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Child-care Policies and Pension in an Endogenous Fertility Model*

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Abstract

Some economically developed countries are suffering from an aging society with fewer children, which has brought about greater burdens imposed by social security. A child allowance and child-care services are provided by the governments in these countries to raise fertility. An increase in fertility pulls up the future labor population. An increase in labor population can subsequently provide sufficient social security benefits in terms of pensions and other transfers. This paper presents consideration of three child-care policies. The first is child allowances. The second is a subsidy for child-care services. The third is a subsidy for child care in the home. These three policies can raise fertility and the future labor population. This paper presents results of a derivation showing that child allowances can raise both the demand for child-care services and child care in the home. Therefore, fertility can always rise. However, a decrease in labor supply time has the effect of reducing the pension benefit. With large substitution between child-care services and child care in the home, the subsidy for child-care services or child care in the home can reduce fertility. Then, the pension benefit can not always increase.

Keywords: Aging society, Child-care service, Fertility, Labor supply, Pay-as-you-go pension

JEL Classifications: H51, H55, J14

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1 Introduction

Fig. 1 presents social expenditures for child-care policies and fertility. In France and Sweden, fertility and social expenditures for child-care policies are higher than those of Japan. Therefore, low fertility in Japan can be raised by sufficient child-care support. However, it is not good that only cash benefits be given for children. A policy is needed that enables a parent to work and care for children simultaneously. In France and Sweden, female labor participation is higher than in Japan.[1]

[Insert Fig. 1 around here.]

High fertility and high labor participation can maintain the pension system because the government can collect sufficient revenue to provide social security benefits such as pensions. In economically developed countries, an aging society is progressing. Providing sufficient social security benefits in the future is an important problem. Figure 2 presents the replacement rate of pension benefits for OECD countries.

[Insert Fig. 2 around here.]

If the replacement rate of the pension benefit is high, then the pension benefit for the income of working generations is large. The replacement rate in Japan is low. Two ways to increase the pension benefit are readily apparent. One is to increase the contribution rate. The other is to increase the population size of working generation and aggregate labor supply. However, the former policy is not good because an aging society is progressing. The burden per capita increases. Therefore, it is not sustainable. The government should expect to increase fertility and labor participation with policies to maintain the pension benefit level.

This paper presents consideration of endogenous fertility in a pay-as-you-go pension and examines whether child-care policies can raise fertility, labor supply, and raise the pension benefit in the future. Concretely, this paper presents an examination of child-care policies of three types: child allowance, the subsidy for child-care services and a subsidy for child care in the home.

[1]The female employment rate from 25 years old to 54 years old in Japan was 69.2% in 2012, which is lower than that in France or Sweden: 76.0% in France, 82.5% in Sweden (OECD Statistics).

Many earlier studies have examined child-care policies. The endogenous fertility model was set by Becker (1960), Becker and Barro (1988) and Barro and Becker (1989). Zhang (1997), van Groezen, Leers and Meijdam (2003), van Groezen and Meijdam (2008), Fanti and Gori (2009), Oshio and Yasuoka (2009) and Yasuoka and Goto (2011) set an endogenous fertility model and examine whether a child allowance can raise fertility or not. Apps and Rees (2004), Martínez and Iza (2004) and Day (2012) examine child-care services and describe a positive relation between fertility and the female labor supply. This result is consistent with those presented in reports by Ahn and Mita (2002) and Sleebos (2003), who described a positive relation between fertility and female labor participation in terms of empirical studies. The positive relation is brought about by child-care services.[2] Momota (2000) reports that child-care policies affect the child-care time or the labor supply. Many earlier studies have been undertaken to examine how a pension policy affects fertility and the labor supply in an endogenous fertility model (Wigger (1999), Lin and Tian (2003), Fenge and Meier (2005), and Hirazawa and Yakita (2009)).

The conclusions presented in this paper are the following. This paper adopts the assumption that fertility is determined by child-care services and child care in the home. The child allowance, which is provided proportionally to the number of children, increases both child-care services and child care in the home. Then, fertility can always rise. However, the labor supply time decreases because of an increase in child care in the home. Therefore, as long as pension benefits to provide for older people are collected from labor income that younger people gain, the pension benefit can not always increase. Without a pension system, the subsidy for child-care services increases the demand for child-care services and decreases child care in the home if high substitution between child-care services and child care in the home exists. Therefore, this subsidy can not always raise fertility even if the labor supply increases. With the pension system, this subsidy can not always raise the demand for child-care services. Finally, a subsidy for child care in the home raises the incidence of child care in the home if the pension system does not exist. With high substitution between child-care inputs of two types, the demand for child-care services

[2]Galor and Weil (1996) set a model in which time is invested in having children. Therefore, a negative correlation prevails between fertility and the female labor supply. Yasuoka and Miyake (2010) show that child-care services do not increase fertility and that they decrease child-care time because the price of child-care services increases, which reduces the use of child-care services.

decreases. Then, fertility does not always increase. With the pension system, this subsidy for child care in the home does not always increase child care in the home. Therefore, these child-care support policies can not always increase the pension benefit: an increase in fertility and a decrease in labor supply exists or a decrease in fertility and an increase in labor supply exists.

This paper includes the following. Section 2 sets the model. Section 3 derives the equilibrium. Section 4 examines child-care policy capabilities for raising fertility, the demand for child-care services, child care in the home, and pension benefits. The final section concludes this paper.

2 The Model

The model economy in this paper is constructed in terms of a two-period (young and old) overlapping generations model. The economy comprises agents of three types: households, firms of two types (one produces child care services; the other produces final goods), and a government. In t period, the population of younger people is N_t; the population of older people is N_{t-1}. We explain the agents in the following subsections.

2.1 Households

Individuals in households exist in two periods: a young period and an old period. Younger people provide labor supply to gain labor income. The labor income is allocated to consumption in the younger period c_{1t}, the savings s_t are consumed during the old period c_{2t+1}. Purchasing child-care services e_t is done at price z_t. The individuals allocate their time to labor time $(1-l_t)$ and child-care time l_t. This paper assumes the number of children n_t as

$$n_t = (Ae_t^{\rho} + Bl_t^{\rho})^{\frac{1}{\rho}}, -\infty < \rho < 1. \tag{1}$$

Apps and Rees (2004) assume a fertility function as having constant returns to scale for l_t and e_t. This paper specifies constant elasticity of substitution between child-care services e_t and child care time l_t.[3] With the wage rate as w_t, the individuals gain labor income $(1-l_t)w_t$. The government provides pension benefits for older people p_t and collects revenues from labor income

[3]Many earlier researchers specify the fertility function and examine how fertility is determined. Martínez and Iza (2004) assume a perfectly substituting function between l_t and e_t. Hirazawa and Yakita (2009) assume a Cobb–Douglas function.

earned by younger people at the contribution rate τ. In addition, the government provides child-care policies: The first policy is a child allowance program in which the government provides an allowance in proportion to the number of children. The second policy is to subsidize child-care services. The third is to provide a benefit if individuals stop working to care for their children. The budget constraint in the young period and that in the old period are shown as

$$s_t + c_{1t} + (1-\gamma)z_t e_t = (1-\tau)(1-l_t)w_t + \epsilon l_t w_t + q n_t - T_t, \tag{2}$$

$$c_{2t+1} = (1+r_{t+1})s_t + p_{t+1}. \tag{3}$$

That is, the household's lifetime budget constraint is shown as

$$c_{1t} + \frac{c_{2t+1}}{1+r_{t+1}} + (1-\gamma)z_t e_t = (1-\tau)w_t - (1-\tau-\epsilon)w_t l_t + \frac{p_{t+1}}{1+r_{t+1}} - T_t + q_t n_t, \tag{4}$$

where $1+r_{t+1}$ denotes the interest rate. The government provides a subsidy for child-care services at γ rate, child allowances at q and child care in the home at ϵ. These policies are financed by a lump sum tax T_t. The household's utility function u_t is assumed as

$$u_t = \alpha \ln c_{1t} + \beta \ln c_{2t+1} + (1-\alpha-\beta)\ln n_t,\ 0<\alpha, 0<\beta, \alpha+\beta<1. \tag{5}$$

The households choose the optimal allocation to maximize their utility (5) subject to the function of quantity of children (1) and the budget constraint (4) as follows.

$$c_{1t} = \alpha\left((1-\tau)w_t + \frac{p_{t+1}}{1+r_{t+1}} - T_t\right), \tag{6}$$

$$c_{2t+1} = (1+r_{t+1})\beta\left((1-\tau)w_t + \frac{p_{t+1}}{1+r_{t+1}} - T_t\right), \tag{7}$$

$$e_t = \frac{(1-\alpha-\beta)\left((1-\tau)w_t - T_t + \frac{p_{t+1}}{1+r_{t+1}}\right)}{(1-\gamma)z_t + (1-\epsilon-\tau)w_t\left(\frac{B(1-\gamma)z_t}{A(1-\epsilon-\tau)w_t}\right)^{\frac{1}{1-\rho}} - q\left(A + B\left(\frac{B(1-\gamma)z_t}{A(1-\epsilon-\tau)w_t}\right)^{\frac{\rho}{1-\rho}}\right)}, \tag{8}$$

$$l_t = \left(\frac{B(1-\gamma)z_t}{A(1-\epsilon-\tau)w_t}\right)^{\frac{1}{1-\rho}} e_t. \tag{9}$$

Considering (1) and (9), the fertility function is given as

$$n_t = \left(A + B\left(\frac{B(1-\gamma)z_t}{A(1-\epsilon-\tau)w_t}\right)^{\frac{\rho}{1-\rho}}\right) e_t. \tag{10}$$

Given (8) and (10), fertility can be determined.

2.2 Firms

This model has two sectors: a final goods sector and an elderly care sector. The production function in the final goods sector is assumed as the following constant returns to scale function:

$$Y_t = F(K_t, L_t),\ \frac{\partial Y_t}{\partial K_t} > 0,\ \frac{\partial Y_t}{\partial L_t} > 0,\ \frac{\partial^2 Y_t}{\partial K_t^2} < 0,\ \frac{\partial^2 Y_t}{\partial L_t^2} < 0,\ \frac{\partial Y_t}{\partial K_t \partial L_t} > 0. \tag{11}$$

Therein, K_t and L_t denote the capital stock and labor input. Defining $\frac{Y_t}{L_t} \equiv f(k_t)$ and $k_t \equiv \frac{K_t}{L_t}$ and assuming a competitive market and a small open economy, an interest rate $1 + r_t = f'(k_t)$ is fixed by the world interest rate r; k is fixed. The capital stock is fully depreciated within a period. Then, the wage rate is given as $w_t = w$, where $w = f(k) - f'(k)k$.

Next, we consider the child-care sector. Child-care services are produced according to the following production function.

$$Y_t^c = \theta L_t^c, \theta > 0. \tag{12}$$

The child-care services are produced solely by labor input. This function is assumed by Yasuoka and Miyake (2010) and Day (2012). In putting labor demand for child-care services L_t^c into the child-care sector, the profit π_t is given as presented below.

$$\pi_t = z_t \theta L_t^c - w_t^c L_t^c \tag{13}$$

The aggregate supply of elderly care services is $X_t = \theta L_t^c$. The wage rate w_t^c is given as

$$w_t^c = \theta z_t. \tag{14}$$

Considering a homogeneous household and complete labor mobility, the wage in the child-care sector w_t^c is given as

$$w_t^c = w \tag{15}$$

or

$$z_t = \frac{w}{\theta} = z. \tag{16}$$

The price of child care z_t is constant over time.[4]

[4]If the aggregate demand for child-care services is large, then the price of child-care services can not be determined by $z_t = \frac{w}{\theta}$ because of a shortage of supply for child-care services. However, this model economy does not consider this case. It is assumed that even if all younger people work in the child-care services sector, the price of child-care services is given as $z_t = \frac{w}{\theta}$. Otherwise, the price of child-care services is more than $z_t = \frac{w}{\theta}$: $z_t = \frac{w}{\theta}$ is assumed when $L_t^c = (1 - l_t)N_t$.

2.3 Government

The government in this model economy provides two child-care policies and a pay-as-you-go pension. First, the government levies taxation on younger people to provide a child allowance and to subsidize elderly care services and child care in the home. Consideration of a balanced budget reduces to

$$T_t = qn_t + \gamma z e_t + \epsilon w l_t, \tag{17}$$

where $0 < \gamma < 1$ and $0 < 1-\tau-\epsilon < 1$. Second, the government provides a pay-as-you-go pension that the government collects the revenue from younger people at t period and gives benefits to older people during the same t period. Considering a balanced budget and $n_{t-1} = \frac{N_t}{N_{t-1}}$, pension benefit p_t is given as

$$p_t = n_{t-1}\tau(1-l_t)w. \tag{18}$$

3 Equilibrium

The equilibrium in this model economy is specified by the dynamics of e_t. Substituting (9) and (10) into (18), the pension benefit in $t+1$ period is

$$p_{t+1} = \tau w e_t \left(A + B \left(\frac{B(1-\gamma)z}{A(1-\epsilon-\tau)w} \right)^{\frac{\rho}{1-\rho}} \right) \left(1 - \left(\frac{B(1-\gamma)z}{A(1-\epsilon-\tau)w} \right)^{\frac{1}{1-\rho}} e_{t+1} \right). \tag{19}$$

Substituting (19) for (8), the following equation is obtained.

$$\frac{\tau w}{1+r} \left(A + B \left(\frac{B(1-\gamma)z}{A(1-\epsilon-\tau)w} \right)^{\frac{\rho}{1-\rho}} \right) \left(\frac{B(1-\gamma)z}{A(1-\epsilon-\tau)w} \right)^{\frac{1}{1-\rho}} e_{t+1} = \frac{((1-\tau)w - T_t)}{e_t} - C, \tag{20}$$

where

$$C = \frac{(1-\gamma)z + (1-\epsilon-\tau)w \left(\frac{B(1-\gamma)z}{A(1-\epsilon-\tau)w} \right)^{\frac{1}{1-\rho}} - q \left(A + B \left(\frac{B(1-\gamma)z}{A(1-\epsilon-\tau)w} \right)^{\frac{\rho}{1-\rho}} \right)}{1-\alpha-\beta}$$
$$- \frac{\tau w}{1+r} \left(A + B \left(\frac{B(1-\gamma)z}{A(1-\epsilon-\tau)w} \right)^{\frac{\rho}{1-\rho}} \right).$$

Equations (17) and (20) give the dynamic path of e_t. Without child-care policies ($T_t = 0$), we obtain the dynamics shown in Fig. 3.

[Insert Fig. 3 around here.]

As shown in Fig. 3, the steady state equilibrium is given uniquely as $e_{t+1} = e_t = e$. $\frac{de_{t+1}}{de_t}$ at the steady state is

$$\frac{de_{t+1}}{de_t} = -\frac{(1-\tau)w}{D}\frac{1}{e^2} < 0. \tag{21}$$

With $-1 < \frac{de_{t+1}}{de_t} < 0$, the steady state equilibrium is locally stable. Child care in the steady state e is

$$e = \frac{-C + \sqrt{C^2 + 4D(1-\alpha-\beta)(1-\tau)w}}{2D}, \tag{22}$$

where

$$D = \frac{\tau w}{1+r}\left(A + B\left(\frac{Bz}{A(1-\tau)w}\right)^{\frac{\rho}{1-\rho}}\right)\left(\frac{Bz}{A(1-\tau)w}\right)^{\frac{1}{1-\rho}}.$$

4 Policy Effect

In economically developed countries, child-care policies are provided to raise fertility and thereby raise the future labor supply. This section presents an examination of whether child-care policies can raise the demand for child care service, child care in the home, fertility and the pension benefit or not in a steady state. This section presents an examination of three child-care policies. The first is child allowances, which give an allowance proportionally with the number of children. The second is a subsidy for child-care services. The third is a subsidy for child care in the home. First, this paper presents an examination of the effects of child allowances.

4.1 Child allowance

Considering (17), (20), $\gamma = 0$, $\epsilon = 0$ and differentiating e by q at the approximation of $q = 0$, the sign of $\frac{de}{dq}$ is positive, as shown in

$$\frac{de}{dq} = \frac{(\alpha+\beta)n}{(1-\alpha-\beta)e^2\left(D + \frac{(1-\tau)w}{e}\right)} > 0. \tag{23}$$

This result is intuitive. Child allowances decrease the child care cost because the government provides a subsidy according to the number of children. Considering (9), the child-care time in home l increases because of $l = \left(\frac{Bz}{A(1-\tau)w}\right)^{\frac{1}{1-\rho}} e$. Therefore, we obtain $\frac{dl}{dq} > 0$. Then fertility increases because both child-care services e and child care in the home l increase. That is, $\frac{dn}{dq} > 0$ is obtained. In addition, this paper presents an examination of whether the pension benefit can

be raised by virtue of child allowances or not. Calculating $\frac{dp}{dq}$, the sign of $\frac{dp}{dq}$ is ambiguous, as shown as

$$\frac{dp}{dq} = \frac{\tau w n}{e}\frac{de}{dq}(1-2l). \tag{24}$$

With $l < \frac{1}{2}$, i.e., care for a small child in the home, the child allowance can raise the pension benefit. Then, the following proposition is established.

Proposition 1 Child allowances can always raise child-care services, child care in the home and fertility in the steady state. The pension benefit can be raised by child allowances if $l < \frac{1}{2}$.

Child allowances can raise fertility and an increase in fertility raises the revenue for pension benefits. However, an increase in child care in the home decreases the labor supply. A decrease in labor supply decreases the revenue for pension benefits. Therefore, child allowances have two offset effects for pension benefits. With $l < \frac{1}{2}$, which means a large labor supply time, child allowances can raise the pension benefit. This proposition is independent of the elasticity of substitution between child-care services and child care in the home.

4.2 Subsidy for Child-Care Service

This subsection presents an examination of whether a subsidy for child-care services can increase child-care services, child care in the home, fertility, and the pension benefit or not. Considering (17), (20), $q = 0$ and $\epsilon = 0$ and differentiating e by γ at the approximation of $\gamma = 0$, the sign of $\frac{de}{d\gamma}$ is positive as shown as

$$\frac{de}{d\gamma} = \frac{\frac{(\alpha+\beta)z}{1-\alpha-\beta} + \frac{(1-\tau)w}{(1-\rho)(1-\alpha-\beta)}\left(\frac{Bz}{A(1-\tau)w}\right)^{\frac{1}{1-\rho}} - E}{D + \frac{(1-\tau)w}{e^2}}, \tag{25}$$

where

$$E = \frac{\tau w B\rho}{(1-\rho)(1+r)}\left(\frac{Bz}{A(1-\tau)w}\right)^{\frac{\rho}{1-\rho}} - \frac{\tau w}{1+r}\left(\frac{\rho B}{1-\rho}\left(\frac{Bz}{A(1-\tau)w}\right)^{\frac{1+\rho}{1-\rho}} + \frac{1}{1-\rho}\left(\frac{Bz}{A(1-\tau)w}\right)^{\frac{1}{1-\rho}}\left(A + B\left(\frac{Bz}{A(1-\tau)w}\right)^{\frac{\rho}{1-\rho}}\right)\right)e.$$

This sign is ambiguous. Without a pension system ($\tau = 0$), we obtain $\frac{de}{d\gamma} > 0$. The effect of the subsidy for child-care services on child care in the home is

$$\frac{dl}{d\gamma} = \left(\frac{Bz}{A(1-\tau)w}\right)^{\frac{1}{1-\rho}} \left(\frac{de}{d\gamma} - \frac{e}{1-\rho}\right). \tag{26}$$

The condition to have $\frac{dl}{d\gamma} > 0$ is $\frac{de}{d\gamma} > \frac{e}{1-\rho}$. Without pension system ($\tau = 0$), the condition to have $\frac{dl}{d\gamma} > 0$ is $\rho < -\frac{1-\alpha-\beta}{\alpha+\beta}$. If the substitution between child-care services and child care in the home is complementary, then an increase in child-care services raises the demand for child care in the home. Fertility is increased if $\rho < -\frac{1-\alpha-\beta}{\alpha+\beta}$. Otherwise, child care in the home decreases because of substitution between e and l. Therefore, fertility can not always be raised by the subsidy for e because of a decrease in l as

$$\frac{dn}{d\gamma} = \frac{n}{e}\frac{de}{d\gamma} - \frac{B\rho}{1-\rho}\left(\frac{Bz}{A(1-\tau)w}\right)^{\frac{\rho}{1-\rho}}. \tag{27}$$

With $\frac{de}{d\gamma} > \frac{e}{n}\frac{B\rho}{1-\rho}\left(\frac{Bz}{A(1-\tau)w}\right)^{\frac{\rho}{1-\rho}}$, the subsidy for child-care services can raise fertility. Without pension system ($\tau = 0$), the condition to have $\frac{dn}{d\gamma} > 0$ is $\frac{de}{d\gamma} > \frac{\frac{B\rho}{1-\rho}\left(\frac{Bz}{A(1-\tau)w}\right)^{\frac{\rho}{1-\rho}}}{A+B\left(\frac{Bz}{A(1-\tau)w}\right)^{\frac{\rho}{1-\rho}}}$: even if the subsidy decreases child care in the home, a large increase in child-care services can raise fertility as long as $\frac{de}{d\gamma}$ is large.

Next, we examine whether pension benefits can rise because of the subsidy for child-care services or not. The sign of $\frac{dp}{d\gamma}$ is not ambiguous because we obtain $\frac{dp}{d\gamma} = \tau w\left((1-l)\frac{dn}{d\gamma} - n\frac{dl}{d\gamma}\right)$. The condition to have $\frac{dp}{d\gamma} > 0$ is

$$(1-2l)\frac{de}{d\gamma} > \frac{e}{1-\rho}\left(\frac{B\rho(1-l)}{n}\left(\frac{Bz}{A(1-\tau)w}\right)^{\frac{\rho}{1-\rho}} - l\right); \tag{28}$$

that is,

$$\frac{de}{d\gamma} \gtrless \frac{e}{(1-\rho)(1-2l)}\left(\frac{B\rho(1-l)}{n}\left(\frac{Bz}{A(1-\tau)w}\right)^{\frac{\rho}{1-\rho}} - l\right) \text{ if } l \lessgtr \frac{1}{2}. \tag{29}$$

Therefore, the following proposition is established.

Proposition 2 Without a pension system, the subsidy for child-care services can raise child-care services. If $\rho < -\frac{1-\alpha-\beta}{\alpha+\beta}$, then child care in the home can be raised by the subsidy. Fertility

increases because of the subsidy if $\frac{de}{d\gamma} > \frac{\frac{B\rho}{1-\rho}\left(\frac{Bz}{A(1-\tau)w}\right)^{\frac{\rho}{1-\rho}}}{A+B\left(\frac{Bz}{A(1-\tau)w}\right)^{\frac{\rho}{1-\rho}}}$ holds. Considering the pension system, the subsidy can raise the pension benefit if (29) holds.

This previous description differs from the case of child allowances. Child allowances decrease the labor supply time (increases child-care time) directly, which decreases the revenues that can be allocated for pension benefits. However, with a subsidy for child-care services, the labor supply increases directly if the elasticity of substitution is large. However, a decrease in child care in the home might reduce fertility. A decrease in fertility has the effect of decreasing pension benefits: even if the labor supply increases, then the pension benefit might be reduced because fertility decreases.

4.3 Subsidy for Child-Care in the Home

This subsection presents an examination of whether the subsidy for child care in the home can increase the use of child-care services, child care in the home, fertility, and pension benefits or not. Considering (17), (20), $q = 0$ and $\gamma = 0$ and differentiating e by ϵ at the approximation of $\epsilon = 0$, the sign of $\frac{de}{d\epsilon}$ is positive as shown as

$$\frac{de}{d\epsilon} = \frac{-\frac{lw}{e} - \frac{\rho w}{(1-\rho)(1-\alpha-\beta)}\left(\frac{Bz}{A(1-\tau)w}\right)^{\frac{1}{1-\rho}} - \frac{\tau w \rho B}{(1-\rho)(1-\tau)(1+r)}\left(\frac{Bz}{A(1-\tau)w}\right)^{\frac{\rho}{1-\rho}}}{D + \frac{(1-\tau)w}{e^2} - \frac{\tau w}{(1-\tau)(1-\rho)(1+r)}\left(\frac{Bz}{A(1-\tau)w}\right)^{\frac{1}{1-\rho}}\left(A + \left(\frac{Bz}{A(1-\tau)w}\right)^{\frac{\rho}{1-\rho}} + \rho B\left(\frac{Bz}{A(1-\tau)w}\right)^{\frac{\rho}{1-\rho}}\right)}. \tag{30}$$

The sign of $\frac{de}{d\epsilon}$ is ambiguous. Without pension system ($\tau = 0$), we obtain $\frac{de}{d\epsilon} < 0$ if $\rho > -\frac{1-\alpha-\beta}{\alpha+\beta}$. Large ρ indicates large substitution between e and l. Therefore, the subsidy for child care in the home increases the relative price of child-care in services and decreases the demand for child-care services. The sign of $\frac{dl}{d\epsilon}$ is

$$\frac{dl}{d\epsilon} = \frac{l}{e}\left(\frac{de}{d\epsilon} + \frac{e}{(1-\rho)(1-\tau)}\right). \tag{31}$$

With $\frac{de}{d\epsilon} > -\frac{e}{(1-\rho)(1-\tau)}$, the subsidy for child care in the home increases the incidence child care in the home. However, if the subsidy greatly decreases the demand for child-care services, then the child care in the home decreases as well. With $\frac{de}{d\epsilon} > 0$ by virtue of complementarity, the child care in the home can always raise the child care in the home. Without a pension system

$(\tau = 0)$, we obtain $\frac{dl}{d\epsilon} = \frac{\frac{z}{1-\rho}\left(\frac{Aw}{Bz}\right)^{\frac{1}{1-\rho}} + (\alpha+\beta)w}{z\left(\frac{Aw}{Bz}\right)^{\frac{1}{1-\rho}} + w} > 0$. Then, the subsidy for child care in the home increases child care in the home and decreases the labor supply time.

Fertility can not always be raised by the subsidy because

$$\frac{dn}{d\epsilon} = \frac{n}{l}\frac{dl}{d\epsilon} - l\frac{\rho A}{1-\rho}\left(\frac{A(1-\tau)w}{Bz}\right)^{\frac{\rho}{1-\rho}}. \tag{32}$$

If $\frac{dl}{d\epsilon} > \frac{l^2}{n}\frac{\rho A}{1-\rho}\left(\frac{A(1-\tau)w}{Bz}\right)^{\frac{\rho}{1-\rho}}$, then we obtain $\frac{dn}{d\epsilon} > 0$. The pension benefit can not always be raised by the subsidy for child care in the home, as shown in

$$\frac{dp}{d\epsilon} = \tau w\left((1-l)\frac{dn}{d\epsilon} - n\frac{dl}{d\epsilon}\right). \tag{33}$$

The condition to have $\frac{dp}{d\epsilon} > 0$ is

$$\frac{1-2l}{l}\frac{dl}{d\epsilon} > \frac{(1-l)l}{n}\frac{\rho A}{1-\rho}\left(\frac{A(1-\tau)w}{Bz}\right)^{\frac{\rho}{1-\rho}}; \tag{34}$$

That is,

$$\frac{dl}{d\epsilon} \begin{matrix} > \\ < \end{matrix} \frac{(1-l)l^2}{(1-\rho)(1-2l)n}\frac{\rho A}{1-\rho}\left(\frac{A(1-\tau)w}{Bz}\right)^{\frac{\rho}{1-\rho}} \text{ if } l \begin{matrix} < \\ > \end{matrix} \frac{1}{2}. \tag{35}$$

Therefore, the following proposition is established.

Even if the subsidy can increase fertility, an increase in child care in the home reduces the labor supply time. A decrease in the labor supply time decreases the revenues available to fund the pension. Therefore, the sign of $\frac{dp}{d\epsilon}$ is ambiguous. Then, the following proposition is established.

Proposition 3 Without a pension system, the subsidy for child care in the home can raise the use of child-care services if $\rho < -\frac{1-\alpha-\beta}{\alpha+\beta}$. Child care in the home can be raised by the subsidy if $\frac{dl}{d\epsilon} = \frac{\frac{z}{1-\rho}\left(\frac{Aw}{Bz}\right)^{\frac{1}{1-\rho}} + (\alpha+\beta)w}{z\left(\frac{Aw}{Bz}\right)^{\frac{1}{1-\rho}} + w} > 0$. Fertility increases if $\frac{dl}{d\epsilon} > \frac{l^2}{n}\frac{\rho A}{1-\rho}\left(\frac{A(1-\tau)w}{Bz}\right)^{\frac{\rho}{1-\rho}}$. Considering the pension system, the subsidy can raise the pension benefit if (35) holds.

An increase in l brought about by the subsidy for child care in the home might not be worse than other two child-care policies. With the large substitution between child-care services and child care in the home, the subsidy for child care in the home reduces the use of child-care

services. Even if child care in the home increases, fertility might decrease because of a decrease in child-care services. Then, both fertility and the labor supply decrease and the pension benefit decreases.

5 Conclusions

This paper presents consideration of endogenous fertility in pay-as-you-go pensions and examines whether child-care policies can raise fertility and labor supply and can raise pension benefits or not. To the extent that the pension benefit is provided by revenues from taxes levied on labor income at the contribution rate, then the pension benefit depends not only on fertility (the intergenerational population ratio), but also on the labor supply. If fertility is determined by both child-care services and child-care time by parents, then child-care support policies such as a child allowance and a subsidy for child-care services or child care in the home can raise fertility because of an increase in demand for child-care services or child care in the home. However, if the labor supply decreases, then the pension benefit decreases.

This paper presents results of a derivation showing that child allowances can raise both the demand for child-care services and child care in the home. Therefore, fertility can always rise. However, a decrease in the labor supply time has the effect of reducing the pension benefit. A subsidy for child-care services does not always increase fertility. Even if this subsidy increases the demand for child-care services, child care in the home decreases in high substitution between child-care services and child care in the home. Therefore, even if this subsidy can increase the labor supply, the pension benefit does not increase because fertility does not always increase. The subsidy for child care in the home can not always increase fertility because a decrease in child-care services is expected to occur. Therefore, this subsidy might reduce not only fertility, but also the labor supply. Consequently, the pension benefit decreases.

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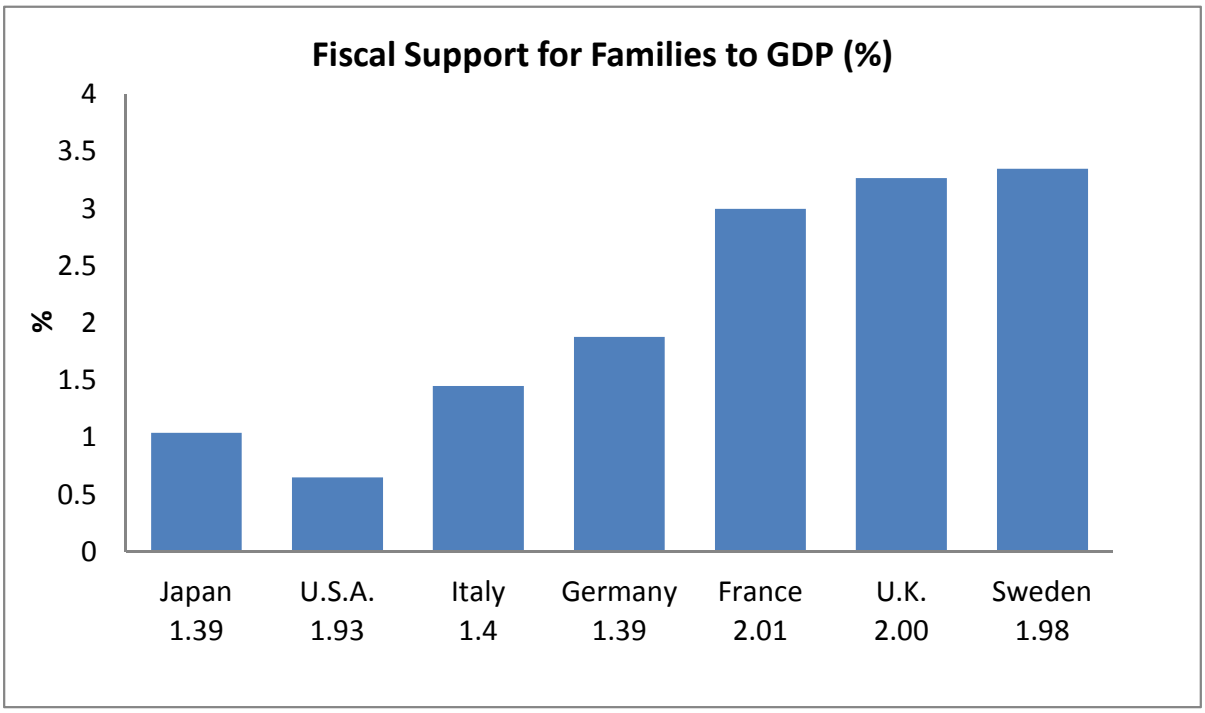

Fig. 1: Fertility (below the country) and Fiscal Support for Family (share of Gross Domestic Product) (Data: OECD Social Expenditure Database (November 2008), A 2012 Declining Birthrate White Paper (2012), Demographic Yearbook (UN) and Vital Statistics in Japan (Ministry of Health, Labour and Welfare (in Japan).) Data of Fiscal Support for Families are those of 2007. Fiscal Support for Family includes benefits in kind (day-care/home help and other benefits in kind) and cash benefits (family allowance, maternity and parental leave and other cash benefit). Data of the Total Fertility Rate are those of 2010.)

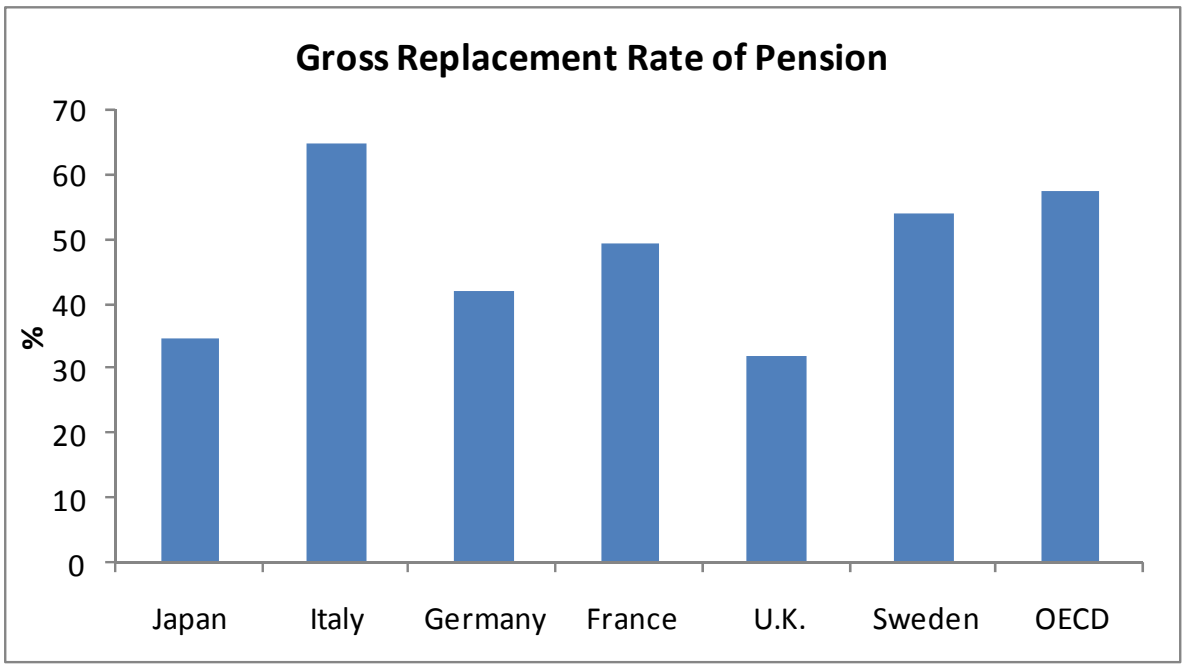

Fig. 2: Gross Replacement Rate of Pension (Data: OECD Statistic Pension at a Glance 2011. Gross Replacement Rate of Pension in Fig. 2 shows the pension that a person earns from an average wage income.)

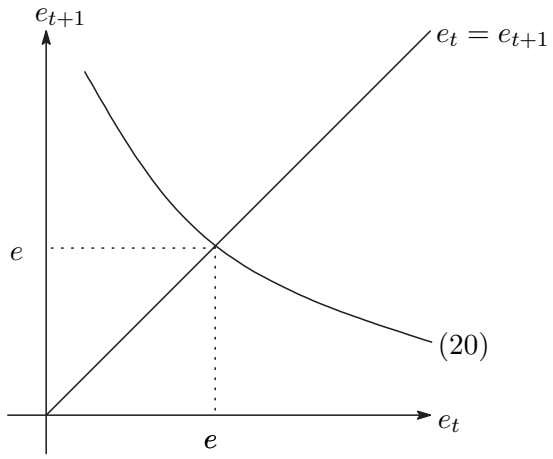

Fig. 3: Dynamics of e_t.