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Cannibalization within the Single Vertically Differentiated Duopoly *

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Abstract

We consider cannibalization in a duopoly model in which firms with different costs supply two vertically differentiated products in the *same market*. We find that an increase in the difference in quality between the two goods or a decrease in the marginal cost of the high-quality goods leads to cannibalization. As a result, these goods keep low-quality goods from the market. Then, as the difference in quality between the two goods increases from a sufficiently small to a sufficiently large level, we find that 1) cannibalization from the low-quality good to the high-quality good of the efficient firm expands, 2) cannibalization from the high-quality good to the low-quality good of the inefficient firm shrinks. Further, we establish that 3) an increase in the production costs of the inefficient firm improves social welfare when the difference in quality between the two goods is sufficiently small.

Keywords: Multi-product firm; Duopoly; Cannibalization; Vertical product differentiation
JEL Classification Codes: D21, D43, L13, L15

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1 Introduction

In a real economy, there are oligopolistic markets in which firms produce and sell multiple products that are vertically differentiated within the same market. For example, GM sells the Chevrolet Cruze and GMC Sierra PU, and Toyota sells the Camry, Corolla Matrix, and Prius—Toyota's hybrid car—in the same segment of the car market. Hyundai also sells the Elantra and Hybrid Sonata in the same segment of the U.S. car market. As another example, Apple sells the iPad Mini and the larger iPad in the tablet market. Similarly, Samsung sells the Galaxy Note and the Galaxy Tab, in both a smaller and a larger variety.[1] Since consumers believe that the quality of the firms' technology differs, each consumer places a different value on the high-quality good of each firm. Thus, these markets are horizontally and vertically differentiated. Such markets present more cases of *cannibalization*.[2] Cannibalization *within the same market* occurs when a firm increases the output of one of its products by reducing the output of a similar competing product in the same market.

The objective of this study is to examine cannibalization within the same market from strategic point of view of the multi-product firm which supplies two goods differentiated in quality.

For the purpose of our analysis, both the quality level and the number of differentiated goods supplied by each firm are given like in Gilbert and Matutes (1993) and unlike in preceding studies. In addition, we do not consider new entries to the market in our model. In our setting, both firms produce and supply two kinds of vertically differentiated goods

[1] See "Samsung's Brand Cannibalization," http://www.indianprice.com/mobiles/articles/15-samsungs-brand-cannibalization.html.

[2] In fact, many reports suggest that the iPad Mini is cannibalizing sales of the larger iPad. See, for example, Seward (2013), "Yes, the iPad Mini is cannibalizing sales of larger iPad."

in a market. Kitamura and Shinkai (2014) show that when a firm (say firm 1) chooses to expand its product line or supply only one type of good, while another firm (firm 2) sells both goods, then firm 1 has an incentive to produce both goods. The present study is an extension of their study. We focus on a model in which both firms supply two vertically differentiated products to the same market. To understand the strategic aspects of cannibalization, we consider two differences: 1) the difference in the quality of the goods; and 2) the difference in the technology of the firms. Here, we characterize the cannibalization resulting from these two differences. Thus, we consider a duopoly with asymmetric marginal costs of a high-quality good.

This study offers three contributions to existing literature. First, we find that cannibalization can be seen as a business strategy characterized by a difference in the quality of vertically differentiated goods and in cost efficiency. Second, we show that, as the difference in quality between the two goods increases from a sufficiently small to a sufficiently large level, cannibalization from the low-quality to the high-quality good of the efficient firm expands, while that from the high-quality to the low-quality good of the inefficient firm shrinks. Third, we show that counter-intuitively, an increase in the production costs of the inefficient firm improves social welfare when the difference in the quality of the two goods is sufficiently small.

In typical models of horizontal or vertical product differentiation, each firm produces only one kind of good, given exogenously, which differs from that of its rival. For example, Ellison (2005), whose study is closely related to the present study, analyzes a market in which each firm sells a high-end and low-end version of the same product. Although each firm produces two differentiated goods, the two goods are sold in different markets, each

with different types of consumers.[3]

In literature on product line design, Desai(2001) considers two segments duopoly markets for high-quality and low-quality goods represented by Hotelling type model. He examines whether the cannibalization problem affects a firm's price and quality decision. He characterises such effects by consumers' differences in quality valuations and in their taste preferences. Gilbert and Matutes (1993) explores competition in the two segment market by focusing the product lines of two spatially differentiated firms with exogenous quality levels. They examine whether both of firms would specialize to serve one segment each and characterize this by the differentiation between two firms.

In existing literature on vertical product differentiation, the quality of goods that firms produce is treated as an endogenous variable. For example, in Bonanno (1986) and Motta (1993), firms initially choose a quality level and then compete in Cournot or Bertrand fashion in an oligopolistic market.

However, all of these studies stated above do not consider firms that sell *multiple products*, differentiated in terms of quality (vertically), in the *same* market. In dealing with cannibalization in such a market, our model needs to allow for a multi-product firm that differs in terms of its features or characteristics. Few previous studies address an oligopolistic market with such firms, although Johnson and Myatt (2003) are a notable exception.[4]

According to Johnson and Myatt (2003), firms that sell multiple quality-differentiated products frequently change their product lines when a competitor enters the market. They explain the common strategies of using "fighting brands" and "pruning" product lines.

[3]This model combines vertical differentiation (two distinct qualities) and horizontal differentiation (two firms located at distinct points in a linear city).

[4]For the sake of simplicity, we focus on a duopoly model.

That is, unlike this study, they endogenize not only the quality level of each good, but also the number of goods that each firm supplies in the market.

Our study's results are also related to those of marketing studies on product segmentation and product distribution strategies. For example, Calzada and Valletti (2012) study a model of film distribution and consumption. They consider a film studio that can release two versions of one film—one for theatres and one for video– although they do not consider oligopolistic competition between film studios. In their model, a film studio decides on its versioning strategy and sequencing strategy. The versioning strategy involves the simultaneous release of the two versions, while the sequencing strategy involves the sequential release of the versions. They show that the optimal strategy for the studio is to introduce versioning if their goods are not close substitutes for each other. In their model, "versioning" and "sequencing" correspond to the simultaneous supply and sequential supply, respectively, of high- and low-quality goods, as in our model. In the case of sequential supply, the film studio supplies the high-quality film version in theatres and then launches the low-quality DVD version to the same market.

We also establish a result which indirectly supports the above result in Calzada and Valletti (2012). Thus, when the difference in quality between the high-quality good H and the low-quality good L is large to some extent and so they are not close substitutes for each other, we show that both of firms had better supply both of goods in the market, that is, they should obey 'versioning strategy.'

The remainder of this paper is organized as follows. In section 2, we present our model and derive a duopoly equilibrium with two vertically differentiated products in a market. Furthermore, we use comparative statistics of the equilibrium output to explore how the quality of goods, cost asymmetry, and cannibalization are related. In section 3,

we conduct a welfare analysis of the duopoly model that we present in section 2. Finally, section 4 concludes the paper and offers suggestions for possible future research.

2 The Model and the Derivation of an Equilibrium

Suppose there are two firms, $i = 1, 2$, and each produce two goods (good H and good L) that differ in terms of quality, where 1 and 2 imply firm 1 and firm 2 in the duopoly case, respectively. Let V_H and V_L denote the quality level of the two goods. Then, the maximum amount consumers are willing to pay for each good is assumed to be $V_H > V_L > 0$. Further, we assume $V_H = (1+\mu)V_L$, where μ represents the difference in quality between the two goods, and we normalize the quality of the low-quality good as $V_L = 1$, for simplicity. Good $\alpha(= H, L)$ is assumed to be homogeneous for any consumer. Moreover, suppose that each firm has constant returns to scale and that $c_{iH} > c_{iL} = c_{jL} = c_L$, where $c_{i\alpha}$ is firm i's marginal and average cost of good α. This implies that a high-quality good incurs a higher cost of production than a low-quality good.[5] Without loss of generality, we also assume that $c_L = 0$. Under these assumptions, each firm's profit is defined in the following manner:

$$\pi_i = (p_{iH} - c_{iH})q_{iH} + p_{iL}q_{iL} \qquad i = 1, 2, \tag{1}$$

where $p_{i\alpha}$ is the price of good α sold by firm i, and $q_{i\alpha}$ is the firm's output of good α. Each firm chooses the quantity to supply that maximizes this profit function in Cournot fashion.

Next, we describe the consumers' behavior in our model.

Following the standard specification in the literature, for example, Katz and Shapiro (1985), we assume there is a continuum of consumers characterized by a taste parameter,

[5] For details on the symmetric costs version of our analysis, see Kitamura and Shinkai (201b).

θ, which is uniformly distributed between 0 and $r(>0)$, with density 1. We further assume that a consumer of type $\theta \in [0, r]$,for $r > 0$, obtains a net surplus from one unit of good α from firm i at price $p_{i\alpha}$. Thus, the utility (net benefit) of consumer θ who buys good α $(= H, L)$ from firm i $(= 1, 2)$ is given by

$$U_{i\alpha}(\theta) = V_\alpha \theta - p_{i\alpha} \qquad i =, 1, 2 \quad \alpha = H, L. \tag{2}$$

Each consumer decides to buy either nothing or one unit of good α from firm i to maximize his/her surplus.

Before deriving the inverse demand of each good, we present three further assumptions about the consumers in our model.

[**Assumption 1**] There exists a consumer, $\hat{\theta}_i \in [0, r]$, who is indifferent between the two goods of the same firm; that is,

$$U_{iH}(\hat{\theta}_i) = U_{iL}(\hat{\theta}_i) > 0, i = 1, 2. \tag{3}$$

[**Assumption 2**] There always exists a consumer, $\underline{\theta}_{iL}, i = 1, 2,$, who is indifferent between purchasing good L and purchasing nothing in the duopoly.

To derive a duopoly equilibrium, we need one other key assumption.

[**Assumption 3**] In the duopoly, for an arbitrary type-θ_α consumer,

$$U_{1\alpha}(\theta_\alpha) = U_{2\alpha}(\theta_\alpha), \alpha = H, L. \tag{4}$$

This last assumption implies that the net surplus of consumer θ_α must be the same whether buying a good produced by firm 1 or a good produced by firm 2, as long as the

two firms produce the same quality of good α and have positive sales.

From these assumptions, we can illustrate the demand for good H and good L using a line segment, as shown in Figure 1.

Insert Figure 1 here.

Here, $\widehat{\theta}$, the threshold between the demand for product H and for L, is given by

$$\widehat{\theta}^* = \frac{1}{\mu}(p_H^* - p_L^*). \tag{5}$$

Thus, the inverse demand functions are obtained in the following manner:

$$\begin{cases} p_H = (1+\mu)(r - Q_H) - Q_L \\ p_L = r - Q_H - Q_L. \end{cases} \tag{6}$$

To maximize profit function (1), each firm determines the quantity of goods to produce, q_{iH} and q_{iL}, in the following manner:

$$\max_{q_{iH}, q_{iL}} \pi_i.$$

Here, we set $c_{2H} > c_{1H} > c_{iL} = 0$, which means that firm 1 is more efficient than firm 2. The first-order conditions for profit maximization are as follows:

$$\begin{aligned} -(1+\mu)q_{1H} + (1+\mu)(r - Q_H) - Q_L - c_{1H} - q_{1L} &= 0 \\ -(1+\mu)q_{2H} + (1+\mu)(r - Q_H) - Q_L - c_{2H} - q_{2L} &= 0 \\ -q_{1H} + r - Q_H - Q_L - q_{1L} &= 0 \\ -q_{2H} + r - Q_H - Q_L - q_{2L} &= 0. \end{aligned}$$

Solving this system, we obtain the following Nash equilibrium quantities:

$$\begin{cases} q_{1H}^* = \frac{r}{3} - \frac{2c_{1H}-c_{2H}}{3\mu}, \quad q_{1L}^* = \frac{2c_{1H}-c_{2H}}{3\mu} \\ q_{2H}^* = \frac{r}{3} - \frac{2c_{2H}-c_{1H}}{3\mu}, \quad q_{2L}^* = \frac{2c_{2H}-c_{1H}}{3\mu}. \end{cases} \tag{7}$$

For q_{iH}^* and q_{iL}^* to be positive, we assume that

$$\mu > \frac{2c_{2H} - c_{1H}}{r} \quad and \quad c_{1H} > \frac{1}{2}c_{2H}. \tag{8}$$

Hence, the total equilibrium output, Q^*, becomes constant:

$$Q^* = Q_1^* + Q_2^* = Q_H^* + Q_L^* = \frac{2}{3}r. \tag{9}$$

From (6) and (7), we obtain the following equilibrium prices of the goods:

$$p_H^* = \frac{(1+\mu)r + c_{1H} + c_{2H}}{3}, \; p_L^* = \frac{r}{3}. \tag{10}$$

We also have the equilibrium profit of firm i:

$$\pi_i^* = \frac{\mu(1+\mu)r^2 - 2\mu(2c_{iH} - c_{jH})r + (2c_{iH} - c_{jH})^2}{9\mu}, \quad i = 1, 2 \; , \quad i \neq j \tag{11}$$

Then, the equilibrium outputs of (7) lead to the following condition for cannibalization: We have

$$
\begin{aligned}
q_{1H}^* - q_{2H}^* &= \frac{1}{3\mu}(2c_{2H} - c_{1H} - (2c_{1H} - c_{2H})) \qquad (12)\\
&= q_{2L}^* - q_{1L}^*\\
&= \frac{1}{\mu}(c_{2H} - c_{1H}) > 0.
\end{aligned}
$$

We also confirm the difference in the profits of the two firms, as follows:

$$
\begin{aligned}
\pi_2 - \pi_1 &= \frac{1}{3\mu}(c_{1H} - c_{2H})(2\mu r - c_{1H} - c_{2H}) < 0, \qquad (13)\\
\text{since } \mu &> \frac{2c_{2H} - c_{1H}}{r} > \frac{c_{1H} + c_{2H}}{2\ r} \text{ and } c_{1H} < c_{2H}.
\end{aligned}
$$

Hence, we can easily establish the following proposition.

Proposition 1 *Although the efficient firm (firm* 1*) produces more of high-quality good* H *than the inefficient firm (firm* 2*), the inefficient firm sells more of the low-quality good* L *than the efficient firm. Furthermore, if the difference in unit costs between the two firms is sufficiently small (i.e., if* $2c_{1H} = c_{2H}$*), then the efficient firm does not produce the low-quality good. The efficient firm* 1 *earns more than the inefficient firm* 2 *does.*

The proposition implies that the efficient firm 1 (inefficient firm 2) produces more of high-quality good H (low-quality good L) than the inefficient firm 2 (efficient firm 1). It also asserts that the efficient firm 1 earns more than the inefficient firm 2 because of cost efficiency of firm 1 over firm 2 on the high-quality good H under the positive outputs assumption (8) in the equilibrium.

Next, we examine under which conditions the cannibalization from one product to another occurs in the equilibrium. Note that we say "a product cannibalizes a similar

product" when a firm increases the output of the product by reducing that of the similar product supplied in the same market.

From (7), we have

$$\begin{aligned} q_{2H}^* - q_{2L}^* &= \frac{1}{3}(r - \frac{2(2c_{2H} - c_{1H})}{\mu}) \gtreqless 0 \\ &\Leftrightarrow \mu \gtreqless \frac{2(2c_{2H} - c_{1H})}{r} \Leftrightarrow q_{2H}^* \gtreqless q_{2L}^* \end{aligned} \tag{14}$$

and

$$\begin{aligned} q_{2H}^* - q_{1L}^* &= \frac{r}{3} - \frac{2c_{2H} - c_{1H}}{3\mu} - \frac{2c_{1H} - c_{2H}}{3\mu} \\ &= q_{1H}^* - q_{2L}^* = \frac{1}{3\mu}(\mu r - (c_{2H} + c_{1H})) \\ &\gtreqless 0 \Longleftrightarrow \mu \gtreqless \frac{c_{2H} + c_{1H}}{r}. \end{aligned} \tag{15}$$

From (8), we also see that

$$\frac{c_{1H} + c_{2H}}{r} > \frac{2c_{2H} - c_{1H}}{r}.$$

Then, from the above inequality, (15), (14), and proposition 1, we immediately obtain

$$
\begin{aligned}
q_{2H}^* &\leq q_{1L}^* < q_{1H}^* \leq q_{2L}^* \text{ for } \frac{2c_{2H} - c_{1H}}{r} < \mu \leq \frac{c_{1H} + c_{2H}}{r}, \\
q_{1L}^* &< q_{2H}^* < q_{2L}^* < q_{1H}^* \text{ for } \frac{c_{1H} + c_{2H}}{r} < \mu < \frac{2(2c_{2H} - c_{1H})}{r}, \\
q_{1L}^* &< q_{2L}^* \leq q_{2H}^* < q_{1H}^* \text{ for } \frac{2(2c_{2H} - c_{1H})}{r} \leq \mu. \qquad (16)
\end{aligned}
$$

Thus, we present the following proposition, without proof.

Proposition 2 *In the duopoly equilibrium derived above, if the difference in the quality of the two goods, μ, is sufficiently small (i.e., $\mu \ni (\frac{2c_{2H}-c_{1H}}{r}, \frac{c_{1H}+c_{2H}}{r}]$), then $q_{2H}^* \leq q_{1L}^* < q_{1H}^* \leq q_{2L}^*$. As μ approaches $\frac{2c_{2H}-c_{1H}}{r}$ from above, product L of firm 2 cannibalizes product H and q_{2H}^* approaches 0. When μ* grows, *product H of both firms always cannibalizes product L. As μ grows and approaches $\frac{c_{1H}+c_{2H}}{r}$, and q_{2H}^* approaches q_{1L}^* . If μ is included in the median value range (i.e., $\mu \ni (\frac{c_{1H}+c_{2H}}{r}, \frac{2(2c_{2H}-c_{1H})}{r})$), then $q_{1L}^* < q_{2H}^* < q_{2L}^* < q_{1H}^*$. As μ grows and approaches $\frac{2(2c_{2H}-c_{1H})}{r}$, q_{2H}^* approaches q_{2L}^* . However, if μ is sufficiently high (i.e., $\mu \ni (\frac{2(2c_{2H}-c_{1H})}{r}, \infty)$), then $q_{1L}^* < q_{2L}^* \leq q_{2H}^* < q_{1H}^*$. As μ approaches ∞ , q_{1L}^* and q_{2L}^* vanish.*

The intuition behind Proposition 2 is straightforward. When the difference in the quality of the two goods is sufficiently small, the inefficient firm produces far more of low-quality good L, with no production cost, than it does of high-quality good H, which has a higher positive cost. In contrast, the efficient firm produces moderately more of its low-quality good L than it does of good H, since its production cost for good H is lower than that of its rival. However, its marginal revenue from good H is not high, because the difference in the quality of the two goods is very small.

Thus, interestingly, as μ approaches $(2c_{2H} - c_{1H})/r$ from (7), q_{2H}^* approaches 0. Thus,

the inefficient firm 2 stops producing the high-quality good H, almost specializing in the low-quality good. Then, in equilibrium, the market approaches a three-goods market. This market is filled with large quantities of the low-quality good L supplied by both of firms, but relatively little of the high-quality good H supplied by firm 1.

This result is consistent with the result in Calzada and Valletti (2012) that the optimal strategy for the film studio is to introduce versioning if their goods are not close substitutes for each other. Thus, when the difference in quality between the high-quality good H and the low-quality good L is large to some extent, we can consider that they are not close substitutes for each other. Then, the result in the above proposition asserts that both of firms had better supply both of goods in the market, that is, to obey 'versioning strategy,' in Calzada and Valletti (2012). On the other hand, if the difference in quality of two goods reduces to nearly zero and they become close substitutes each other, the best strategy of the inefficient firm 2 is to vanish the output of its high-quality goods H and to specialize in the low-quality good L!

The efficient firm can produce its high-quality good H at a lower cost than its rival. When the difference in the quality of the two goods becomes high, the efficient firm produces far more of the high-quality good than it does of the low-quality good, because it is profitable to do so. However, the inefficient firm also reduces the output of its low-quality good and increases that of its high-quality good, because the profitability of good H becomes large, even though the inefficient firm's production cost is higher than that of its rival.

In this case, as μ approaches $(c_{1H} + c_{2H})/r$ from (7), q^*_{2H} approaches q^*_{1L}. As μ increases further over $(c_{1H} + c_{2H})/r$, the cannibalization from the low-quality good to the high-quality good of efficient firm 1 increases, since the benefit to the efficient firm 1 of

supplying the high-quality good over the low-quality good increases. However, the same benefit to the inefficient firm 2 decreases, until the former surpasses the latter. Then, as μ approaches $2(2c_{2H} - c_{1H})/r$, q^*_{2H} approaches q^*_{2L}. Lastly, as μ increases further over $2(2c_{2H} - c_{1H})/r$ to infinity, q^*_{1L} and q^*_{2L} vanish and both firms only produce their high-quality goods H.

Next, we analyze the comparative statics of the equilibrium outputs and profits of the firms for differences in the quality and in the marginal costs of good H.

Proposition 3 *In the duopoly equilibrium derived above, when the difference in the quality of the two goods, μ, or the marginal cost of high-quality good H of competitor c_{jH} increases (decreases), then cannibalization occurs in the outputs of firm i such that the high-quality (low-quality) good H (L) keeps the low-quality (high-quality) good L (H) from the market. However, if the marginal cost of its own high-quality good H, c_{iH}, increases (decreases), then cannibalization occurs in the outputs of firm i such that the low-quality (high-quality) good L (H) keeps the high-quality (low-quality) good H (L) from the market.*

We illustrate the intuitive reasoning behind this proposition in relation to the current tablet PC market. When the difference in the quality of the goods, μ, is sufficiently large, or the marginal cost of the high-quality good H of its rival, c_{jH}, is high, the efficient firm, for example Apple, increases its output of the high-quality iPad. In contrast, if its rival, the inefficient firm (for example, Samsung), can produce a high-quality tablet (owing to its research and development efforts) at a lower cost, c_{jH}, than that of Apple, or if the difference in the quality of the goods, μ, becomes small, then Apple expands production of the lower-quality iPad Mini, which cannibalizes the larger iPad. Then, Samsung's new tablet cannibalizes sales of its existing 10.1-inch tablet. However, unless the market

has goods that are extremely differentiated or extremely similar in terms of quality, cannibalization does not keep one of the firms' products from the market.[6]

From (11), we have

$$\frac{\partial \pi_i^*}{\partial \mu} = \frac{(\mu r + 2c_{iH} - c_{jH})(\mu r - (2c_{iH} - c_{jH}))}{9\mu^2} > 0, i = 1, 2. \tag{17}$$

Furthermore, we also check the effects of production costs on profit. From (11) and (8), we have

$$\frac{\partial \pi_i^*}{\partial c_{iH}} = -\frac{4}{9}(r - \frac{2c_{iH} - c_{jH}}{\mu}) < 0, \ \frac{\partial \pi_i^*}{\partial c_{jH}} = \frac{2}{9}(r - \frac{2c_{iH} - c_{jH}}{\mu}) > 0 \ .$$

Thus, we obtain the following proposition.

Proposition 4 *When the difference in the quality of the two goods increases, the equilibrium profits of both firms increase. Furthermore, a decrease in the marginal cost of a firm's own good H or an increase in the marginal cost of the competitor's good H increases the profit of the firm.*

This proposition is plausible. When the difference in the quality between two goods is sufficiently small, the inefficient firm produces more of the low-quality good than it does of the high-quality good, from equation (16), to avoid suffering from the positive marginal cost of producing the high-quality good. Then, an increase in the difference in the quality of the two goods, μ, or a decrease in the unit cost of a firm's own good H or an increase in

[6] From the article in the web news,"Samsung's Brand Cannibalization," Samsung occasionally improves its products, which kills its existing product in the market. The launch of the 10.1 inch Galaxy Note (Samsung's latest tablet) will most likely cannibalize sales of the existing 10.1 inch tablet. However, Samsung does not mind, as one of the best ways to continue to exist in a competitive market is to eradicate your own goods. See http://www.indianprice.com/mobiles/articles/15-samsungs-brand-cannibalization.html
for more detail.

the unit cost of its competitor's good H helps this firm to produce more of the high-quality good. Thus, it reduces the quantity of the low-quality good L because of cannibalization. However, from equations (7) and (16), the proportion of the cannibalization from the low-quality good to the high-quality good in both firms is different. That of the defensive efficient firm 1 is lower than that of the aggressive inefficient firm 2 because of the cost efficiency of firm 1 for the high-quality good.[7] Similarly, if the difference in quality is sufficiently small, a decrease in a firm's own unit cost of good H or an increase in the unit cost of the rival firm has a similar effect on both firms' proportions of cannibalization from the low-quality good to the high-quality good.

However, if the difference in quality between the goods μ becomes sufficiently large, the efficient firm 1 aggressively produces more of the high-quality good and reduces the quantity of the low-quality good because of its cost efficiency in the case of the high-quality good. Then, the inefficient firm 2 defensively reduces the quantity of the low-quality good and increases the output of the high-quality good to limit the reduction in its profit owing to the cannibalization from the low-quality good to the high-quality good. In the case of a decrease in a firm's own unit cost of good H or an increase in the unit cost of the rival firm when the difference in quality between the goods, μ, is large, the effect is similar to the effect on both firms' proportions of cannibalization from the low-quality good to the high-quality good. The changes in μ, c_{iH},and c_{iH} mean that the increase in the profit of

[7]From (7), the proportions of the cannibalization for firm 1 and firm 2 from the low-quality good to high-quality good owing to an increase in the difference in quality are expressed by
$\Delta Canniba^1_{qL\to H(\mu)} \equiv \partial q^*_{1H}/\partial\mu - \partial q^*_{1L}/\partial\mu = ((2c_{1H} - c_{2H}) - (2c_{2H} - c_{1H}))/(3\mu^2)$
$= 2(2c_{1H} - c_{2H})/(3\mu^2)$,
and
$\Delta Canniba^2_{qL\to H(\mu)} \equiv \partial q^*_{21H}/\partial\mu - \partial q^*_{2L}/\partial\mu = ((2c_{2H} - c_{1H}) - (2c_{2H} - c_{1H}))/(3\mu^2)$
$= 2(2c_{2H} - c_{1H})/(3\mu^2)$, respectively. Hence,
$\Delta Canniba^1_{qL\to H(\mu)} - \Delta Canniba^2_{qL\to H(\mu)} = 2(c_{2H} - c_{1H})/\mu^2 > 0$.
Furthermore, from (16), we see that
$q^*_{1H} - q^*_{1L} < q^*_{2L} - q^*_{2H}$ if $\frac{2c_{2H}-c_{1H}}{r} < \mu < \frac{c_{1H}+c_{2H}}{r}$.

firm 1 surpasses that of firm 2.[8]

3 Welfare Analysis with Asymmetric Cost

In this section, we describe the comparative statics of the social welfare in the equilibrium.

The social surplus in equilibrium, derived in the preceding section, is given by

$$\begin{aligned} W^* &= \int_{\frac{r}{3}}^{\hat{\theta}^*} \theta d\theta + \int_{\hat{\theta}^*}^{r} (1+\mu)\theta d\theta - c_{1H}q_{1H}^* - c_{2H}q_{2H}^* \\ &= -\frac{\mu}{2}(\hat{\theta}^*)^2 - \frac{r^2}{18} + \frac{(1+\mu)r^2}{2} - c_{1H}q_{1H}^* - c_{2H}q_{2H}^*. \end{aligned} \tag{18}$$

First, we explore the effect of a change in unit cost on social welfare. From (5) and (7)

$$\frac{\partial W^*}{\partial c_{iH}} = \frac{11c_{iH} - 7c_{jH} - 4\mu r}{9\mu} \quad i = 1, 2.$$

Thus,

$$\begin{cases} \frac{\partial W^*}{\partial c_{1H}} < 0 & \\ \frac{\partial W^*}{\partial c_{2H}} > 0 & if \ \frac{2c_{2H}-c_{1H}}{r} \leq \mu < \frac{11c_{2H}-7c_{1H}}{4r} \\ \frac{\partial W^*}{\partial c_{2H}} \leq 0 & if \ \frac{11c_{2H}-7c_{1H}}{4r} \leq \mu \end{cases} . \tag{19}$$

Finally, we show that a change in the difference in quality between the two goods always has a positive effect on social welfare, as follows:

$$\frac{\partial W^*}{\partial \mu} = \frac{8\mu^2 r^2 - 11c_{1H}^2 - 11c_{2H}^2 + 14c_{1H}c_{2H}}{18\mu^2} \tag{20}$$

[8] For an increase in μ, we see that $\frac{\partial \pi_1^*}{\partial \mu} - \frac{\partial \pi_2^*}{\partial \mu} = (c_{1H} + c_{2H})(c_{2H} - c_{1H})/(3\mu^2) > 0$, since $c_{2H} > c_{1H} > 0$, from (17). The argument is similar for a decrease in c_{iH} and an increase in c_{jH}.

The sign of $\partial W^*/\partial\mu$ is determined by the sign of the numerator of (20), where we define the numerator by $W_\mu^n(r)$, and $W_\mu^n(r)$ is a quadratic in r. Evaluating $W_\mu^n(r)$ at $r = (2c_{2H} - c_{1H})/\mu$, we have

$$\begin{aligned} W_\mu^n(\frac{2c_{2H} - c_{1H}}{\mu}) &= 3(7c_{2H}^2 - c_{1H}^2 - 6c_{1H}c_{2H}) \\ &= 3(c_{2H} - c_{1H})(7c_{2H} + c_{1H}) > 0, (\because c_{2H} > c_{1H}) \end{aligned} \tag{21}$$

and we see that the slope of $W_\mu^n(r)$ with respect to r is

$$\left.\frac{\partial W_\mu^n(r)}{\partial r}\right|_{r=\frac{2c_{2H}-c_{1H}}{\mu}} = 16(2c_{2H} - c_{1H}) > 0.$$

Then, we obtain

$$\frac{\partial W^*}{\mu} > 0. \tag{22}$$

Thus, we show that an increase in the difference between the two goods improves social welfare.

Proposition 5 *The social surplus in equilibrium increases with*

1. *a decrease in the marginal cost of the efficient firm for the high-quality good.*
2. *a decrease (increase) in the unit cost of the inefficient firm when producing the high-quality good if the difference in quality is sufficiently large (small).*

Moreover, an increase in the difference between the two goods always increases the social surplus in equilibrium.

The second part of this proposition is both interesting and counter-intuitive, because we may think that an increase in the production cost would lead to a decrease in social

welfare. However, a case exists in which social welfare improves if there is an increase in the marginal cost of the high-quality good. The reason is that when the difference in quality is small, the increase in the marginal cost of the inefficient firm leads to a reduction in the total cost; $(\partial Total\ cost)/\partial c_{2H} < 0$. This has a positive effect on social welfare. On the other hand, the effect on total consumer utility is always negative; $(\partial Total\ utility)/\partial c_{2H} < 0$. Thus, when the positive effect of the former dominates the negative effect of the latter, the social surplus in equilibrium increases because the unit cost to the inefficient firm of producing good H increases and the difference in quality is sufficiently small.

4 Concluding Remarks

In this study, we considered and proposed a duopoly model of cannibalization in which two firms each produce and sell two distinct products that are differentiated vertically in the same market. Then, we showed that in the market equilibrium, the efficient firm produces more of the high-quality good and the inefficient firm produces more of the low-quality good. When the difference in the quality of the two types of goods is small (large), cannibalization for firm 2 (firm 1) is stronger than that for firm 1 (firm 2).

Furthermore, we presented several comparative statics and established that an increase in the difference in the quality of the two types of goods (a reduction in the marginal cost of producing its own high-quality good) leads to cannibalization such that the high-quality good drives the low-quality good out of the market. Similarly, a decrease in the difference in the quality of the two goods (an increase in the marginal cost of the high-quality good of the competitor) causes cannibalization such that the low-quality good L drives the high-quality good H out of the market. However, unless the market

has goods that are extremely differentiated or extremely similar in terms of quality, cannibalization does not keep one product of a firm from the market, and firms supply both goods.

We also presented an intuitive explanation for these comparative statics. In relating to the results in marketing studies on product segmentation and product distribution strategies, we also establish a result which is consistent with the result in Calzada and Valletti (2012) that the optimal strategy for the film studio is to introduce versioning if their goods are not close substitutes for each other. Thus, when the difference in quality between the high-quality good H and the low-quality good L is large to some extent and so they are not close substitutes for each other, we show that both of firms had better supply both of goods in the market, that is, they should obey 'versioning strategy.'

Then, we conducted a welfare analysis and showed that an increase in the difference between the two goods and a decrease in the production costs of the high-quality good for the efficient firm always increase social welfare. However, an increase in the marginal cost of producing the high-quality good for the inefficient firm does not always harm social welfare. In particular, if the difference in quality is sufficiently small, rather counter-intuitively, an *increase in the unit cost* of the high-quality good for the inefficient firm *improves social welfare.*

Extensions to this study in future research are possible. For example, it would be useful to analyze a case in which each firm can choose its quality level as well as the number of goods it produces. In addition, in this study, we do not consider a market with network externality, which would be worth studying if we consider a market such as the tablet PC industry described in section 2. Indeed, we are analyzing such a market in another study.

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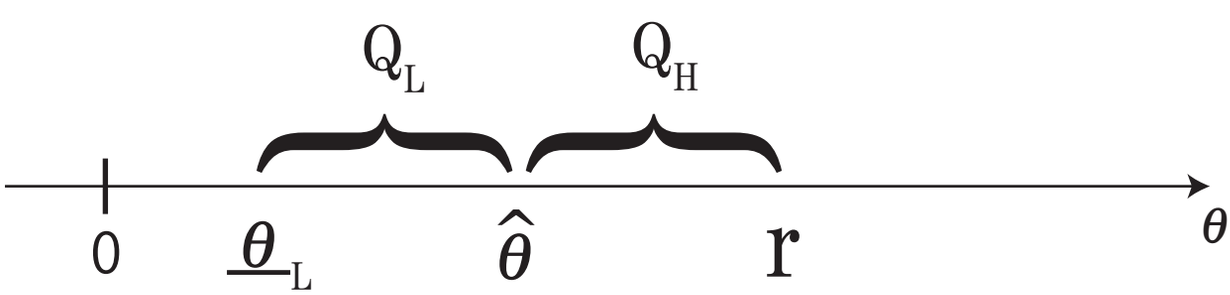

Figure 1