging society does not always raise the share of labor dedicated to the elderly care sector. The following proposition can be established as shown by the proof above.

6 Because of the local stability condition, the sign of the denominator is negative. See the Appendix for a detailed proof of the local stability condition.

7 The demand curve and supply curve in Figs. 3 and 4 are not linear. However, this paper depicts the linearization at the approximation of the equilibrium.

Proposition 1

An increase in life expectancy increases the capital stock and decreases the labor share of the elderly care services sector.

This proposition shows that even if the wage rate in the elderly care services sector rises by virtue of an increase in demand for elderly care services because of an aging society, the labor supply does not always increase because the relative wage of the elderly care services sector to the final goods sector remains low. An increase in life expectancy raises household savings because the individual can live a long period and the savings for consumption during the old period is needed. An increase in savings raises the capital stock and therefore the labor demand for final goods and services. Then, the relative wage rate of the elderly care sector can remain low; the labor supply for elderly care services might not increase.[8] This result is obtained in this model of a closed economy: an increase in life expectancy raises household savings. With a small open economy with an interest rate and wage rate in the final goods sector, an increase in life expectancy always raises the labor share for the elderly care services sector.

Aisa and Pueyo (2013) set the closed economy considering capital accumulation and derive that the effect of an aging society on the labor supply to elderly care service is ambiguous because of the effect of capital accumulation. However, this paper can derive the negative effect of an aging society on the labor supply to elderly care service in the model of wage inequality, which is not considered by Aisa and Pueyo (2013).[9]

This paper can examine wage inequality between the final goods sector and the elderly care services sector, as shown below.

$$\frac{w}{w^c} = \frac{w}{w - \sigma^*} = \frac{1}{1 - \frac{\sigma^*}{w}}, \tag{35}$$

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[8] An aging society raises capital accumulation. Then the labor share of final goods sector rises. However, the labor share of elderly care service sector decreases. No labor supply occurs in the elderly care sector. Regarding the demand for elderly care service, the positive labor supply in the elderly care sector exists even if the price of the elderly care service is very high.

[9] A large open economy is considered between a small open economy and a closed economy. Then, the effect derived by a closed economy can be nonexistent in a large open economy. The effect of an increase in the survival rate does not always reduce the labor supplied to the elderly care sector.

Because of $\frac{d\frac{w}{\sigma^*}}{d\sigma^*} = \frac{\frac{\theta(1-\beta)}{1-\theta}}{\bar{\sigma}\left(1-\left(1+\frac{\theta(1-\beta)}{1-\theta}\right)\frac{\sigma^*}{\bar{\sigma}}\right)^2} > 0$, an increase in σ^* reduces $\frac{\sigma^*}{w}$. Then, the wage inequality $\frac{w}{w^c}$ decreases. Then, the following proposition can be established.

Proposition 2

An increase in life expectancy reduces the wage inequality $\frac{w}{w^c}$.

This proposition shows that an increase in life expectancy has the effect of both a decrease in labor share of elderly care service and an increase in relative wage rate of the elderly care sector. As shown in Fig. 4.b, we confirm that the wage rate of the elderly care sector rises. Then, the price of elderly care service rises too because of (11).

I would like to compare the results presented in this paper with those presented by Caselli (1999), who considers an increase in the technological parameter, with workers moving from the old technology sector to new technology sector and wage rate of new technology sector rises. As described in this paper, an increase in p plays the role of an increase in technology parameters in Caselli (1999). However, an increase in p affects both sectors. In the final goods sector, an increase in p raises capital accumulation. Also, the wage rate of the final goods sector rises. An increase in p raises the demand for elderly care services and the wage rate of elderly care services. However, the former effect is larger than the latter effect. Therefore, an increase in p has the effect of an increase in technology parameter in the analysis presented by Caselli (1999).

5. Subsidy for Elderly Care Sector

This section presents an examination of how subsidies for elderly care sector affect the labor share of the elderly care sector. The government collects tax revenues and provides subsidies for elderly care services. The tax is levied as lump-sum taxation T_t; the subsidy rate is given as ε $(0 < \varepsilon < 1)$. The budget constraints are shown below.

$$\bar{w}_t - T_t = c_{1t} + s_t, \tag{36}$$

$$R_{t+1}s_t = c_{2t+1} + (1-\varepsilon)z_{t+1}e_{t+1}, \tag{37}$$

In the steady state equilibrium, (25) and (26) can be changed as

$$K = \frac{p(1-\alpha)}{\alpha + p(1-\alpha)}\left(w - \sigma^* + \frac{\sigma^{*2}}{2\bar{\sigma}} - T\right), \tag{38}$$

$$\frac{(1-\varepsilon)(w-\sigma^*)(\bar{\sigma}-\sigma^*)}{\bar{\sigma}} = (1-\beta)(1+r)K. \tag{39}$$

The government budget constraint in the steady state is given as

$$T = \varepsilon pze. \tag{40}$$

Therefore, the comparative statics at the approximation of $\varepsilon = 0$ is derived as

$$\begin{pmatrix} 1-A & -B \\ C & -D \end{pmatrix}\begin{pmatrix} \frac{dK}{d\varepsilon} \\ \frac{d\sigma^*}{d\varepsilon} \end{pmatrix} = \begin{pmatrix} -\frac{(1-\alpha)p}{\alpha + p(1-\alpha)}\frac{dT}{d\varepsilon} \\ (w-\sigma^*)\left(1 - \frac{\sigma^*}{\bar{\sigma}}\right) \end{pmatrix}.$$

Then, $\frac{d\sigma^*}{d\varepsilon}$ are derived as

$$\frac{d\sigma^*}{d\varepsilon} = \frac{(1-A)(w-\sigma^*)\left(1-\frac{\sigma^*}{\bar{\sigma}}\right) + \frac{(1-\alpha)p}{\alpha+p(1-\alpha)}\frac{dT}{d\varepsilon}C}{D\left(A + \frac{BC}{D} - 1\right)}. \tag{41}$$

The sign of (41) is ambiguous. The sign of $\frac{dK}{d\varepsilon}$ is ambiguous because

$$\frac{dK}{d\varepsilon} = \frac{(w-\sigma^*)\left(1-\frac{\sigma^*}{\bar{\sigma}}\right)B + \frac{(1-\alpha)p}{\alpha+p(1-\alpha)}\frac{dT}{d\varepsilon}D}{D\left(A + \frac{BC}{D} - 1\right)}. \tag{42}$$

This result is intuitive. The subsidy for elderly care services, which is a transfer from younger people to older people, can be regarded as a pay-as-you-go transfer. An increase in the lump-sum tax reduces savings because the household disposable income decreases. However, an increase in the subsidy raises demand for elderly care services and raises the wage rate of elderly care services, which raises household disposable income.

The subsidy for elderly care services requires tax revenues. Even if the subsidy raises demand for elderly care services directly, the lump-sum tax reduces the demand indirectly. As long as the effect of the latter is greater than that of the former, the demand for elderly care services decreases; the price and wage rate for elderly care services decreases. Finally, the labor share in the elderly care service decreases. Therefore, the effects of the subsidy include ambiguous effects on the share of labor working for elderly care services and the wage inequality.

6. Numerical Examples

This section presents examination of the effect of an aging society and the subsidy for elderly care on the labor market with numerical examples. This paper sets the parameter as follows.[10]

[Insert Table 1 around here.]

In this section, the product function (7) is defined as $Y_t = AK_t^{\theta}L_t^{1-\theta}$. Then, the dynamics of K_t is depicted as the following figure. As shown by Fig. 5, the dynamics of this model is locally stable.

[Insert Fig. 5 around here.]

6.1 Steady State

Based on the parameter setting that is consistent with the realistic economic situation, this paper presents examination of the change of parameters p, β and ε. The result is shown in the following table.

[Insert Table 2 around here.]

Table 2 presents the result obtained in each case at the steady state. Case 1 is the basic case. Case 2 shows the case in which the survival rate increases and the ratio of older people to total population increases. Then, the labor share of final goods sector σ^* increases. The wage inequality between final goods sector and elderly care sector $\frac{w}{w^c}$ decreases. This result demonstrates that an aging society promotes capital accumulation and the wage rate of final goods sector increases. Then, the labor supply moves from the elderly care sector to the final goods sector. However, the wage rate of the elderly care

[10] Data for the parameter settings are presented in the Appendix.

sector rises because of a decrease in labor supply in the elderly care sector. Therefore, thanks to this effect, the wage inequality $\frac{w}{w^c}$ reduces.

Case 3 shows the case in which that the preference parameter for elderly care service increases. Compared with Case 1, the labor share of elderly care sector increases; wage inequality $\frac{w}{w^c}$ decreases. An increase in demand for elderly care services raises the wage for elderly care services. The labor supply moves from the final goods sector to the elderly care sector.

Case 4 shows the subsidy for elderly care service. Compared with Case 1, the labor share of elderly care service increases and wage inequality reduces. The utility level of the final goods sector worker that has average education cost $\frac{\sigma^*}{2}$ and elderly care service sector workers are defined respectively as $U(F)$ and $U(E)$. By virtue of the subsidy for elderly care services, both $U(F)$ and $U(E)$ increase. The subsidy for elderly care service raises the utility levels of workers of both types.

However, as shown by Case 5, if the preference parameter of the elderly care is large, the subsidy for elderly care services can raise only the utility of elderly care workers.

The results and the data demand some explanation. As shown by Fig. 1, the elderly care workers are positively correlated with the elderly people share. However, Prop 1 shows a negative correlation between the elderly care workers and the elderly people share: an increase in the elderly people share reduces the elderly care workers purely. As shown by the numerical examples in Case 4, the true factor of the positive correlation is the subsidy for elderly care services. Elderly care insurance in Japan started in 2000. After starting the subsidy for elderly care service, one can check the positive correlation.

In addition, one must explain the wage inequality between two sectors $\frac{w}{w^c}$. Prop 2 shows that an aging society reduces wage inequality $\frac{w}{w^c}$ as a pure effect. In Japan, the wage rate of the elderly care sector has increased in recent years. In fact, the low wage rate poses severe difficulties. The government has raised subsidies for the wage rate of the elderly care sector. Case 4 presents the case. Results presented in this paper are consistent with the real data.

6.2 Transitional Path

This paper presents examination of the transitional path of the effect of an aging society and the subsidy for the elderly care service. Effects on the labor share and the wage inequality between the final goods sector and the elderly care sector are shown by the following figures.

[Insert Fig. 6 around here.]

Fig. 6.a presents the case in which p increases by 10%. That is, p changes 0.9 to 0.99. Period -1 shows the steady state before changing p. During the 0 period, the shock occurs. An increase in p increases the wage inequality and decreases the labor supply of elderly care service. However, capital accumulation effects weaken the positive effect on the wage inequality and the negative effect on the labor supply of elderly care services. An increase in capital accumulation raises the wage rate of the final goods sector and the demand for elderly care service increases. By virtue of an increase in the demand for elderly care services, the wages of the elderly care sector increase.

Fig. 6.b shows the case in which ε changes from 0 to 0.1. Subsidies raise the labor supply of elderly care services and reduce wage inequality. Because of decreased capital accumulation, demand for elderly care services decreases; the wage rate of elderly care sector decreases. Consequently, wage inequality is magnified.

7. Conclusions

This paper presents an examination of how an aging society affects the labor market of the elderly care services sector. An aging society raises demand for elderly care services. Then, the share of labor dedicated to elderly care services sector increases. However, an aging society brings with it savings by which households can subsist for a longer time. Those savings are necessary for consumption. Therefore, an aging society raises the wage rate of the final goods sector because of the capital accumulation. Then, the labor supply is small in the elderly care services sector.

This paper presents an examination of wage inequality between the elderly care services sector and the final goods sector. Results show that the relative wage rate of the elderly care services sector increases. Moreover, this paper presents an examination of subsidy effects on the elderly care services market. Results of the numerical example show that the subsidy raises the labor supplied to the elderly care services sector and shrink the wage inequality between final goods sector and the elderly care service sector.

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Appendix

Local Stability Condition

Total differentiation of (25) with respect to K_{t+1}, K_t, σ_t^* gives

$$dK_{t+1} = \frac{p(1-\alpha)}{\alpha + p(1-\alpha)}\left(\frac{\partial w}{\partial K} dK_t + \left(\frac{\partial w}{\partial \sigma_t^*} - \frac{\bar{\sigma} - \sigma_t^*}{\bar{\sigma}}\right) d\sigma_t^*\right). \quad \text{(A.1)}$$

Considering $\frac{\alpha + p(1-\alpha)}{p(1-\alpha)} K_{t+1} = \int_0^{\sigma_t^*} (w_t - \sigma) \frac{1}{\bar{\sigma}} d\sigma + \int_{\sigma_t^*}^{\bar{\sigma}} w_t^c \frac{1}{\bar{\sigma}} d\sigma$ and (21), (26) is obtained. From total differentiation of (26) with respect to K_t, σ_t^*, the following equation is obtained.

$$\left(\left(1 - \frac{\sigma^*}{\bar{\sigma}}\right)\frac{\partial w}{\partial K} - \beta\theta(1+r)\right) dK_t = \left(\frac{(1-\beta)\theta w}{\bar{\sigma}} - \frac{\partial w}{\partial \sigma^*}\left(1 - \frac{\sigma^*}{\bar{\sigma}}\right) + 1 + \frac{w}{\bar{\sigma}} - \frac{2\sigma^*}{\bar{\sigma}}\right) d\sigma_t^*. \quad \text{(A.2)}$$

Then, $\frac{dK_{t+1}}{dK_t}$ are derived as shown below.

$$\frac{dK_{t+1}}{dK_t} = A + \frac{BC}{D}. \quad \text{(A.3)}$$

$-1 < A + \frac{BC}{D} < 1$ is needed because of the local stability condition.

Parameter Setting

The discount factor of the consumption for quarter period is regarded as 0.99 in the literature of real business cycle theory (de la Croix and Doepke (2003)). Considering one period in the overlapping generations model as 30 years, 0.99^{120} is about 0.3. Then, $\alpha = 0.7$ is set.

This paper sets $(1-\alpha)\beta = 0.25$, that is, $\beta = \frac{0.25}{1-0.7} = 0.83$. The preference parameter for elderly care is given as $(1-\alpha)(1-\beta) = 0.05$. The data of Statistics Japan "System of National Accounts" shows the household total expenditure and the expenditure for elderly care and medical care. The share of this expenditure to total expenditure is 4% at 2015.

As shown by the data of Ministry of Health, Labour and Welfare, Japan "Simple Life Table," the survival rate that zero-year-old people can survive to 60 years old is about 0.9. Then, $p = 0.9$ is set.

In the most recent 30 years, the interest rate in Japan has been approximately 2%. $A = 3.64$ and $\bar{\sigma} = 1$ are set to hold the interest rate at $1 + r = 1.8$, as calculated using 1.02^{30}.

ρ is set as unity. The capital income share θ is set as 0.3.

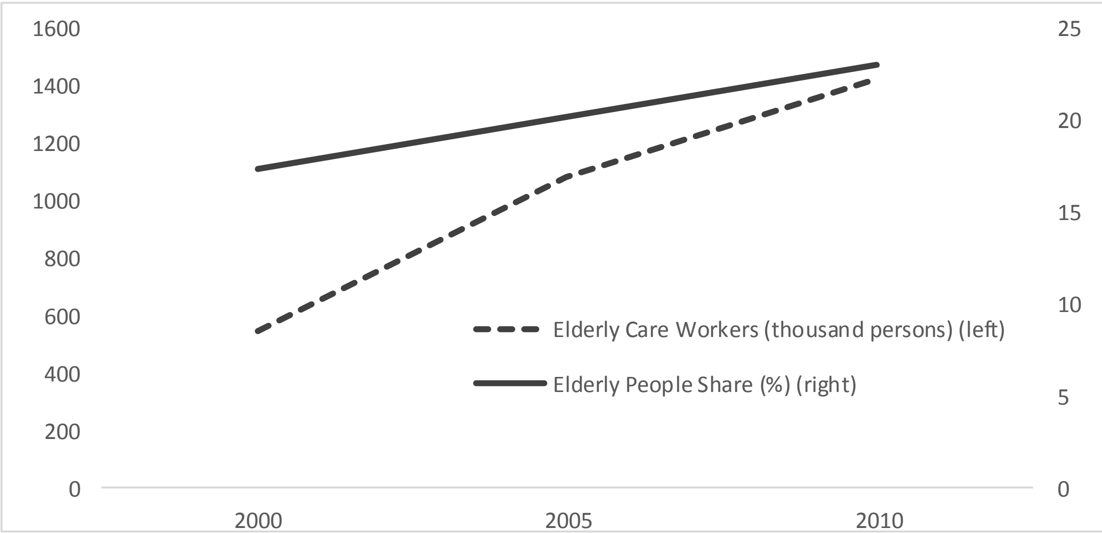


Fig. 1: Elderly Care Workers and Elderly People's Share of the Population.

(Data: Population Statistics, National Institution of Population and Social Security Research, Ministry of Health, Labour and Welfare, Japan)

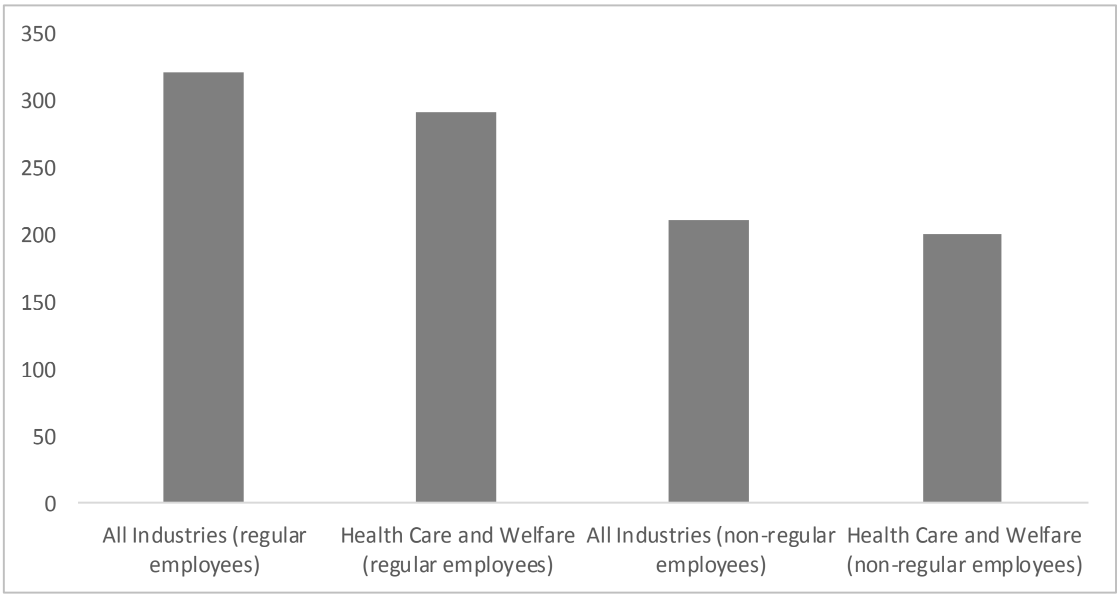
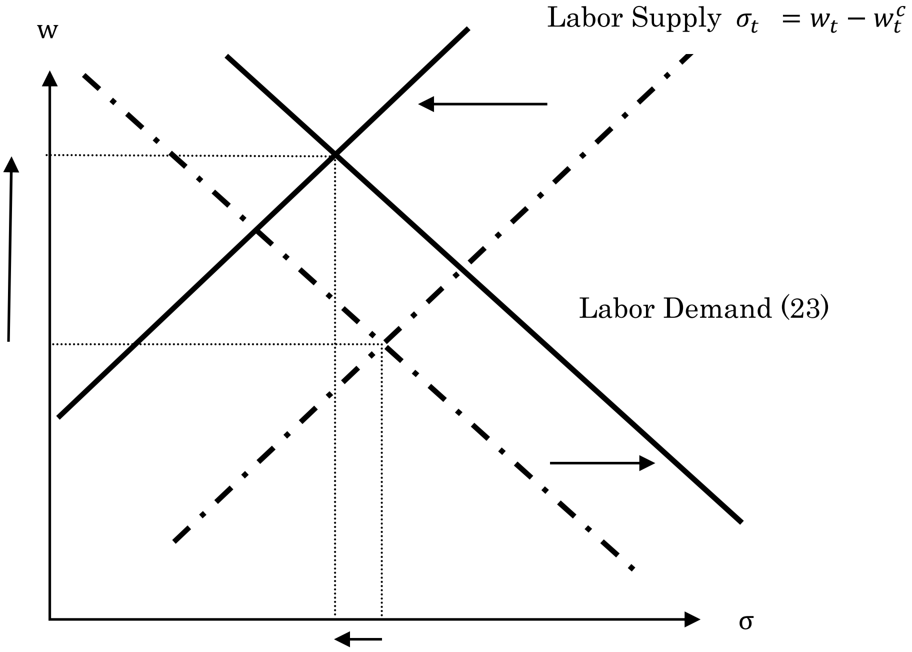


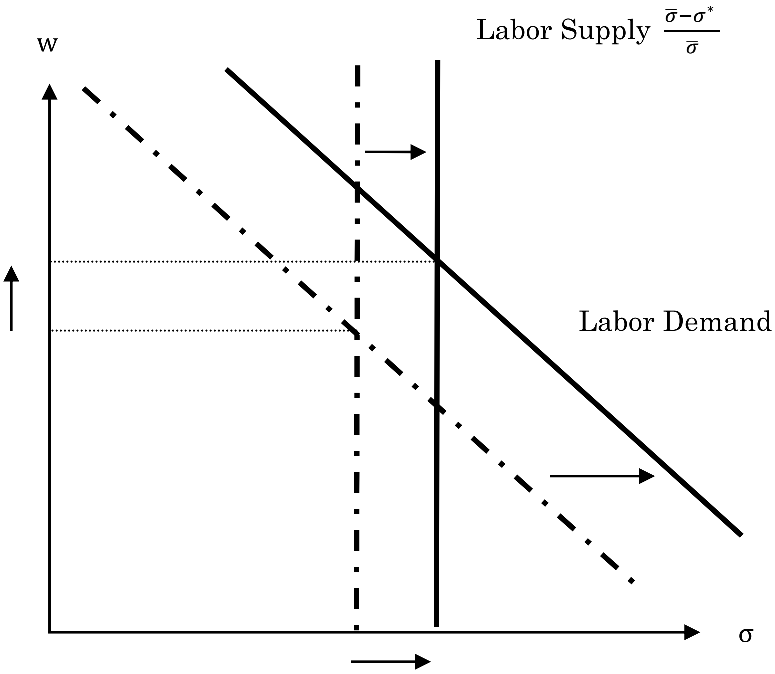
Fig. 2: Salaries of Elderly Care Workers (Thousand yen per month).

(Data: Ministry of Health, Labour and Welfare, Japan)



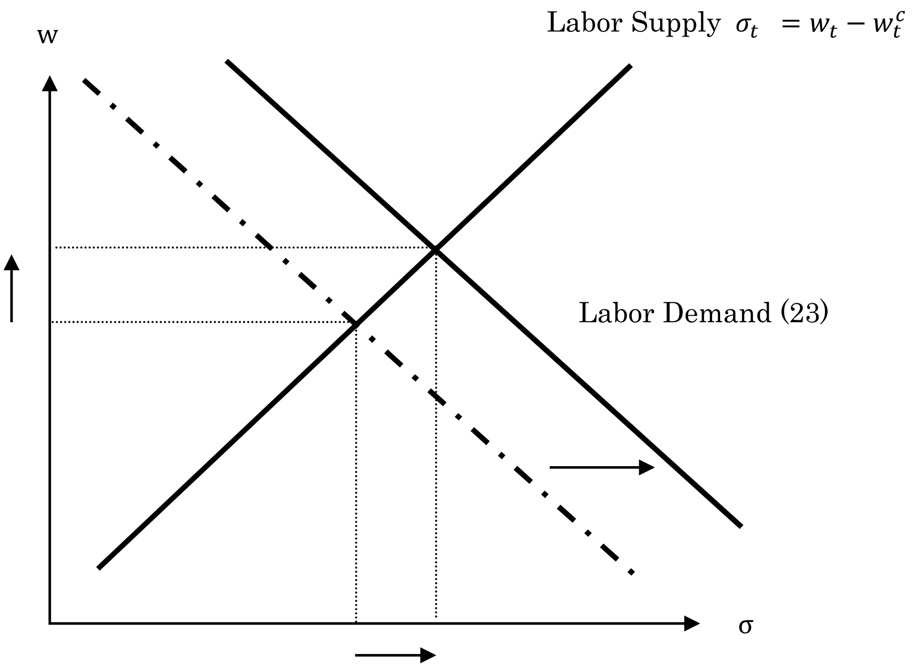
a) Labor market of final goods sector

Fig. 3a: Aging society and the labor market.



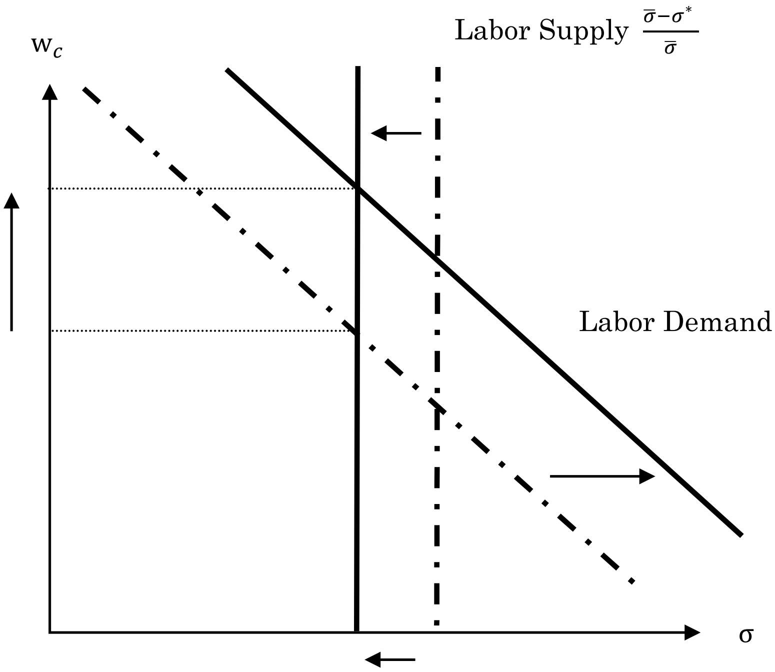
b) Labor market of elderly care services sector

Fig. 3b: Aging society and the labor market.



a) Labor market of final goods sector

Fig. 4a: Aging society and the labor market.



b) Labor market of elderly care services sector

Fig. 4b: Aging society and the labor market.

p	0.9
α	0.7
β	0.83
$\bar{\sigma}$	1
θ	0.3
A	3.64
ρ	1

Table 1: Parameter Setting

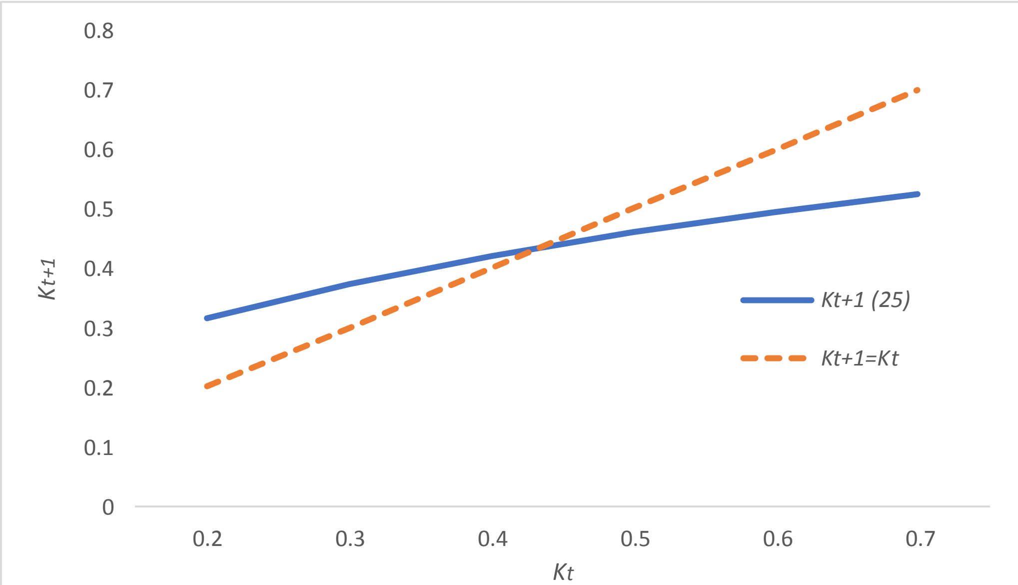


Fig. 5: Dynamics of K_t.

	p	α	$(1-\alpha)\beta$	ε	K	$\sigma*$	w	$1+r$	w/w^c	$U(F)$	$U(E)$
①	0.9	0.7	0.25	0	0.4348	0.8883	2.0557	1.8	1.7609	−0.0522	−0.3649
②	1	0.7	0.25	0	0.4909	0.8907	2.1302	1.6566	1.7185	−0.0837	−0.3906
③	0.9	0.7	0.2	0	0.4653	0.8133	2.1542	1.6138	1.6066	−0.0728	−0.3297
④	0.9	0.7	0.25	0.1	0.4325	0.8784	2.0594	1.7925	1.7438	−0.0517	−0.3613
⑤	0.9	0.7	0.2	0.1	0.4607	0.7988	2.1595	1.6047	1.5870	−0.0729	−0.3266

Table 2: Effects of an Aging Society and Elderly Care Subsidy

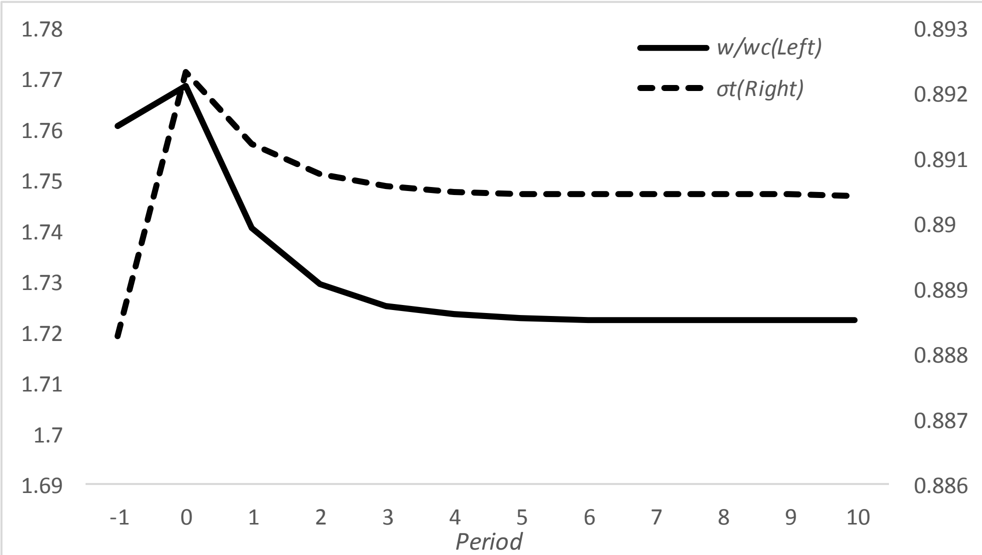


Fig. 6.a: Transition Path for Increased Life Expectancy.

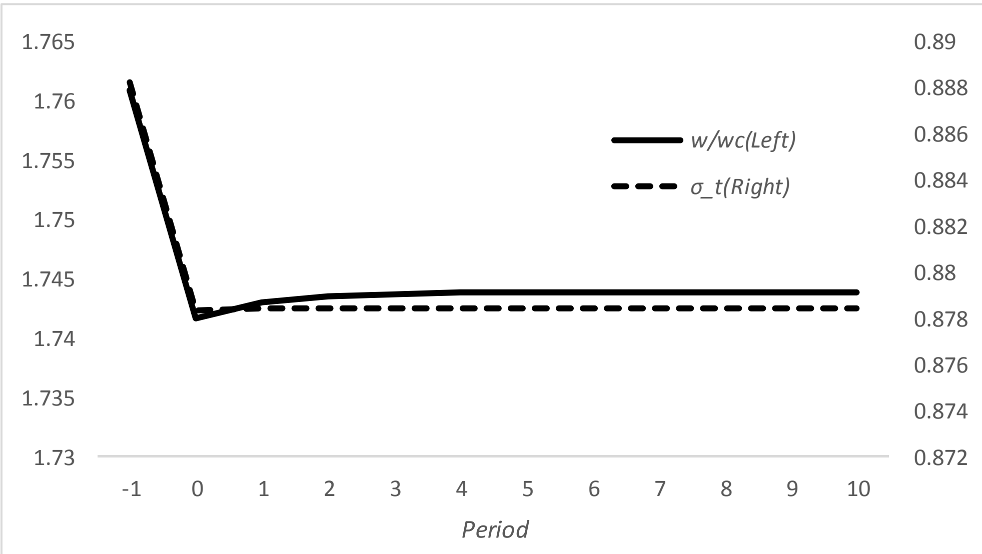


Fig. 6.b: Transition Path in an Increase in Subsidy for Elderly Care Service.