DISCUSSION PAPER SERIES

Discussion paper No. 120

Strategic Choice on Product Line in Vertically Differentiated Duopoly

Ryoma Kitamura
Graduate School of Economics, Kwansei Gakuin University

Tetsuya Shinkai
School of Economics, Kwansei Gakuin University

August, 2014

MASTERY FOR SERVICE

SCHOOL OF ECONOMICS

KWANSEI GAKUIN UNIVERSITY

1-155 Uegahara Ichiban-cho
Nishinomiya 662-8501, Japan

Strategic Choice on Product Line in Vertically Differentiated Duopoly[1]

Ryoma Kitamura[2]
Graduate School of Economics, Kwansei Gakuin University
1-155, Uegahara Ichiban-cho, Nishinomiya, Hyogo 662-8501, Japan

Tetsuya Shinkai[3]
School of Economics, Kwansei Gakuin University
1-155, Uegahara Ichiban-cho, Nishinomiya, Hyogo 662-8501, Japan

August 29, 2014

Abstract

In a real oligopoly, firms often supply multiple products differentiated by quality in the same market. To examine why they do so, we consider a duopoly model in which firms can choose between supplying two vertically differentiated products and selling a single product in the same market. By deriving equilibriums for possible games and comparing their outcomes with each other, we explored the conditions in which firms strategically determine their product lines, choosing to sell between a single product and two products. The first three are the cases in which both firms supply both products, or they supply either homogeneous product of the two in the same market. The last two are those in which one firm supplies both but another firm does either of the two. We find that a firm producing only one product has an incentive to launch another product as long as it can do so.

Keywords: Multi-product firm; Duopoly; Strategic choice of product line; Vertical product differentiation, Cannibalization, Launch of product

JEL Classification Codes: D21, D43, L13, L15

---

[1] The authors are grateful to Tommaso Valletti, Federico Etro, Noriaki Matsushima, Toshihiro Matsumura, Kenji Fujiwara, and Keizo Mizuno for their useful comments on an earlier version of this paper.
The second author was supported by Grants-in-Aid for Scientific Research (Nos. 23330099 and 24530255) MEXT.
[2]Email: aja83048@kwansei.ac.jp, Phone +81-798-54-6204, Fax +81-798-51-0944.
[3]Corresponding author, School of Economics, Kwansei Gakuin University , 1-155, Uegahara Ichiban-cho, Nishinomiya, Hyogo 662-8501, Japan, E-mail: shinkai@kwansei.ac.jp, Phone +81-798-54-6967, Fax: +81-798-51-0944.

Introduction

In a real oligopolistic market, firms strategically choose their product line. That is, they often produce and sell multiple products vertically differentiated in a same market. For example, GM sells Chevrolet Cruze and GMC Sierra PU and Toyota sells Camry, Corolla Matrix, and Prius---the hybrid car---in the same segment of the car market. Since the quality of technology of each firm differs according to the viewpoint of each consumer, each consumer places different value on a high-quality good of each firm. As another example, Apple sells the iPad Mini and the larger iPad in the tablet market. Similarly, Samsung sells the Galaxy Note and the Galaxy Tab, both in a smaller and larger variety.

Why does any oligopolistic firms supply only one product of them and other firms does multiple products in a same oligopolistic market? A natural reason for this different behavior of firms is that the former firms strategically decide to meet demand of different types of consumers by supplying multi products differentiated in quality, but the latter firms choose to meet demand a type of consumers instead. Thus, these markets may be horizontally as well as vertically differentiated. In such markets, there are more cases of *cannibalization or launch of other products in the market*.[4]

In this paper, we explore under what conditions firms strategically choose their product line and sell only one product or multi products vertically differentiated in the same market by focusing on the consumer's assessment value of the difference of the quality and costs difference between firms and between vertically differentiated goods .

In typical existing models of horizontal or vertical product differentiation, each firm produces only one kind of good, given exogenously, which differs from that of its rival. In Ellison's (2005) study, which is closely related to the present study, he analyzes a market in which each firm sells a high-end and low-end version of the same product. Although each firm produces two differentiated goods, the two goods are sold in different markets, each with different types of consumers.[5]

In the existing literature on vertical product differentiation, the quality of goods that firms produce is treated as an endogenous variable. For example, in Bonanno (1986) and Motta (1993), firms initially choose a quality level and then compete in Cournot or Bertrand fashion in an oligopolistic market. However, these studies do not consider firms that sell *multiple products* in the *same* market that are differentiated in terms of quality (vertically). In dealing with cannibalization and launch of multi products in such a market, our model needs to allow for a multi-product firm that differs in terms of its features or characteristics. Few previous studies address an oligopolistic market with such firms, although Johnson and Myatt (2003) are a notable exception.[6]

[4]Actually, many reports suggest that the iPad Mini is cannibalizing sales of the larger iPad. See, for example, Seward (2013). According to a web business news site, ‘With a launch of 10.1-inch Galaxy Note (Samsung’s latest tablet) it [Samsung] would most likely to [*sic*] cannibalize sales of the existing 10.1-inch Tablet’ (Samsung’s Brand Cannibalization, http://www.indianprice.com/mobiles/articles/15-samsungs-brand-cannibalization.html).

[5]His model combines vertical (two distinct qualities) and horizontal (two firms located at distinct points in a linear city) differentiation.

[6]For the sake of simplicity, we focus on a duopoly model.

According to Johnson and Myatt (2003), firms that sell multiple quality-differentiated products frequently change their product lines when a competitor enters the market. They explain the common strategies of using fighting brands and pruning product lines. That is, they endogenize not only the quality level of each good, but also the number of goods that each firm supplies in the market. By choosing strategically quality and outputs of multiple products differentiated in quality, they suggest that firms can identify different segments of the market demand.

For the purpose of our analysis, i.e. characterization of firms' cannibalization and launch of multi products, we consider cases in which both the quality level and the number of differentiated goods supplied by each firm are given, unlike in preceding studies. In addition, we do not consider new entries to the market in our model. In our setting, at first both firms produce and supply two kinds of vertically differentiated goods in a market and derive equilibrium. Then, we consider the other four cases in which both firms sell only product H, or product L, one of two firms sells both products and another firm sells only product H or product L and derive four equilibria in these cases. By comparing equilibriums one the other in five cases including first case, we explore under what conditions firms strategically choose their product line and sell only one product or multi products vertically differentiated in the same market.

Notably, this study's results are related to those in the marketing literature on product segmentation and the distribution strategy of products. For example, Calzada and Valletti (2012) study a model of film distribution and consumption. They consider a film studio that can release two versions of one film--one in theatres and the others as a video-- although they do not consider oligopolistic competition between film studios. In their model, a film studio decides on its versioning and sequencing strategy. The former involves simultaneous release of the versions, and the latter, sequential release. They show that the optimal strategy of the studio is to introduce versioning if their goods are not close substitutes for each other. This versioning and sequencing in their model respectively correspond to a simultaneous and sequential supply (launch) of high- and low-quality goods to the same market in our model. It implies the big difference of the quality of the two differentiated of goods in our model that two goods are not too substitute in their model.

The remainder of this paper is organized as follows. In section 2, we present the model. In section 3, at first we derive a duopoly equilibrium with two vertically differentiated products in a market. Furthermore, we also derive duopoly equilibria in other four cases in which both firms sell only product H, or product L, one of two firms sells both products and another firm sells only product H or product L. In section 4, we compare equilibria one the other in five case and examine firms' strategic choice of product line. Finally, section 5 concludes the paper and offers suggestions for possible future research.

The Model

In this section, at first, we present our basic model. Then, we consider the case in which one firm can alter its product lines in response to competition and derive equilibriums in three cases. That is, the first case is the one in which each of two firms supplies vertically differentiated two products, the second is the one in which Suppose that there are two firms, ($i=1,2$), each producing two goods (good H and good L) that differ in terms of quality, where $1,2$ imply firm 1 and firm 2 in the duopoly case, respectively. Let V_H and V_L

denote the quality level of the two goods. Then, the maximum amount consumers are willing to pay for each good is assumed to be $V_H > V_L > 0$. Further, we assume $V_H = (1+\mu)V_L$, where μ represents the difference in quality between the two goods, and we normalize the quality of the low-quality good as $V_L = 1$, for simplicity. Good k $= H, L$) is assumed to be homogeneous for any consumer. Moreover, suppose that each firm has constant returns to scale and that $c_{iH} > c_{iL} = c_{jL} = c_L$, where c_{ik} is firm i 's marginal and average cost of good k. This implies that a high-quality good incurs a higher cost of production than a low-quality good.[7] Without loss of generality, we also assume that $c_L = 0$.

Under these assumptions, each firm's profit is defined in the following manner:

$$\pi_i = (p_{iH} - c_{iH})q_{iH} + p_{iL}q_{iL} \qquad i = 1, 2, \tag{1}$$

where p_{ik} is the price of goods k sold by firm i, and q_{ik} is the firm's output. Each firm chooses the quantity to supply that maximizes this profit function in Cournot fashion.

Next, we describe the consumers' behavior in our model.
Following the standard specification in the literature, for example, Katz and Shapiro (1985), we assume that there is a continuum of consumers characterized by a taste parameter, θ, which is uniformly distributed between 0 and $r(>0)$, with density 1. We further assume that a consumer of type $\theta \in [0, r]$, for $r > 0$, obtains a net surplus from one unit of good k from firm i at price p_{ik}. Thus, the utility (net benefit) of consumer θ who buys good k $(= H, L)$ from firm i $(= 1, 2)$ is given by

$$U_{ik}(\theta) = V_k\theta - p_{ik} \qquad i =, 1, 2 \quad k = H, L. \tag{2}$$

Each consumer decides to buy either nothing or one unit of good k from firm i to maximize his/her surplus.
Under some regular conditions, we can derive the following inverse demand functions:[8]

$$\begin{cases} p_H = (1+\mu)(r - Q_H) - Q_L \\ p_L = r - Q_H - Q_L. \end{cases} \tag{3}$$

Derivation of an Equilibrium

In this section, we firstly consider the duopoly in which each firm supplies both of vertically differentiated two products in the same market as a bench mark. Then, we consider the case when one of firms can alter its product lines in response to competition. For example, suppose that a firm (say, firm i) can choose to produce both goods or one type of good but another

[7] For the symmetric costs version of our analysis, see Kitamura and Shinkai (2013) in details.

[8] See the detailed discussion in Kitamura and Shinkai (2014) for the derivation of this inverse demand function under the following assumptions. First, the consumer, $\hat{\theta}_i \in (-\infty, r], (r > 0)$, is indifferent between the two goods of the same firm. Second, the consumer, $\underline{\theta}_{iL}, i = 1, 2,$ is always indifferent between purchasing good L and purchasing nothing in a duopoly. Finally, the net surplus of consumer θ_α must be the same regardless of whether the good is produced by firm 1 or firm 2 as long as the two firms produce goods of the same quality α and have positive sales in the duopoly.

firm (firm j) produces both goods in oligopoly. One real example of this situation is the fact observed in the Japanese smart phone industry---Softbank, which is a Japanese mobile service provider, sells both android smart phones and iPhones.[9] On the other hand, KDDI---also a Japanese mobile service provider---sold only android smart phones. However, in 2011, KDDI determined to sell the iPhone as well. Is it favorable for KDDI to sell both android smart phones and the iPhone? In the real economy, there are several examples of the above mentioned models.

We explore under what conditions firms strategically choose their product line and sell only one product or multi products vertically differentiated in the same market. Starting the case in which each firm supplies both goods into the market, we consider the following five situations. To save the space in the paper, we omit derivation of outputs and profits of firms and prices of goods in the equilibriums of these five case games and we summarize these in Table 1 and Table 2.

- Case a

  In this case, both firms produce both goods. This situation has already been considered in Kitamura and Shinkai (2014), and we use it's all assumptions as benchmark of our model. Thus, the inverse demand functions are obtained in the following manner:

$$\begin{cases} p_H = (1+\mu)(r - Q_H) - Q_L \\ p_L = r - Q_H - Q_L. \end{cases} \tag{4}$$

Then, firm i chooses its outputs of each product so as to maximize its profit (1). For the equilibrium outputs of the goods, q_{iH}^* and q_{iL}^* to be positive, from Table 1 we assume that

$$r > \frac{2c_{iH} - c_{jH}}{\mu} \quad and \quad c_{iH} > \frac{1}{2} c_{jH} \; i, j = 1, 2, \;\; i \neq j. \tag{5}$$

- Case b

  In this case, firm i produces only good H and firm j supplies both goods in the market. Then, by setting $Q_L = q_{jL}$ in (4), the inverse demands of two goods can be rewritten as

$$\begin{cases} p_H = (1+\mu)(r - Q_H) - q_{jL} \\ p_L = r - Q_H - q_{jL}, \quad i \neq j. \end{cases}$$

  Then, the profit of each firm is rewritten from (1) as

$$\begin{cases} \pi_i = (p_H - c_{iH}) q_{iH} \\ \pi_j = (p_H - c_{jH}) q_{jH} + p_L q_{jL}, \quad i, j = 1, 2 \;\; i \neq j. \end{cases} \tag{6}$$

  Then, each firm chooses its outputs of each product so as to maximize its profit (6).

For the equilibrium outputs of the goods, q_{iH}^* and q_{jH}^* to be positive, from Table 1 we

[9] Geekbench indicates quality differences between the iPhone and Android smartphones. See http://browser.primatelabs.com/geekbench2/1030202 for the iPhone and http://browser.primatelabs.com/android-benchmarks for Android smartphones.

assume that

$$r > \frac{2\mu(2c_{iH} - c_{jH})c_{iH} + 3c_{jH}}{2\mu(1+\mu)} \tag{7}$$

- Case c

  In this case, firm i produces only good L. Therefore by setting $Q_H = q_{jH}$ in (4), the inverse demands of two goods are rewritten as

$$\begin{cases} p_H = (1+\mu)(r - q_{jH}) - Q_L \\ p_L = r - q_{jH} - Q_L \end{cases} \tag{8}$$

  The profit of each firm is rewritten from (1) as

$$\begin{cases} \pi_i = p_L q_{iL} \\ \pi_j = (p_H - c_{jH})q_{jH} + p_L q_{jL}, \quad i, j = 1, 2 \;\; i \neq j. \end{cases} \tag{9}$$

Each firm chooses its outputs of each product so as to maximize its profit (9).
For the equilibrium outputs of the goods, q_{iH}^{c*} and q_{jH}^{c*} to be positive from Table 1, we see that the inequality

$$r > \frac{c_{jH}}{\mu} > \frac{1}{3}r \tag{10}$$

holds.

- Case d

In this case, both firms produce only product H. So by setting $Q_L = 0$ in (4), the inverse demands of two goods are presented from as

$$p_H = (1+\mu)(r - Q_H). \tag{11}$$

So from (1) the profit of each firm is given by

$$\pi_i = (p_H - c_{iH})q_{iH} = ((1+\mu)(r - Q_H) - c_{iH})q_{iH}, i = 1, 2. \tag{12}$$

Each firm chooses its outputs of each product so as to maximize its profit (profit of case d).
For the positivity of equilibrium output, we see from Table1 that

$$r > \frac{2c_{iH} - c_{jH}}{1+\mu}, i, j(\neq i) = 1, 2. \tag{13}$$

- Case e

In this case, both firms produce only product L. Hence by setting $Q_H = 0$ in (4), the inverse demands of two goods can be rewritten as

$$p_L = r - Q_L.$$

Then, from (1) the profit of each firm is

$$\pi_{iL} \equiv p_L q_{iL} = (r - Q_L) q_{iL}.$$

To maximize its profit give above, each firm chooses its outputs of each product.

[Insert here Table 1]

Strategic Choice of Product Line

In this section, we compare five equilibriums one the other and explore how each firm strategically chooses its product line and supply only one product or both ones in a same market. By comparison case c and case e, we can obtain the conditions under which firm j in case e expands its product line and supplies not only product L but also H in the same market thus moves to case c, and we obtain the following proposition. For the proof of the proposition, see Appendix.

Proposition 1 *Suppose that each firm supplies only product* L (*case e*). *If the evaluation of the value of the one unit difference of high and low quality product for the consumer with the highest preference is higher than firm* j *'s unit cost of high quality product, i.e.* $r > c_{jH} / \mu$ ($r \le c_{jH} / \mu$)*, then firm* j *supplies both quality products* (only product L) *in the same market and the state of the market moves to case c* (*remains case e*).

The intuition for the proposition is obvious. If the maximal value obtained for the consumer with the highest preference from replacing a unit consumption of product H with that of product L is larger than the unit cost of the unit cost of product H of firm j, then the demand for product H of firm j is at least positive and the maximum margin for a unit of product H is positive and a launch of product H into the market is better off for firm j and the state moves from case e to case c. But conversely, the maximum margin for a unit of product H is negative, i.e. $r\mu \le c_{jH}$, then firm j cannibalizes the product H and supplies only product L into the same market.

Next, we compare case b with case d and we explore the conditions under which firm j in case d supplies not only product L but also H in the same market, thus moves to case b. For the proof of the proposition, see Appendix.

[Insert here Table 2]

Proposition 2 *Suppose that each firm supplies only product* H (*case d*). *If the evaluation of the value of the one unit of high quality product for the consumer with the highest preference is higher than firm* i 's *substantial unit cost of high quality product* H*, which is compensated by its cost advantage over firm* j*, i.e.* $(1+\mu)\ r > 2c_{iH} - c_{jH}$*, then firm* j

supplies both quality products in the same market.

The intuition for the proposition is interesting. The condition for firm j to supply both quality products given in the proposition can be rewritten into $r(1+\mu) > (2c_{iH} - c_{jH}) = c_{iH} - (c_{jH} - c_{iH})$. The left hand side of this inequality is the evaluation for the product H of the consumer with the highest preference, and $(c_{jH} - c_{iH})$ in the right hand side of it can be interpreted as how much does have firm j disadvantage in unit cost of high quality product compared to firm i. Note that both firms supply only high quality product in case d. Thus, as the evaluation for the product H of the consumer with the highest preference is higher, or as firm j 's cost disadvantage for the product H is higher, the more this inequality facilitates to hold. So firm j starts to produce the output of product L with *no cost* and cannibalizes product H with own cost disadvantage!
Finally we examine under which conditions firm i in case b or c expands its product line and supplies not only one product but also another in the same market and moves to case a when its rival firm j produces and sell both products H and L.

Proposition 3 *When firm i chooses to expand its product line or supply only one type of goods, while firm j sells both goods, then firm i has an incentive to produce both goods.*

Proof: From the third column in Table 2,

$$\pi_i^{a*} - \pi_i^{b*} = \frac{1}{9\mu(1+\mu)}(2c_{iH} - c_{jH})^2 > 0,$$

and

$$\pi_i^{a*} - \pi_i^{c*} = \frac{1}{9\mu}(\mu r - (2c_{iH} - c_{jH}))^2 > 0,$$

where both of inequalities in the above hold from positive output condition in case a, (5). ■

To understand the intuition of this proposition, we consider the increments of firm i 's profit of each good from case (b) to case (a) and from case (c) to case (a). For this, we define π_{ik}^{n*} and $\Delta\pi_k^n$ as

$$\pi_{ik}^{n*} \equiv (p_k^{n*} - c_{ik})q_k^{n*}, i = 1, 2, n = b, c, k = H, L, \qquad (14)$$

and

$$\Delta\pi_k^n \equiv \pi_{ik}^{a*} - \pi_{ik}^{n*}, i = 1, 2, \quad n = b, c, \ k = H, L, \qquad (15)$$

respectively.

Then, we can immediately obtain each incremental quantity of profits:

$$\begin{cases} \Delta\pi_H^b = \frac{-(2c_{iH}-c_{jH})((1+\mu)r-2c_{iH}+c_{jH})}{9\mu(1+\mu)} < 0, \\ \Delta\pi_L^b = \frac{r(2c_{iH}-c_{jH})}{9\mu} > 0, \\ \Delta\pi_H^c = \frac{((1+\mu)r-2c_{iH}+c_{jH})(\mu r-2_{iH}+c_{jH})}{9\mu} > 0, \\ \Delta\pi_L^c = -\frac{r}{9}(r-\frac{(2c_{iH}-c_{jH})}{\mu}) < 0. \end{cases}$$

And we can show that

$$\begin{cases} \Delta\pi_H^b + \Delta\pi_L^b > 0 \\ \Delta\pi_H^c + \Delta\pi_L^c > 0. \end{cases} \tag{16}$$

Moreover, it is easy to see that

$$p_H^{a*} = p_H^{b*} \quad and \quad p_L^{a*} = p_L^{c*}.$$

From the above, we reach following intuition to proposition 3: firm i makes a profit by expand its product line and the profit is larger than it of the baseline product because selling two products gives him the positive effects of both price and quantity on profit, while it results in the negative effect of only quantity of original good. Equation (16) implies the positive effects always dominate the negative effect on firm i 's profit. Thus, firm i supply two differentiated goods.

In the Japanese smart phone industry, Proposition 3 shows that the introduction of the iPhone is very natural because KDDI makes a profit by selling it.

It is interesting that firm i determines to compete in two goods to maximize its profit (that is, both firms supply two differentiated goods). Thus, in the future, it is possible that both Softbank and KDDI sell both goods so long as they can supply high quality product H in Japan.

Concluding Remarks

In this study, we considered a duopoly model in which two firms choose cannibalization or launch of other products that are differentiated as well as vertically in the same market. We explored under what conditions firms strategically choose their product lines and sell only one product or two products in the market.

At first, we derived duopoly equilibrium in which each firm sells two vertically differentiated products H and L in a same market. Next, we also derived equilibriums in the other four cases; the first two cases in which both firms sell only homogeneous product H or product L, the next two in which one of the firms sells both products but another firm sells only product H or product L.

Then, starting from the case in which both firms only product L in a duopolistic market, we explored the condition under which a firm whether or not expands its product line and launches product H in the same market provided that its rival firm remains to sell only product L. In consequent, we showed that if the evaluation of the value of the one unit difference of high quality product for the consumer with the highest preference is higher than it own firm's unit cost of high quality product, then the firm supplies both quality products in the same market.

Next, when both firms only product H in a duopolistic market, we explored the condition under which a firm whether or not expands its product line and launches product L

in the same market provided that its rival firm remains to sell only product H. Then, we show that if the evaluation of the value of the one unit of high quality product for the consumer with the highest preference is higher than its rival firm's substantial unit cost of high quality product H, which is compensated by its cost advantage over own firm $c_{iH} - (c_{jH} - c_{iH})$, then the firm supplies both quality products in the same market.

Finally, starting from the case in which one of the firms sells both products but another firm sells only product H or product L, we examined the condition under which a firm whether or not expands its product line and launches another product in the same market provided that its rival firm remains to sell both products. Consequently, we showed that the firm always has an incentive to expand its product line and launch another good.

Extensions to this study in future research are possible. For example, it would be useful to analyze a case in which each firm can choose its quality level as well as the number of goods it produces. In addition, in this study, we do not consider a market with network externality, which would be worth studying if we consider a market such as smart phone industry or the tablet PC industry described in section 2. Indeed, we are investigating such a market in another study.

References

Bonanno, Giacomo. 1986. Vertical Differentiation with Cournot Competition. *Economic Notes* 15 (2): 68–91.

Calzada, Joan and Tommaso. M. Valletti. 2012. Intertemporal Movie Distribution: Versioning When Customers Can Buy Both Versions. *Marketing Science* 31 (4): 649–667.

Ellison, G. 2005. A Model of Add-on Pricing. *Quarterly Journal of Economics* 120 (2): 585–637.

Johnson, Justin, P. and David, P. Myatt. 2003. Multiproduct Quality Competition: Fighting Brands and Product Line Pruning. *American Economic Review* 93 (3): 3748–3774.

Katz Michel, and Carl Shapiro. 1985. Network Externalities, Competition, and Compatibility, *American Economic Review* 75 (3): 424–440.

Kitamura, Ryoma and Tetsuya Shinkai. 2014. Cannibalization May Allow a Cost-inefficient Firm to Earn More Than a Cost-efficient Firm in a Duopoly with Two Vertically Differentiated Goods. *Discussion Paper Series No.113, School of Economics, Kwansei Gakuin University*, Nishinomiya, 16 pages.

Kitamura, Ryoma and Tetsuya Shinkai. 2013. The Economics of Cannibalization: A Duopoly in Which Firms Supply Two Vertically Differentiated Products. *Discussion Paper Series No.100, School of Economics, Kwansei Gakuin University*, Nishinomiya, 21 pages.

Motta, Massimo 1993. Endogenous Quality Choice: Price vs. Quantity Competition. *Journal of Industrial Economics* 41 (2): 113–131.

Seward, Z. M. 2013. Yes, the iPad Mini Is Cannibalizing Sales of the Larger iPad. Last accessed December 25, 2013.
http://qz.com/47265/apple-ipad-mini-is-cannibalizing-sales-of-the-larger-ipad/.

Appendix

Proof of Proposition 1: From the third column in Table 2,
when case c can occur,

$$\begin{aligned}\pi_j^{c*}-\pi_j^{e*} &= (p_H^{c*}-c_{jH})q_{jH}^{c*}+p_L^{c*}q_{jL}^{c*}-p_L^{e*}q_{jL}^{e*}\\ &=\frac{1}{2}\{\frac{9\mu+4}{18}r^2-rc_{jH}+\frac{1}{2\mu}c_{jH}^2\}-\frac{r^2}{9}\\ &=\frac{1}{4\mu}\left(\mu r-c_{jH}\right)^2>0\end{aligned}$$

holds, since $r>\frac{c_{jH}}{\mu}>\frac{1}{3}r$ in the aquarium in case c from (10). If $r\le\frac{c_{jH}}{\mu}$, then firm j never supplies high quality product H and supplies only product L, i.e. $q_{jH}^{*}=0$ and $q_{jL}^{e*}=r/3$. Hence $\pi_j^{c*}-\pi_j^{e*}>(\le)0,$ the result follows. ■

Proof of Proposition 2: From the third column in Table 2, we have

$$\begin{aligned}\pi_j^{b*}-\pi_j^{d*} &= \frac{1}{9(1+\mu)}\{(1+\mu)r+(c_{iH}-2c_{jH})\}^2+\frac{1}{4\mu}c_{jH}^2\\ &\quad-\frac{1}{9}\{(1+\mu)r^2+2\left(c_{iH}-3c_{jH}\right)r+\frac{1}{1+\mu}(c_{iH}-4c_{jH})(2c_{jH}-c_{iH})\}\\ &=\frac{1}{36\mu(1+\mu)}\left[8\mu(1+\mu)rc_{jH}+8\mu c_{iH}^2-40\mu c_{jH}c_{iH}+(57\mu+9)c_{jH}^2\right]\ge(<)0 \quad \text{iff.}\end{aligned} \tag{A1}$$

$$r\ge(<)\frac{1}{8\mu(1+\mu)c_{jH}}\left[-8\mu c_{iH}^2+40\mu c_{jH}c_{iH}-(57\mu+9)c_{jH}^2\right].$$

Looking $-8\mu c_{iH}^2+40\mu c_{jH}c_{iH}-(57\mu+9)c_{jH}^2$ upon as a quadratic function of c_{iH} in the square brackets of the last line and denoting its discriminant by D, since the coefficient of c_{iH}^2 is negative i.e. $-8\mu<0$ and

$$\frac{D}{4}=-7\mu(8\mu+1)c_{jH}^2<0,$$

We see that $-8\mu c_{iH}^2+40\mu c_{jH}c_{iH}-(57\mu+9)c_{jH}^2<0$ and the right-hand side of the last inequality (A1), thus we have $\frac{1}{8\mu(1+\mu)c_{jH}}\left[-8\mu c_{iH}^2+40\mu c_{jH}c_{iH}-(57\mu+9)c_{jH}^2\right]<0$. So taking this into consideration with the fact that $r>0$ and (13) when $r>\frac{2c_{iH}-c_{jH}}{1+\mu}$ holds, we see that $\pi_j^{b*}-\pi_j^{d*}>0$ and the case b is always better off for firm j than the case d. If $r\le\frac{2c_{iH}-c_{jH}}{1+\mu}$ holds, then firm i never supplies product H from (13) and it implies that even case d cannot occur, so the difference $\pi_j^{b*} - \pi_j^{d*}$ itself is meaningless. ■

Case	Inverse Demand	Equilibrium outputs: q_{iH}^*, q_{iL}^*
a	$\begin{cases} p_H = (1+\mu)(r - Q_H) - Q_L, \\ p_L = r - Q_H - Q_L. \end{cases}$	$\begin{cases} q_{iH}^* = \frac{1}{3}\left(r - \frac{2c_{iH} - c_{jH}}{\mu}\right) \text{ and} \\ q_{iL}^* = \frac{2c_{iH} - c_{jH}}{3\mu}, \quad i,j = 1,2, \quad i \neq j. \end{cases}$
b	$\begin{cases} p_H = (1+\mu)(r - Q_H) - q_{jL}, \\ p_L = r - Q_H - q_{jL}, \quad i \neq j. \end{cases}$	$\begin{cases} q_{iH}^{b*} \equiv q_{iH}^* = \frac{1}{3}\left(r - \frac{2c_{iH} - c_{jH}}{(1+\mu)}\right), q_{iL}^{b*} = 0, \\ q_{jH}^{b*} \equiv q_{jH}^* = \frac{1}{3}\left(r - \frac{2\mu(2c_{iH} - c_{jH}) + 3c_{jH}}{2\mu(1+\mu)}\right), \\ q_{jL}^{b*} \equiv q_{jL}^* = \frac{c_{jH}}{2\mu}, \quad j = 1,2. \end{cases}$
c	$\begin{cases} p_H = (1+\mu)(r - q_{jH}) - Q_L, \\ p_L = r - q_{jH} - Q_L. \end{cases}$	$\begin{cases} q_{iL}^{c*} \equiv q_{iL}^* = \frac{r}{3}, q_{iH}^{c*} = 0, \\ q_{jH}^{c*} \equiv q_{jH}^* = \frac{1}{2}(r - \frac{c_{jH}}{\mu}), \\ q_{jL}^{c*} \equiv q_{jL}^* = \frac{1}{2}(\frac{c_{jH}}{\mu} - \frac{r}{3}). \end{cases}$
d	$p_H = (1+\mu)(r - Q_H)$	$q_{iH}^{d*} = \frac{1}{3}(r - \frac{2c_{iH} - c_{jH}}{1+\mu}), i, j(\neq i) = 1,2$
e	$p_L = r - Q_L$	$q_{iL}^{e*} = \frac{1}{3}r, i = 1,2$

Table 1 Inverse Demand and Equilibrium outputs of firms in each case

Case	p_H^*, p_L^*	π_i^*, π_j^*
a	$\begin{cases} p_H^{a*} = \frac{(1+\mu)r+c_{iH}+c_{jH}}{3} \\ p_L^{a*} = \frac{1}{3}r \end{cases}$	$\pi_i^{a*} = \frac{\mu(1+\mu)r^2-2\mu(2c_{iH}-c_{jH})r+(2c_{iH}-c_{jH})^2}{9\mu}$, $i, j = 1, 2, i \neq j$
b	$\begin{cases} p_H^{b*} = \frac{1}{3}((1+\mu)r + c_{iH}+c_{jH}) \\ p_L^{b*} = \frac{1}{3}\left(r - \frac{c_{jH}}{2(1+\mu)} + \frac{c_{iH}}{1+\mu}\right) \end{cases}$	$\begin{cases} \pi_i^{b*}=\frac{1}{9}(1+\mu)(r - \frac{2c_{iH}-c_{jH}}{(1+\mu)})^2, i,j = 1,2 \ \ i \neq j \\ \pi_j^{b*}=\frac{1}{9(1+\mu)}\{(1+\mu)r + (c_{iH}-2c_{jH})\}^2+\frac{1}{4\mu}c_{jH}^2 \end{cases}$
c	$\begin{cases} p_H^{c*} = \frac{1}{2}(\frac{(3\mu+2)}{3}r + c_{jH}) \\ p_L^{c*} = \frac{1}{3}r \end{cases}$	$\begin{cases} \pi_i^{c*}=\frac{r^2}{9}, \quad i = 1,2 \\ \pi_j^{c*}=\frac{1}{2}\{\frac{(9\mu+4)}{18}r^2-rc_{jH}+\frac{1}{2\mu}c_{jH}^2\} \end{cases}$
d	$p_H^{d*} = \frac{1}{3}(1+\mu)(r + \frac{c_{jH}-c_{iH}}{1+\mu})$	$\pi_i^{d*}=\frac{1}{9}\{(1+\mu)r^2+2(c_{jH} - 3c_{iH})r$ $+\frac{1}{1+\mu}(c_{jH}-4c_{iH})(2c_{iH}-c_{jH})\}, i,j = 1,2$
e	$p_L^{c*} = \frac{1}{3}r$	$\pi_i^{e*}=\frac{1}{9}r^2, i = 1,2$

Table 2 Equilibrium prices and profit of each firm in each case