Trade and FDI Liberalization in Multiple Oligopolies

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Abstract

This paper examines welfare effects of trade and FDI liberalization in a model where several oligopolistic industries compete for a common factor. With this general equilibrium effects, trade and FDI liberalization has a contrasting effect. While it is unclear whether trade liberalization benefits each country, it necessarily raises world welfare. When FDI is liberalized, the host country loses and the source country gains, resulting in no effect on world welfare. These results suggest that simultaneous liberalization of trade and FDI becomes welfare-improving for the world.

Keywords: trade liberalization, FDI liberalization, oligopoly, factor price, general equilibrium.

JEL Classifications: F12, F13, F23.
1 Introduction

Multinational firms are playing an important role in the modern world economy, and have a considerable impact on the host country’s employment and income, on the one hand. On the other hand, the choice between exporting and FDI is a great concern from the source country’s point of view. These observations on multinational firms are recognized in the latest report of UNCTAD (2015, p. 18), stating that ‘Indicators of international production—production of MNE foreign affiliates—show a rise in sales by 7.6 per cent, while employment of foreign affiliates reached 75 million. Exports of foreign affiliates remained relatively stable, registering a 1.5 per cent rise.’ Then, a natural question arises; what are the effects of liberalization of trade and/or FDI when the presence of multinational firms is substantial?

To answer this question, this paper develops an oligopoly model that allows a coexistence of trade and FDI, and examines the welfare effects of trade and FDI liberalization on the host country, the source country and the world. It is usually assumed in an oligopoly model that a factor price, e.g. wage rate, is fixed through the profit maximization condition of the perfectly competitive numeraire sector. In this setting, trade liberalization (tariff reduction) and FDI liberalization (investment tax reduction) have the same effect; these liberalization policies shift the profit from the host country to the source country by lowering the source firm’s marginal cost. In contrast, supposing multiple oligopolistic industries that employ a common factor, the factor price is endogenously determined so that the factor market clears. By taking into account this general equilibrium effect, we show that the effects of trade and FDI liberalization are quite different from those in the partial equilibrium model. Concretely, we establish the following results. Trade liberalization increases consumer surplus and profits in the host country, and improves world welfare.\textsuperscript{1} If FDI is liberalized, the host country’s firms lose,\textsuperscript{1}

\textsuperscript{1}Welfare effects on the host and source countries are regrettably unclear. For this
but the consumer utility in the host country and world welfare are unchanged. These results illuminate the importance of general equilibrium considerations in theoretical analyses and in practical applications.

There is an extensive literature on multinational firms, which is mainly driven by the development of new trade theory in 1980s. While this literature comprises a variety of issues, the choice between exporting and FDI receives much attention. Dei (1990), Horstmann and Markusen (1992) and Brainard (1997) propose, in an oligopoly model, a so-called proximity-concentration hypothesis of FDI, according to which FDI is preferred to exporting if the fixed cost to set up a foreign plant is small relative to trade costs. Extending the model of heterogeneous firms by Melitz (2003), Helpman et al. (2004) augment this hypothesis, demonstrating that the firm with a sufficiently high productivity chooses FDI.

One common assumption in these works is that the source firm chooses either exporting or FDI. However, one can easily find that some multinationals undertake both exporting and FDI. Glass and Saggi (1999) are the first to model such a coexistence of exporting and FDI. Specifically, incorporating an FDI subsidy into the oligopoly model of Dixit and Grossman (1986), Glass and Saggi (1999) characterize the optimal FDI policies for the host and source countries. The works of Glass and Saggi (1999, 2004, 2014) greatly contribute to literature by assuming that multiple oligopolistic industries use a common factor, the price of which is endogenously determined. Our model heavily relies on their approach.

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2Two earliest contributions in this field are Markusen (1984) and Helpman (1984), each of whom explains horizontal and vertical FDI, respectively. See Markusen (1995, 2002, 2011) for a comprehensive survey.

3The latest survey of FDI and multinationals is found in Antras and Yeaple (2014).

4Extending their own work to multiple source and host countries, Glass and Saggi (2004, 2014) further discuss the effects and coordination of FDI taxes.

5It should be noted that we allow domestic consumption whereas both Dixit and Gross-
In addition to the literature above, there is a growing interest in the interaction between trade and FDI liberalization. Developing an oligopoly model with free entry, Markusen (1997, 2002) finds that trade and FDI liberalization has a quite different effect on welfare. Egger et al. (2004, 2007a, 2007b), based on Markusen’s (1997) framework, numerically solve the more complicated model than Markusen’s (1997, 2002), and examine in detail the welfare effects of trade and FDI liberalization. Incorporating service FDI into a duopoly model, Ishikawa et al. (2010, 2014) show that FDI liberalization has to be accompanied in order to ensure successful trade liberalization. Furthermore, Liu and Qiu (2013) and Ahn (2014) extend the model of Helpman et al. (2004) to address the effects of trade and FDI liberalization. Liu and Qiu (2013) demonstrate that trade and FDI liberalization has a contrasting effect on welfare, and Ahn (2014) theoretically point out a possibility of welfare-reducing liberalization of trade and/or FDI.

This paper is organized as follows. Section 2 constructs a model, and Section 3 states and discusses the main results. Section 4 concludes, and Appendix provides a formal proof of the main results.

2 Model

Suppose a consumer of a host country named Home who consumes \( m + n \geq 2 \) duopolized goods and a numeraire good, and has a utility function

\[
\sum_{i=1}^{m+n} u(c_i) + z, \quad u'(\cdot) > 0, \quad u''(\cdot) < 0,
\]

where \( c_i, i = 1, \ldots, m + n \) is consumption of Good \( i \), and \( z \) is consumption of the numeraire good. Then, denoting by \( p_i \) the price of Good \( i \) measured by Good \( z \), utility maximization yield the following inverse demand function:

\[
p_i = p(x_i + x_i^*), \quad p'(\cdot) < 0, \quad \text{where } x_i \text{ and } x_i^* \text{ are output of the Home and Foreign firms, respectively.}
\]
We divide $m + n$ duopolistic industries into $m \geq 1$ industries in which the Foreign firm engages in both exporting and FDI and $n \geq 1$ industries in which the Foreign firm just exports. In what follows, we use subscripts $i$ and $j$ to refer to a representative industry in the former and latter set of industries, respectively. In all the duopolistic industries, one unit of specific factor, named capital, is required to produce one unit of good. In addition, the Home government levies an import tariff $t$ and an investment tax, both of which are of specific (per-unit) type. With these assumptions, the profit of the Home and Foreign firms in industries $i$ and $j$ is defined as follows.

$$\pi_i \equiv p(x_i + x_i^*)x_i - rx_i$$ (1)

$$\pi_i^* \equiv p(x_i + x_i^*)x_i^* - [(r + \tau)\theta + (r^* + t)(1 - \theta)]x_i^*$$ (2)

$$\pi_j \equiv p(x_j + x_j^*)x_j - rx_j$$ (3)

$$\pi_j^* \equiv p(x_j + x_j^*)x_j^* - r^*x_j^*$$ (4)

where $\pi$ is the profit, $r$ and $r^*$ are capital rental in Home and Foreign, and $\theta \in [0, 1]$ represents a fraction of FDI. Eq. (2) states that the FDI incurs Home capital rental $r$ plus investment tax $\tau$ while exporting involves Foreign capital rental $r^*$ plus tariff $t$.

The first-order conditions for profit maximization are as follows.

$$x_ip'(x_i + x_i^*) + p(x_i + x_i^*) - r = 0$$ (5)

$$x_i^*p'(x_i + x_i^*) + p(x_i + x_i^*) - (r + \tau)\theta - (r^* + t)(1 - \theta) = 0$$

$$x_jp'(x_j + x_j^*) + p(x_j + x_j^*) - r = 0$$

$$x_j^*p(x_j + x_j^*) + p(x_j + x_j^*) - r^* = 0.$$

Noting here that $r + \tau = r^* + t$ must hold in order to ensure the interior solution of $\theta \in [0, 1]$, these equations can be rewritten as

$$x_ip'(x_i + x_i^*) + p(x_i + x_i^*) - r = 0$$ (5)

$$x_i^*p'(x_i + x_i^*) + p(x_i + x_i^*) - r - \tau = 0$$ (6)
by eliminating \( r^* \)

The capital rentals are determined in the capital market-clearing condition in each country:

\[
mx_i + nx_j + m\theta x_i^* = k \tag{9}
\]

\[
nx_j^* + m(1 - \theta)x_i^* = k^*, \tag{10}
\]

where \( k \) and \( k^* \) are the factor endowment of Home and Foreign, respectively. While our model consists of six equations (Eqs. (5)-(10)) that determine six variables \( (x_i, x_i^*, x_j, x_j^*, r \text{ and } \theta) \), it can be further simplified by summing (9) and (10) together:

\[
m(x_i + x_i^*) + n(x_j + x_j^*) = k + k^*. \tag{11}
\]

Then, our model consists of Eqs. (5)-(8) and (11) in which \( x_i, x_i^*, x_j, x_j^* \) and \( r \) are endogenously determined.

We close this section by making comparative statics of the above model. Totally differentiating Eqs. (5)-(8) and (11), we have

\[
\begin{bmatrix}
    x_i p''_i + 2p'_i & x_i p''_i + p'_i & 0 & 0 & -1 \\
    x_i p''_i + p'_i & x_i p''_i + 2p'_i & 0 & 0 & -1 \\
    0 & 0 & x_j p''_j + 2p'_j & x_j p''_j + p'_j & -1 \\
    0 & 0 & x_j p''_j + p'_j & x_j p''_j + 2p'_j & -1 \\
    m & m & n & n & 0
\end{bmatrix}
\begin{bmatrix}
dx_i \\
dx_i^* \\
dx_j \\
dx_j^* \\
dr
\end{bmatrix}
\] 

\[
= \begin{bmatrix}
    0 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 \\
    -1 & 0 & 1 & 0 \\
    0 & 0 & 0 & 1
\end{bmatrix} dt + \begin{bmatrix}
    0 \\
    1 \\
    0 \\
    0
\end{bmatrix} d\tau,
\]

where \( p'_i \equiv p'(x_i + x_i^*) \) and the other derivatives are defined in the same way. Following the standard oligopoly theory, we make:
\[ x_i p''_i + p'_i, x_i^* p''_i + p'_i, x_j p''_j + p'_j \quad \text{and} \quad x_j^* p''_j + p'_j \] are all negative.

Under this assumption, the effects of a tariff are obtained as follows.

\[ \frac{\partial x_i}{\partial t} = \frac{n \left[ - (x_i - x_i^*) p''_i + p'_i \right] p'_j}{\Delta}, \quad \frac{\partial x_i^*}{\partial t} = \frac{n \left[ (x_i - x_i^*) p''_i + p'_i \right] p'_j}{\Delta} \] (12)

\[ \frac{\partial x_j}{\partial t} = \frac{n \left[ (X_j p''_j + 3 p'_j) + 2 m (x_j p''_j + p'_j) \right] p'_i}{\Delta} < 0 \] (13)

\[ \frac{\partial x_j^*}{\partial t} = \frac{n \left[ (X_j p''_j + 3 p'_j) + 2 m (x_j p''_j + 2 p'_j) \right] p'_i}{\Delta} > 0 \] (14)

\[ \frac{\partial r}{\partial t} = \frac{n (X_i p''_i + 3 p'_i) p'_j p'_j}{\Delta} > 0, \] (15)

where \( X_i \equiv x_i + x_i^* \) is total output of Good \( i \), and \( \Delta \equiv 2n (X_j p''_j + 3 p'_j) + m (X_j p''_j + 3 p'_j) \) is the determinant of the coefficient matrix of the totally-differentiated system. Analogously, the effects of an investment tax become

\[ \frac{\partial x_i}{\partial \tau} = -\frac{1}{2 p'_i} > 0, \quad \frac{\partial x_i^*}{\partial \tau} = \frac{1}{2 p'_i} < 0 \] (16)

\[ \frac{\partial x_j}{\partial \tau} = -\frac{1}{2 p'_j} > 0, \quad \frac{\partial x_j^*}{\partial \tau} = \frac{1}{2 p'_j} < 0 \] (17)

\[ \frac{\partial r}{\partial \tau} = -\frac{1}{2} \] (18)

From these outcomes, the effects on total supply are

\[ \frac{\partial X_i}{\partial t} = \frac{2 m p'_j p'_j}{\Delta} < 0, \quad \frac{\partial X_j}{\partial t} = -\frac{2 m p'_j p'_j}{\Delta} > 0, \quad \frac{\partial X_i}{\partial \tau} = \frac{\partial X_j}{\partial \tau} = 0. \] (19)

We now seek the intuitions behind these comparative statics outcomes, which will be important in discussing the welfare effects of trade and FDI liberalization. When the tariff \( t \) is reduced, the non-arbitrage condition breaks down and becomes \( r + \tau > r^* + t \). Since this means that FDI is more costly than exporting, Foreign firm \( i \) shifts production from the host country (Home) to the source country (Foreign). This production shift lowers capital demand in Home, and \( r \) falls. In contrast, Foreign capital rental \( r^* \) rises.
because capital demand in Foreign increases.\textsuperscript{6} Thus, the fall in \( r \) encourages all the Home firms to produce more, and the rise in \( r^* \) discourages all the Foreign firms’ production.\textsuperscript{7}

The effects of FDI liberalization can be interpreted in a parallel way. When the investment tax \( \tau \) is reduced, it holds that \( r + \tau < r^* + t \), and Foreign firm \( i \) replaces exporting with FDI. This production shift puts upward pressure on \( r \) and downward pressure on \( r^* \).\textsuperscript{8} The resulting increase in \( r \) induces the Home firms to contract outputs, and decrease in \( r^* \) induces the Foreign firms to expand outputs.

It is noteworthy that ‘the increase in the source output exactly offsets the decrease in the host output, leaving total output of each industry unchanged.’ (Glass and Saggi, 1999, p. 319) This finding will play an important role in welfare effects discussed later.

3 Welfare Effects

Based on the foregoing arguments, this section examines the welfare effects of trade and FDI liberalization. Since the effects on welfare of each country are ambiguous, we consider the effects on Home consumer surplus, firm profits in each country and the factor income in each country, following the strategy of Ishikawa et al. (2010).

\textsuperscript{6}The effect on \( r^* \) is given by

\[
\frac{\partial r^*}{\partial t} = -\frac{n (X_i p''_i + 3 p'_i) + 2m (X_j p''_j + 3 p'_j)) p'_i p'_j}{\Delta} < 0.
\]

\textsuperscript{7}As shown in Eq. (12), trade liberalization increases both the Home and Foreign firms’ output if demand is linear (\( p''_i = 0 \)). However, under non-linear demand, the sign of \( \partial x_i/\partial t \) and \( \partial x^*_i/\partial t \) is ambiguous, but total output necessarily increases, i.e. \( \partial X_i/\partial t < 0 \).

\textsuperscript{8}The effect on \( r^* \) is simply \( \partial r^*/\partial \tau = 1/2 > 0 \).
3.1 Effects on Consumer Surplus

We begin with the effects on Home consumer surplus. Using the definitions $X_i \equiv x_i + x_i^*$ and $X_j \equiv x_j + x_j^*$, Home consumer surplus $CS$ is defined by

$$CS \equiv m \left[ \int_0^{X_i} p(Q)dQ - X_i p(X_i) \right] + n \left[ \int_0^{X_j} p(Q)dQ - X_j p(X_j) \right].$$ (20)

Then, the effects of trade and FDI liberalization are summarized as follows.\(^9\)

**Proposition 1.** Trade liberalization raises Home consumer surplus if $t < 2(p_i - p_j)$. FDI liberalization has no effect on Home consumer surplus.

The former part claims that the host country’s consumer gains from trade liberalization if the initial tariff is sufficiently small. From (19), a tariff reduction increases $X_i$ but decreases $X_j$, and so it is generally ambiguous whether trade liberalization benefits the consumer. However, the effect on consumer utility can be positive if the initial tariff is sufficiently small because if $t$ is low enough, the Foreign firm has a stronger incentive to produce more after the tariff reduction. Therefore, the positive effect on $X_i$ dominates the negative effect on $X_j$, leading to the consumer’s gain. If, in contrast, $t$ is sufficiently high, output contraction in industry $j$ dominates output expansion in industry $i$, thereby leaving the consumer worse off.

It is trivial that FDI liberalization has no effect on Home consumer surplus just by invoking that FDI liberalization keeps $X_i$ and $X_j$ unchanged.

3.2 Effects on Firm Profits

Let us next address the effects on the firm profits in each country. We derive the effects on the Home firms’ profits, and then turn to those on the Foreign firms’ profits. Relating the comparative statics outcomes (12)-(18) to (1) and (3), we find:

\(^9\)The proofs of all propositions are in Appendix.
**Proposition 2.** Trade liberalization raises the profits of all the Home firms. FDI liberalization lowers the profits of all the Home firms.

As indicated in (12), it is unclear whether the Home firm $i$ increases output as a result of trade liberalization. However, a tariff reduction benefits Home firm $i$ since its marginal cost, which equals $r$, definitely falls after trade liberalization. In addition, a lower tariff raises the profit of the Home firm $j$ by encouraging its production (see (13)) and reducing its marginal cost (see (14)). That is, the decline in $r$ caused by trade liberalization plays a crucial role in the positive effect on the aggregate profits in Home.

The negative effects of FDI liberalization on firm profits can be interpreted analogously. Looking at Eqs. (16), (17) and (18), an investment tax reduction decreases output of all the Home firms while it increases the Home capital rental. Therefore both of these effects are detrimental to all the Home firms.

While Proposition 2 focuses on the effects on the Home firms’ profits, the effects on the profits of the Foreign firms are now examined. The results are more complicated, which are stated as follows.

**Proposition 3.** Trade liberalization raises the profits of the Foreign firms that undertake both exporting and FDI, and lowers the profits of the Foreign firms that just export. FDI liberalization raises the profits of all the Foreign firms.

Whether trade and FDI liberalization is profitable for the Foreign firms is determined by its effect on output and capital rental. From Eq. (12) and Footnote 7, it is unclear whether the tariff reduction increases output of

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10It is easily confirmed that the Home firm increases output if demand is linear.
the Foreign firm \( i \), but it necessarily reduces its marginal cost \( r^* + t \), which increases the profit. In contrast, as a result of trade liberalization, Foreign capital rental rises and output of Foreign firm \( j \) declines. Hence, these effects both lead Foreign firm \( j \) to reduce profits.

As in Proposition 2, the effects of FDI liberalization are straightforward. As the investment tax is lower, all the Foreign firms produce more, and Foreign capital rental falls. Thus, FDI liberalization allows all the Foreign firms to increase profits.

### 3.3 Effects on Factor Income

In a conventional partial equilibrium model, no policy change affects the factor income because it is equal to the fixed factor supply. However, the present model allows a change in factor prices, and hence changes in factor income significantly influence welfare. Noting that \( r \) falls (resp. rises) and \( r^* \) rises (resp. falls) after trade liberalization (resp. FDI liberalization), it follows that:

**Proposition 4.** Trade liberalization lowers the Home factor income, and raises the Foreign factor income. FDI liberalization raises the Home factor income, and lowers the Foreign factor income.

Propositions 2, 3 and 4 offer two important implications. First, both liberalization policies give rise to a conflict of interest between the duopolistic firms and the capital owner in the sense that in Home trade liberalization (resp. FDI liberalization) benefits the firms (resp. capital owner), but harms the capital owner (resp. firms).\(^{11}\) Second, trade and FDI liberalization has a contrasting effect on each welfare component. In particular, the Home oligopolistic firms gain from trade liberalization, but lose from FDI liberal-

\(^{11}\)Therefore, the welfare effect on each country inevitably becomes ambiguous as is shown in Section 4.
ization. Similarly, the Home capital owner loses from trade liberalization, but gains from FDI liberalization. Combined liberalization of trade and FDI is naturally called for in order to solve this income distribution problem, which will be addressed later.

### 3.4 Effects on World Welfare

Home welfare is by summing up consumer surplus $CS$ and national income $I$, which is

$\begin{align*}
I & \equiv m \pi_i + n \pi_j + rk + tm(1 - \theta)x_i^* + \tau m \theta x_i^* \\
& = m(p_i - r)x_i + n(p_j - r)x_j + rk + tm(1 - \theta)x_i^* + \tau m \theta x_i^* \\
& = mp_i x_i + np_j x_j + r(k - mx_i - nx_j) + tmx_i^* - (t - \tau)(k - mx_i - nx_j) \\
& = mp(X_i)x_i + np(X_j)x_j + (r - t + \tau)(k - mx_i - nx_j) + tmx_i^*,
\end{align*}$

where $tm(1 - \theta)x_i^* + \tau m \theta x_i^*$ in the first line represents government revenue from trade and investment taxes, and the third line uses (9). In a parallel way, Foreign welfare $W^*$, which is equal to Foreign’s national income, is derived as

$\begin{align*}
W^* & \equiv m \pi_i^* + n \pi_j^* + r^* k^* \\
& = m(p_i - r - \tau)x_i^* + n(p_j - r + t - \tau)x_j^* + (r - t + \tau)k^* \\
& = m(p_i - r - \tau)x_i^* + n(p_j - r + t - \tau)x_j^* + (r - t + \tau)(mx_i + mx_i^* + nx_j + nx_j^* - k) \\
& = mp(X_i)x_i^* + np(X_j)x_j^* - (r - t + \tau)(k - mx_i - nx_j) - tmx_i^*,
\end{align*}$

where the third line uses (11). From (21) and (22), world welfare becomes

$W + W^* = CS + mp(X_i)X_i + np(X_j)X_j.$

Differentiating (23) with respect to $t$ and $\tau$ and using (19), we can establish:

**Proposition 5.** Trade liberalization raises world welfare, and FDI liberalization has no effect on world welfare.
From the findings so far, trade and FDI liberalization necessarily leads to a conflict of interest within each country and across countries. However, Proposition 5 suggests that simultaneous liberalization of trade and FDI may be a solution to such problems. If both trade and FDI are liberalized, the world enjoys an efficiency gain, and thus it is possible for both countries to gain from this liberalization through international income transfers.

**Remark.** We have unexplored the effects on each country’s welfare since nothing definite can be said. But, we now briefly address how trade and FDI liberalization affects each country. The following proposition concerns the welfare effect of trade and FDI liberalization on each country.

**Proposition 6.** The welfare effects of trade liberalization on each country are ambiguous. FDI liberalization lowers Home welfare, but raises Foreign welfare.

It is no surprise that the welfare effects of tariff reductions are unclear by recalling Propositions 2, 3 and 4. On the contrary, we can claim that reductions in investment tax are necessarily harmful to the host country and beneficial to the source country. The reason is that in the host country, the firms’ losses from FDI liberalization end up being larger than the capital owner’s gains, and exactly the opposite holds in the source country.

### 4 Concluding Remarks

We have formulated a model in which multiple oligopolistic industries employ the same factor to examine the welfare effects of trade and FDI liberalization. The model, which basically follows Dixit and Grossman (1986) and Glass and Saggi (1999), is capable of considering the general equilibrium effects in a simply way. It is shown that trade and FDI liberalization has a contrasting
effect on the consumer utility, firm profits and factor income in each country. In addition, we have demonstrated that trade liberalization positively affects world welfare, but that FDI liberalization has no effect on it.

While these results may provide a useful policy implication, they admittedly rest on a number of simplifying assumptions. Among others, the assumption of a quasi-linear preference has allowed us to use the sum of consumer surplus and national income as a welfare measure. It is undoubtedly restrictive to assume no income effect of the demand of oligopolistic goods. In order to overcome this difficulty, Neary (2009) proposes an ‘general oligopolistic equilibrium’ model in which full general equilibrium effects and oligopolistic competition are combined.\textsuperscript{12} It is an interesting and important task to examine the effects of trade and FDI liberalization in such a richer model.

\section*{Appendix}

\textbf{Proof of Proposition 1}

Differentiating (20) with respect to $t$ and relating (19) to the resulting expression, the consumer surplus effect of trade liberalization becomes

\[
\frac{\partial CS}{\partial t} = -mX_i\pi_i\frac{\partial X_i}{\partial t} - nX_j\pi_j\frac{\partial X_j}{\partial t} = \frac{2mn\pi_i\pi_j(X_j\pi_j - X_i\pi_i)}{\Delta} = \frac{2mn\pi_i\pi_j[2(p_i - p_j) - t]}{\Delta},
\]

where the last equation follows from the first-order conditions (5)-(8). Given that $\Delta$ is negative, we find that trade liberalization improves consumer surplus, i.e. $\partial CS/\partial t < 0$, if $2(p_i - p_j) - t > 0$. In contrast, FDI liberalization has no effect on consumer surplus because $X_i$ and $X_j$ are unchanged.

\textsuperscript{12}Colacicco (2014) provides a comprehensive survey on Neary’s (2009) approach and the subsequent works.
**Proof of Proposition 2**

Differentiating (1) and (3) with respect to \( t \) and \( \tau \), and using the comparative statics outcomes (12)-(15), we have

\[
\frac{\partial \pi_i}{\partial t} = \left( p_i' \frac{\partial x_i^*}{\partial t} - \frac{\partial r}{\partial t} \right) x_i = -\frac{2nx_i'p_j'(x_i'p_j'' + p_j')}{\Delta} < 0
\]

\[
\frac{\partial \pi_j}{\partial t} = \left( p_j' \frac{\partial x_j^*}{\partial t} - \frac{\partial r}{\partial t} \right) x_j = -\frac{2x_j'p_j'[n(X_ip_i'' + 3p_i') + m(x_jp_j'' + 2p_j')]}{\Delta} < 0
\]

\[
\frac{\partial \pi_i}{\partial \tau} = \left( p_i' \frac{\partial x_i^*}{\partial \tau} - \frac{\partial r}{\partial \tau} \right) x_i = x_i > 0
\]

\[
\frac{\partial \pi_j}{\partial \tau} = \left( p_j' \frac{\partial x_j^*}{\partial \tau} - \frac{\partial r}{\partial \tau} \right) x_j = x_j > 0,
\]

where the envelope property such that \( \partial \pi_i/\partial x_i = 0 \) and \( \partial \pi_j/\partial x_j = 0 \) is utilized. The former two results suggest that trade liberalization raises all the Home firms’ profits, and the latter two results suggest that FDI liberalization lowers them.

**Proof of Proposition 3**

Looking at Eqs. (2) and (4) and making manipulations similar to those in the proof of Proposition 2, we obtain

\[
\frac{\partial x_i^*}{\partial t} = \left( p_i' \frac{\partial x_i}{\partial t} - \frac{\partial r}{\partial t} \right) x_i^* = -\frac{2nx_i'p_j'(x_i'p_j'' + p_j')}{\Delta} < 0
\]

\[
\frac{\partial x_j^*}{\partial t} = \left( p_j' \frac{\partial x_j}{\partial t} - \frac{\partial r}{\partial t} \right) x_j^* = \frac{2x_j'p_j'[n(X_ip_i'' + 3p_i') + m(x_jp_j'' + 2p_j')]}{\Delta} > 0
\]

\[
\frac{\partial x_i^*}{\partial \tau} = \left( p_i' \frac{\partial x_i}{\partial \tau} - \frac{\partial r}{\partial \tau} - 1 \right) x_i^* = -x_i^* < 0
\]

\[
\frac{\partial x_j^*}{\partial \tau} = \left( p_j' \frac{\partial x_j}{\partial \tau} - \frac{\partial r}{\partial \tau} - 1 \right) x_j^* = -x_j^* < 0.
\]

These sign patterns lead to Proposition 3 in the main text.
Proof of Proposition 5

Differentiating the definition of world welfare (23) with respect to $t$ and $\tau$ leads to

$$\frac{\partial (W + W^*)}{\partial t} = \frac{mp_i \partial X_i}{\partial t} + np_j \frac{\partial X_j}{\partial t} = \frac{2mnp_i'p_j'(p_i - p_j)}{\Delta} < 0$$

$$\frac{\partial (W + W^*)}{\partial \tau} = 0.$$

4.1 Proof of Proposition 6

Home welfare is defined by $W \equiv CS + I$, where $CS$ and $I$ are given by Eqs. (20) and (21), respectively. Differentiating this with respect to $t$ and making lengthy manipulations yield

$$\frac{\partial W}{\partial t} = -mX_i p_i' \frac{\partial X_i}{\partial t} - nX_j p_j' \frac{\partial X_j}{\partial t} + m \left( x_i p_i' \frac{\partial X_i}{\partial t} + p_i \frac{\partial x_i}{\partial t} \right) + n \left( x_j p_j' \frac{\partial X_j}{\partial t} + p_j \frac{\partial x_j}{\partial t} \right) + \left( \frac{\partial r}{\partial t} - 1 \right) \left( k - mx_i - nx_j \right) - \left( r - t + \tau \right) \left( m \frac{\partial x_i}{\partial t} + n \frac{\partial x_j}{\partial t} \right) + mx^*_i + tm \frac{\partial x^*_i}{\partial t}$$

$$= \frac{mn p_i'}{\Delta} \left[ -(p_i - r - \tau)(x_i - x_i^*)p_i'' + (3p_i - 3r + 2t - 3\tau)p_i' \right] + \frac{np_j' \left( X_j p_j'' + 3p_j' \right)}{\Delta} \left[ n(X_i p_i'' + 3p_i') + 2mx_j p_j'' \right] + \frac{np_j' \left( X_j p_j'' + 3p_j' \right)(k - mx_i - nx_j)}{\Delta} + mX_i + nx_j - k,$$

from which the effect of trade liberalization on Home welfare is ambiguous.

In contrast, the effect of FDI liberalization is shown to be negative since differentiating $W$ with respect to $\tau$ becomes

$$\frac{\partial W}{\partial \tau} = mp_i \frac{\partial x_i}{\partial \tau} + np_j \frac{\partial x_j}{\partial \tau} + \left( \frac{\partial r}{\partial \tau} \right) \left( k - mx_i - nx_j \right) - (r - t + \tau) \left( m \frac{\partial x_i}{\partial \tau} + n \frac{\partial x_j}{\partial \tau} \right) + tm \frac{\partial x^*_i}{\partial \tau}$$

$$= -m(2p_i - r - \tau) - n \left( \frac{p_j - r + t - \tau}{2p_j'} \right) + k - mx_i - nx_j > 0.$$
The effects on Foreign welfare can be analogously obtained. Differentiating $W^*$ in (22) with respect to $t$ and $\tau$, we have

$$\frac{\partial W^*}{\partial t} = m \left( x^*_i p'_i \frac{\partial X_i}{\partial t} + p'_i \frac{\partial x^*_i}{\partial t} \right) + n \left( x^*_j p'_j \frac{\partial X_j}{\partial t} + p'_j \frac{\partial x^*_j}{\partial t} \right) - \left( \frac{\partial r}{\partial t} - 1 \right) (k - mx_i - nx_j)$$

$$+ (r - t + \tau) \left( m \frac{\partial x_i}{\partial t} + n \frac{\partial x_j}{\partial t} \right) - mx^*_i - tm \frac{\partial x^*_i}{\partial t}$$

$$= \frac{mnp'_i}{\Delta} \left[ (p_i - r - \tau)(x_i - x^*_i)p''_i - (p_i - 3r + 2t - 3\tau)p'_i \right]$$

$$+ \frac{n p'_j}{\Delta} \left[ -n(p_j - r + t - \tau)(X_j p''_i + 3p'_i) - 2m(p_j - r + t - \tau)x_j p''_j - 2mp'_j \right]$$

$$- \frac{n p'_j (X_i p''_i + 3p'_i)(k - mx_i - nx_j)}{\Delta} + k - mX_i - nx_j$$

$$\frac{\partial W^*}{\partial \tau} = \frac{mp_i}{2p'_i} \frac{\partial x^*_i}{\partial \tau} + \frac{np_j}{2p'_j} \frac{\partial x^*_j}{\partial \tau} - \left( \frac{\partial r}{\partial \tau} + 1 \right) (k - mx_i - nx_j) + (r - t + \tau) \left( m \frac{\partial x_i}{\partial \tau} + n \frac{\partial x_j}{\partial \tau} \right)$$

$$- tm \frac{\partial x^*_i}{\partial \tau}$$

$$= \frac{m(p_i - r - \tau)}{2p'_i} + \frac{n(p_j - r + t - \tau)}{2p'_j} - \frac{k - mx_i - nx_j}{2} < 0.$$ 

Therefore, it is unclear whether trade liberalization improves Foreign welfare while Foreign necessarily gains from FDI liberalization.

References


