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Abstract

In economically developed countries, aging of the population with fewer children is progressing. Social security benefits such as pensions and elderly care are increasing. In a society with fewer children, it is difficult for a government to provide sufficient pension benefits for older people if pay-as-you-go pensions are adopted because a decrease in the working population reduces tax revenues to provide pension benefits. Therefore, the pension contribution rate must be increased to provide sufficient pension benefits. This paper demonstrates that an increase in the pension contribution rate can not always raise pension benefits. However, if a government provides a subsidy for elderly care services and if aggregate demand for elderly care services increases, then the pension benefit can always increase because younger people purchase elderly care services and increase the labor supply instead of performing elderly care with their time. Moreover, this paper presents an examination of whether a subsidy for elderly care can raise the level of social welfare or not and shows that the subsidy can raise the social welfare level thanks to an increase in pension benefits.

Keywords: Aging society, Elderly care service, Pay-as-you-go pension

JEL Classifications: H51, H55, J14

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1 Introduction

Japan's aging society is progressing. Older people who need elderly care services will increase. Colombo et al. (2011) surveys a recent OECD study of long-term care. Average public spending on long-term care was 1.5% of GDP in 2008. This public spending increases in an aging society. The ratio of the workforce in the elderly care service to total workforce was 1.3% at 2008. Cremer and Pestieau (2012) explain that elderly care service will necessarily increase in OECD countries. In France and Germany, which provide sufficient elderly care services in the market, informal care given by the family will not increase greatly.[1] However, in Italy and Poland, informal care given by the family is expected to increase greatly. Informal care provided by the family brings about negative effects on the labor supply. Younger people care for elderly parents with their time. Therefore, younger people must cut their working time, which engenders a decrease in the aggregate labor supply. In OECD countries, especially in Japan, fertility is low. The working generations might decrease in the future. This fact presents a problem because the decrease in labor supply reduces the tax revenue. Consequently, the government can not provide sufficient social security benefits.

The study of long-term care can be examined from many perspectives. Many studies have been conducted to assess the insurance of long-term care. Pauly (1990) reports that public long-term care insurance is necessary because of adverse selection. Miyazawa, Moudoukoutas and Yagi (2000) also consider public long-term care insurance. Cremer and Pestieau (2011) and Cremer and Roeder (2012) report that the subsidy for private long-term care insurance or the public provision of long-term care is needed. However, Richter and Ritzberger (1995) show that the public long-term care insurance brings about moral hazard: people do not make an effort to avoid needing elderly care service. The risk of elderly care service brings about precautionary saving, which reduces the utility, compared with the perfect foresight economy. Although long-term care has negative effects such as moral hazard, insurance has a positive effect on utility, as shown by Smith and Witter (2004).

Some studies have examined how the insurance of long-term care affects economic growth, in-

[1] Data from EC (2009).

come per capita, and welfare. Tabata (2005) shows that the subsidy for long-term care increases the utility level in present generations instead of decreasing the future generation's utility for preventing economic growth. Mizushima (2009) derives the optimal subsidy level of long-term care. Miyazawa (2010) considers elderly care as a provision in kind and shows that is better than a provision in cash in terms of welfare and economic growth. In OECD countries, long-term care expenditure and the workforce related to elderly care services increase in an aging society. This situation in OECD countries is explained by Hashimoto and Tabata (2010).

Moreover, some studies examine how the subsidy for elderly care service affects the formal care purchased in the market and the informal care given by the family. Korn and Wrede (2012) examine how the long-term care service provided in the market affects the female labor supply. However, they do not consider the subsidy for elderly care service. Mou and Winer (2012) considers the subsidy for the formal care purchased by the parents that long-term care is given and examines how this subsidy affects the formal care and informal care given by their children.

This paper presents an examination of whether the subsidy for elderly care service can raise the labor supply and then increase the pension benefit or not. In an aging society, it is difficult to provide a certain level of pension benefits because of a lack of revenue caused by a decrease in the number of younger people. However, even if younger people become fewer, the government can gain revenue by virtue of an increase in labor supply. Therefore, if the subsidy for elderly care services promotes the purchase of formal care in the market and reduces informal care time, this subsidy raises the labor supply and the revenue to provide pension benefits. This paper presents derivation of these results. As described herein, even if the government increases the contribution rate to increase pension benefits, the pension benefits can not always increase. However, a subsidy for elderly care can always increase pension benefits. Therefore, both pensions and the subsidy for elderly care services should be provided simultaneously.

The government set child-care support policies to raise fertility, supporting the working population of the future. Child-care policies are intended to raise the working population to provide sufficient social security benefits for future generations. However, this paper shows that an increase in pension benefits will occur even if the government does not provide child-care support policies.

This paper consists of the following sections. Section 2 sets the model and Section 3 derives the equilibrium in the model economy. Section 4 examines how the subsidy affects the labor supply and that pension benefits can rise by virtue of this subsidy. Section 5 evaluates the policy effects in terms of social welfare. The final section presents results obtained through the analyses presented in this paper.

2 The Model

The model economy presented herein is constructed in terms of a two-period (young and old) overlapping generations model. The economy comprises agents of three types: households, firm of two types (one produces elderly care services; the other produces final goods), and a government. In t period, the population of younger people is N_t and the population of older people is N_{t-1}. Then, the gross population growth rate is given as $n = \frac{N_t}{N_{t-1}}$, which represents an intergenerational population ratio. The population growth rate is assumed to be constant over time. We explain the agents in the following subsections.

2.1 Households

Individuals in households exist in two periods: a young period and old period. Younger people provide labor supply to gain labor income. The labor income is allocated to consumption in a younger period and savings to consume in the old period. In addition, younger people provide elderly care. There are two means to provide elderly care. One is time. Younger people have a unit of time. This paper assumes that ϕ unit of time is necessary to provide a unit of elderly care. The other is to buy elderly care provided by an elderly care service market. Then, it is assumed that younger people pay a price p_t for a unit of the service if they want to use the service. Then, a household's lifetime budget constraint is shown as presented below.

$$c_{1t} + \frac{c_{2t+1}}{1+r_{t+1}} = (1-\tau)aw_t(1-\phi e_t) - T + \frac{S_{t+1}}{1+r_{t+1}}, \text{Not Using Care Service} \tag{1}$$

$$c_{1t} + \frac{c_{2t+1}}{1+r_{t+1}} + (1-\epsilon)p_t e_t = (1-\tau)aw_t - T + \frac{S_{t+1}}{1+r_{t+1}}, \text{Using Care Service} \tag{2}$$

Therein, c_{1t} and c_{2t+1} respectively denote the consumption of young people in t period and in the old period in $t+1$ period. The younger people provide elderly care e_t for their parents. A ϕ unit of time is needed if they provide elderly care with their time. Then, the labor supply is $1-\phi e_t$. However, if the younger people purchase elderly care services at price p_t, then the younger people can provide a unit of time as the labor supply. w_t and $1+r_t$ denote the wage rate per an effective labor unit and the interest rate. This paper assumes that the households have productivity a, which is uniformly distributed between $\underline{a}$ and $\bar{a}$ ($\bar{a} > \underline{a}$). Then, the density function is given as $\frac{1}{\bar{a}-\underline{a}}$. Also, τ denotes the pension contribution rate; S_{t+1} shows pension benefits in the old period at $t+1$ period. T denotes the lump-sum taxation to provide a subsidy for elderly care. This subsidy rate is assumed as ϵ $(0 < \epsilon < 1)$.

The household's utility function is assumed as

$$u = \alpha \ln c_{1t} + \beta \ln c_{2t} + (1-\alpha-\beta) \ln e_t, \; 0 < \alpha, 0 < \beta, \alpha+\beta < 1. \tag{3}$$

The households decide the optimal allocation to maximize their utility (3) subject to their budget constraint (1) or (2) as

$$c_{1t} = \alpha \left((1-\tau)aw_t - T + \frac{S_{t+1}}{1+r_{t+1}} \right), \tag{4}$$

$$c_{2t+1} = (1+r_{t+1})\beta \left((1-\tau)aw_t - T + \frac{S_{t+1}}{1+r_{t+1}} \right), \tag{5}$$

$$e_t^n = \frac{(1-\alpha-\beta)\left((1-\tau)aw_t - T + \frac{S_{t+1}}{1+r_{t+1}}\right)}{\phi(1-\tau)aw_t}, \; or \tag{6}$$

$$e_t^c = \frac{(1-\alpha-\beta)\left((1-\tau)aw_t - T + \frac{S_{t+1}}{1+r_{t+1}}\right)}{(1-\epsilon)p_t}. \tag{7}$$

Therein, e_t^n denotes the elderly care provided by younger people who do not purchase care services; e_t^c denotes the elderly care service provided in the market. As long as $(1-\epsilon)p_t < \phi(1-\tau)aw_t$, younger people purchase elderly care service in the market. Unless the younger people provide elderly care by themselves.

2.2 Firms

In this model, two sectors exist: one for the final goods sector and the other for the elderly care sector. The production function in the final goods sector is assumed as

$$Y_t = F(K_t, L_t),\ \frac{\partial Y_t}{\partial K_t} > 0,\ \frac{\partial Y_t}{\partial L_t} > 0,\ \frac{\partial^2 Y_t}{\partial K_t^2} < 0,\ \frac{\partial^2 Y_t}{\partial L_t^2} < 0,\ \frac{\partial Y_t}{\partial K_t \partial L_t} > 0. \tag{8}$$

In those expressions, K_t and L_t respectively denote capital stock and effective labor. Assuming a competitive market, the wage rate and the interest rate are shown as

$$w_t = f(k_t) - f'(k_t)k_t, \tag{9}$$

$$1 + r_t = f'(k_t), \tag{10}$$

where $\frac{Y_t}{L_t} \equiv f'(k_t)$ and $k_t \equiv \frac{K_t}{L_t}$. This paper assumes full capital depreciation in one period. Moreover, for these analyses, we assume a small open economy. The wage rate and the interest rate are given exogenously as w and $1 + r$, respectively. An individual has productivity a gain wage rate aw because w is the wage rate per effective labor unit. In addition to the final goods sector, the elderly care service sector exists in this model. The production function in elderly care service is shown as

$$Y_t^c = \rho L_t^c, \rho > 0. \tag{11}$$

The elderly care service is produced only by labor input. This function is assumed by Hashimoto and Tabata (2010) and Yasuoka and Miyake (2010).[2] Then, the profit function is shown as

$$\pi_t = p_t \rho L_t^c - w_t^c L_t^c. \tag{12}$$

The wage rate w_t^c is given as

$$w_t^c = \rho p_t. \tag{13}$$

This paper defines $\hat{a}$ as $\hat{a} = \frac{(1-\epsilon)p_t}{\phi(1-\tau)w_t}$ and assumes that if individuals work in the elderly care service sector, then they receive the labor income w_t^c in spite of their productivity a. Therefore, individuals that have the ability $w_t^c > aw$, they work in elderly care service sector. However,

[2]Hashimoto and Tabata (2010) assume this function form and examine elderly care service market with population aging. Yasuoka and Miyake (2010) assume the same function form as child care service sector.

given $w_t^c < aw$, they work in the final goods sector. Therefore, defining $\tilde{a} \equiv \frac{w_t^c}{w}$, the younger people who have productivity in $[\underline{a}, \tilde{a}]$ work in the elderly care service market. Younger people who have productivity in $(\tilde{a}, \bar{a}]$ work in the final goods market.[3]

Considering (13) and $\tilde{a}$, we obtain

$$p_t = \frac{\tilde{a}w}{\rho}. \tag{14}$$

Therefore, an increase in $\tilde{a}$, which means an increase in the labor population in elderly care service market, raises price p_t.

The analysis presented in this paper assumes $\tilde{a} < \hat{a}$, i.e., $\frac{1-\epsilon}{\rho\phi(1-\tau)} > 1$. This assumption is set for simplicity. Even if this paper were not to set this assumption, the proposition obtained in this study might not change. The budget constraint (1) and the demand for elderly care (6) of individuals that work in elderly care service are given as

$$c_{1t} + \frac{c_{2t+1}}{1+r_{t+1}} = (1-\tau)w_t^c(1-\phi e_t) - T + \frac{S_{t+1}}{1+r_{t+1}}, \tag{15}$$

$$e_t^n = \frac{(1-\alpha-\beta)\left((1-\tau)w_t^c - T + \frac{S_{t+1}}{1+r_{t+1}}\right)}{\phi(1-\tau)w_t^c}. \tag{16}$$

2.3 Government

The government in this model economy provides a subsidy for elderly care service provided in the market and pay-as-you-go pension. First, the government levies lump-sum taxation on younger people to subsidize elderly care services. Consideration of the balanced budget reduces to the following equation:

$$\int_{\hat{a}}^{\bar{a}} \frac{\epsilon p_t e_t^c}{\bar{a} - \underline{a}} da = T. \tag{17}$$

Substituting (7) into (17), we obtain

$$\frac{1-\alpha-\beta}{\bar{a}-\underline{a}} \frac{\epsilon}{1-\epsilon} \left(\frac{(1-\tau)(\bar{a}^2 - \hat{a}^2)w}{2} + \left(-T + \frac{S_{t+1}}{1+r}\right)(\bar{a} - \hat{a}) \right) = T. \tag{18}$$

Second, the government provides a pay-as-you-go pension by which the government collects revenue from younger people at t period and gives the benefit for the older people in same t

[3] Meckl and Zink (2004) consider productivity a and assume that the individual gain in wage a^2w if he works in skilled labor sector. He gains wage aw if he works in the unskilled labor sector. The elderly care service described in this paper is similar to that of the unskilled labor sector introduced by Meckl and Zink (2004).

period. With the balanced budget, the budget constraint is shown as

$$n\tau\left(\int_{\hat{a}}^{\bar{a}}\frac{aw}{\bar{a}-\underline{a}}da+\int_{\tilde{a}}^{\hat{a}}\frac{(1-\phi e_t^c)aw}{\bar{a}-\underline{a}}da+\int_{\underline{a}}^{\tilde{a}}\frac{(1-\phi e_t^c)w_t^c}{\bar{a}-\underline{a}}da\right)=S_t. \tag{19}$$

3 Equilibrium

The market equilibrium condition in elderly care service market is given as (7) and (11). We obtain the following equation:

$$\frac{1-\alpha-\beta}{1-\epsilon}\int_{\hat{a}}^{\bar{a}}\left((1-\tau)aw-T+\frac{S_{t+1}}{1+r}\right)\frac{1}{\bar{a}-\underline{a}}da=p_t\frac{\rho(\tilde{a}-\underline{a})}{\bar{a}-\underline{a}}. \tag{20}$$

Therefore, the price of elderly care service p_t is reduced by

$$p_t=\frac{1-\alpha-\beta}{\rho(\tilde{a}-\underline{a})(1-\epsilon)}\left(\frac{(1-\tau)w(\bar{a}^2-\hat{a}^2)}{2}+\left(-T+\frac{S_{t+1}}{1+r}\right)(\bar{a}-\hat{a})\right), \tag{21}$$

where $\hat{a}=\frac{(1-\epsilon)p_t}{\phi(1-\tau)w}$ and $\tilde{a}=\frac{\rho p_t}{w}$. Given S_{t+1}, the price p_t is given to hold this equation. The dynamics of S_t is reduced by substituting (19) into (6) and (16) as

$$\begin{aligned}\frac{n\tau}{\bar{a}-\underline{a}}\Bigg(&\frac{w(\bar{a}^2-\hat{a}^2)}{2}+\int_{\tilde{a}}^{\hat{a}}awda-\int_{\tilde{a}}^{\hat{a}}\frac{(1-\alpha-\beta)\left((1-\tau)aw-T+\frac{S_{t+1}}{1+r}\right)}{1-\tau}da\\&+\int_{\underline{a}}^{\tilde{a}}\left(w_t^c-\frac{(1-\alpha-\beta)\left((1-\tau)w_t^c-T+\frac{S_{t+1}}{1+r}\right)}{1-\tau}\right)da\Bigg)=S_t.\end{aligned} \tag{22}$$

Given p_t and S_{t+1}, we obtain S_t. Then, considering (21) and (22), we obtain the elderly care service price p_t.

4 Subsidy for Elderly Care Services

This section presents examination of the subsidy for elderly care e_t. The government gives the subsidy to younger people to buy elderly care services supplied in the market. Such a subsidy can stimulate labor supply and raise tax revenue to provide pension benefits. This section presents an examination of whether the subsidy can raise the pension benefit or not thanks to an increase in labor supply. First, this paper shows that the subsidy can decrease $\hat{a}$, i.e., the subsidy can increases the younger people that buy elderly care in the market and work in full time. Differentiating (7) and (17) by ϵ and T at the approximation at $\epsilon = 0$ and $T = 0$, we obtain the following equation,

$$dT = \frac{1-\alpha-\beta}{\bar{a}-\underline{a}}\left(\frac{(1-\tau)(\bar{a}^2-\hat{a}^2)}{2}+\frac{(\bar{a}-\hat{a})S}{1+r}\right)d\epsilon. \tag{23}$$

Substituting $\tilde{a} = \frac{\rho p_t}{w}$ into $\hat{a} = \frac{(1-\epsilon)p_t}{\phi(1-\tau)w}$ and completely differentiating at $\epsilon = 0$, the following equation is derived as

$$d\tilde{a} = \phi(1-\tau)\rho d\hat{a} + \tilde{a}d\epsilon. \tag{24}$$

Substituting $p_t = \frac{\phi(1-\tau)\hat{a}w}{1-\epsilon}$ into (21) and completely differentiating at $\epsilon = 0$ and $T = 0$, we obtain

$$\begin{aligned}&\left(\phi(1-\tau)w\rho(2\tilde{a}-\underline{a})+(1-\alpha-\beta)\left((1-\tau)w+\frac{S_{t+1}}{1+r}\right)\right)d\hat{a}\\ &= -\tilde{a}^2wd\epsilon-(\bar{a}-\hat{a})(1-\alpha-\beta)dT.\end{aligned} \tag{25}$$

With (23)–(25), we obtain the negative sign of $\frac{d\hat{a}}{d\epsilon} < 0$ as

$$\frac{d\hat{a}}{d\epsilon} = -\frac{\tilde{a}^2w+\frac{(\bar{a}-\hat{a})(1-\alpha-\beta)^2}{\bar{a}-\underline{a}}\left(\frac{(1-\tau)(\bar{a}^2-\hat{a}^2)w}{2}+\frac{(\bar{a}-\hat{a})S}{1+r}\right)}{\phi w\rho(1-\tau)(2\tilde{a}-\underline{a})+(1-\alpha-\beta)\left((1-\tau)w+\frac{S}{1+r}\right)} < 0. \tag{26}$$

Then, the following proposition is established.

Proposition 1 The subsidy for the elderly care service in the market can increase the number of younger people who purchase elderly care services and work full time.

When the number of younger people who buy elderly care services in the market increases, this subsidy has the effect of an increase in the labor supply. However, this subsidy does not change the labor supply of individuals who have productivity $[0, \hat{a}]$. They provide $(1 - \phi e_t^n)$ units of time for labor supply. The lump-sum tax T affects e_t^n. As shown by (6) and (16), the lump-sum tax decreases elderly care time and increases labor time $1 - \phi e_t^n$. These increases in labor supply raise the pension benefit S_{t+1}. However, this effect increases e_t^n and decreases labor supply $1 - \phi e_t^n$. Therefore, they do not always increase their labor supply.

This paper presents an examination of how the government can increase pension benefits. We consider two means to raise pension benefits. One is an increase in contribution rate. The other is the subsidy for elderly care services. First, we examine whether an increase in contribution rate can pull up pension benefits or not. Now, we consider the pension benefit $S_t = S_{t+1} = S$ in the steady state. Differentiating (21) and (22) by τ and S, we obtain the following equation.[4]

$$\frac{dS}{d\tau} = \frac{\frac{S}{\tau} + \frac{(1-\alpha-\beta)(\hat{a}-\underline{a})S}{(\tilde{a}-\underline{a})(1-\tau)^2(1+r)} - \frac{(1-\alpha-\beta)\hat{a}}{(\tilde{a}-\underline{a})(1-\tau)}\left(\hat{a}w + \frac{S}{(1-\tau)(1+r)}\right)}{1 + \frac{(1-\alpha-\beta)(\hat{a}-\underline{a})}{(\tilde{a}-\underline{a})(1-\tau)(1+r)}}$$
$$+ \frac{\frac{(\alpha+\beta)(\tilde{a}-\underline{a})\rho}{\tilde{a}-\underline{a}} - \frac{(1-\alpha-\beta)\hat{a}}{(\tilde{a}-\underline{a})p}\left(\hat{a}w + \frac{S}{(1-\tau)(1+r)}\right)}{1 + \frac{(1-\alpha-\beta)(\hat{a}-\underline{a})}{(\tilde{a}-\underline{a})(1-\tau)(1+r)}} \frac{dp}{d\tau}, \tag{27}$$

where

$$\frac{dp}{d\tau} = \frac{\frac{1-\alpha-\beta}{\rho(\tilde{a}-\underline{a})}\left(\frac{\tilde{a}-\hat{a}}{1+r}\frac{dS}{d\tau} - \frac{\hat{a}}{1-\tau}\left((1-\tau)w\hat{a} + \frac{S}{1+r}\right) - \frac{w(\bar{a}^2-\hat{a}^2)}{2}\right)}{1 + \frac{\tilde{a} + \frac{(1-\alpha-\beta)\hat{a}}{\rho p}\left((1-\tau)w\hat{a} + \frac{S}{1+r}\right)}{\tilde{a}-\underline{a}}}.$$

The sign of $\frac{dS}{d\tau}$ is ambiguous. The first term of (27) shows the direct effect of an increase in τ. This effect directly raises the pension benefit S. However, an increase in τ reduces the opportunity cost for elderly care and the household's labor supply decreases. This effect decreases the pension benefit S. The second term of (27) shows the effect of an increase in τ on elderly care service price p. An increase in τ reduces the demand for care because an increase in τ reduces the opportunity cost to provide elderly care with their time. Moreover, even if the younger people purchase elderly care services in the market, the demand for care in the market decreases because of a decrease in disposable household income. Therefore, tax revenue to provide pensions decreases because a decrease in p_t is the same as a decrease in the wage rate w^c. Considering $\hat{a} = \frac{(1-\epsilon)p_t}{\phi(1-\tau)w}$, the younger people who provide elderly care with time increase because of an increase in τ: an increase in τ reduces the opportunity cost for elderly care and decreases the younger people who work in full time. Consequently, the labor supply decreases. Revenue for pension benefits decreases, too. Therefore, an increase in the contribution rate of pensions can not always raise pension benefits, although an increase in the contribution rate directly raises the pension benefit.

Next, we examine whether the subsidy for elderly care service in the market can pull up the pension benefit or not. If the subsidy for elderly care service increases the aggregate demand for elderly care services, the supply of elderly care service increases, i.e., $\frac{d\tilde{a}}{d\epsilon} > 0$.[5] Completely

[4] The locally stable condition of the steady state is obtained in the Appendix.
[5] See the Appendix for a detailed proof.

differentiating (21) and (22) in the steady state by T and S, because $\frac{d\hat{a}}{d\epsilon} < 0$ and $\frac{dT}{d\epsilon} > 0$, we obtain the following.

$$\frac{dS}{d\epsilon} = \frac{-\frac{(1-\alpha-\beta)S}{(1+r)(1-\tau)w}\frac{d\hat{a}}{d\epsilon} + \frac{(1-\alpha-\beta)(\hat{a}-\underline{a})\frac{dT}{d\epsilon}+(\alpha+\beta)(\tilde{a}-\underline{a})\frac{d\tilde{a}}{d\epsilon}}{(1-\tau)w}}{\frac{(1-\alpha-\beta)(\hat{a}-\underline{a})}{w(1-\tau)} + \frac{\bar{a}-\underline{a}}{wn\tau}} > 0 \tag{28}$$

Then, we can establish the following proposition.

Proposition 2 If the subsidy for elderly care service increases the aggregate demand for elderly care services, then pension benefits increase by virtue of an increase in labor supply.

This proposition presents an important policy implication. Some economically developed countries suffer from an aging society with fewer children. If the pension benefit is managed using a pay-as-you-go pension, then fewer children, which decreases the younger people in future, decreases pension benefits in the future. Therefore, the government must increase the contribution rate of pensions because pension benefits are kept at a certain level.[6] However, this paper presents derivation that an increase in the contribution rate can not always raise pension benefits. Therefore, the government must consider another way to increase pension benefits. The subsidy for elderly care can increase then labor supply. Then, by virtue of an increase in the labor supply, the government can collect revenue to pay for older people as pension benefits.

5 Welfare Analysis

The subsidy for elderly care services can raise the pension benefit S in a steady state. However, social welfare can not always be raised by a subsidy for elderly care services because of the greater tax burden. This section presents an examination of whether the subsidy for elderly care can raise the level of social welfare or not.

This paper defines u^i as type i's utility and the following social welfare function W in the steady state.

$$W = \int_{\hat{a}}^{\bar{a}} u^x \frac{1}{\bar{a}-\underline{a}} da + \int_{\tilde{a}}^{\hat{a}} u^y \frac{1}{\bar{a}-\underline{a}} da + + \int_{\underline{a}}^{\tilde{a}} u^z \frac{1}{\bar{a}-\underline{a}} da. \tag{29}$$

[6] In Japan, the replacement rate of pension benefits is chosen to maintain fifty percent in the future at 2004 pension reform.

Type x are households that have the ability $[\hat{a},\bar{a}]$; type y are households that have the ability $[\tilde{a},\hat{a}]$. Also, type z are households that have the ability $[\underline{a},\tilde{a}]$.

Substituting (4)–(7) and (16) into (29), we obtain the following social welfare function of

$$
\begin{aligned}
W = & \int_{\tilde{a}}^{\bar{a}} \ln\left((1-\tau)aw - T + \frac{S}{1+r}\right)\frac{1}{\bar{a}-\underline{a}}da + \int_{\underline{a}}^{\tilde{a}} \ln\left((1-\tau)\tilde{a}w - T + \frac{S}{1+r}\right)\frac{1}{\bar{a}-\underline{a}}da \\
& - (1-\alpha-\beta)\int_{\tilde{a}}^{\hat{a}} \ln a \frac{1}{\bar{a}-\underline{a}}da \\
& - (1-\alpha-\beta)\int_{\underline{a}}^{\tilde{a}} \ln \tilde{a} \frac{1}{\bar{a}-\underline{a}}da - (1-\alpha-\beta)\int_{\hat{a}}^{\bar{a}} \ln(1-\epsilon) \frac{1}{\bar{a}-\underline{a}}da \\
& - (1-\alpha-\beta)\int_{\hat{a}}^{\bar{a}} \ln p \frac{1}{\bar{a}-\underline{a}} - (1-\alpha-\beta)\int_{\underline{a}}^{\hat{a}} \ln \phi(1-\tau)w \frac{1}{\bar{a}-\underline{a}}da \\
& + \int_{\underline{a}}^{\bar{a}} \frac{C}{\bar{a}-\underline{a}}da,
\end{aligned}
\tag{30}
$$

where $C \equiv \alpha\ln\alpha + \beta\ln\beta + (1-\alpha-\beta)\ln(1-\alpha-\beta) + \beta\ln(1+r)$. Differentiating (30) by T, we obtain $\frac{dW}{dT}$ as follows.

$$
\begin{aligned}
\frac{dW}{dT} = & \underbrace{\frac{1}{(1-\tau)(\bar{a}-\underline{a})}\left(\ln\frac{(1-\tau)\bar{a}w + \frac{S}{1+r}}{(1-\tau)\tilde{a}w + \frac{S}{1+r}}\left(-1 + \frac{1}{1+r}\frac{dS}{dT}\right) - (1-\tau)w\ln\left((1-\tau)\tilde{a}w + \frac{S}{1+r}\right)\frac{d\tilde{a}}{dT}\right)}_{W_1} \\
& + \underbrace{\frac{\tilde{a}-\underline{a}}{\bar{a}-\underline{a}}\frac{-1+\frac{1}{1+r}\frac{dS}{dT}}{(1-\tau)\tilde{a}w + \frac{S}{1+r}} + \frac{(1-\tau)\tilde{a}w + \frac{S}{1+r}}{\bar{a}-\underline{a}}\frac{d\tilde{a}}{dT}}_{W_2} \\
& - \underbrace{(1-\alpha-\beta)\frac{\ln\hat{a}\frac{d\hat{a}}{dT} - \ln\tilde{a}\frac{d\tilde{a}}{dT}}{\bar{a}-\underline{a}}}_{W_3} - \underbrace{(1-\alpha-\beta)\frac{\frac{\tilde{a}-\underline{a}}{\tilde{a}} + \ln\tilde{a}}{\bar{a}-\underline{a}}\frac{d\tilde{a}}{dT}}_{W_4} + \underbrace{(1-\alpha-\beta)\frac{(\bar{a}-\hat{a})}{\bar{a}-\underline{a}}\frac{d\epsilon}{dT}}_{W_5} \\
& - \underbrace{\frac{1-\alpha-\beta}{\bar{a}-\underline{a}}\left(-\frac{d\hat{a}}{dT}\ln p + \frac{\bar{a}-\hat{a}}{p}\frac{dp}{dT}\right)}_{W_6} - \underbrace{\frac{1-\alpha-\beta}{\bar{a}-\underline{a}}\ln\phi(1-\tau)w\frac{d\hat{a}}{dT}}_{W_7}.
\end{aligned}
\tag{31}
$$

It remains ambiguous whether the subsidy for elderly care services can raise the social welfare or not. W_1 and W_2 show that the subsidy affects the household income. W_3 shows that the subsidy increases social welfare because the aggregate opportunity cost decreases thanks to an increase in the subsidy to use more elderly care services. W_4 shows that the subsidy decreases social welfare because the opportunity cost to the individuals working in elderly care services increases because of an increase in wage rates in elderly care service. W_5 and W_6 show that the subsidy can raise social welfare because of subsidy incentives to use elderly care services. W_7

shows that the subsidy can raise social welfare because individuals who use elderly care services increase and the aggregate opportunity cost decreases.

Therefore, it is apparent that the subsidy for elderly care service affects social welfare in many ways. The subsidy reduces the consumer's price of elderly care and increases social welfare, as shown by $W_5 + W_6$. Social welfare increases as shown by W_1 if the tax burden is smaller than the increase in pension benefit. However, one must consider that individuals can use elderly care services by virtue of the subsidy and social welfare increases. Then, even if the household's income decreases because of the tax burden and the small increase in pension benefits, social welfare can increase by virtue of an increase in the individuals who use elderly care services. The subsidy raises the wage rate of elderly care services and this increases the opportunity cost to care for their parents by their time. We note that social welfare decreases if this effect is large.

6 Conclusions

This paper presents an examination of how the government can increase pension benefits in a pay-as-you-go pension. The results derived in this paper are the following. An increase in the pension contribution rate can not always increase pension benefits because an increase in the contribution rate decreases the opportunity cost of providing elderly care services with time and decreases the labor supply, which decreases the revenue for pension benefits. However, if the government provides a subsidy for elderly care service in the market, then the younger people stop elderly care with their time and purchase elderly care services and increase the labor supply. As a result, the revenue for pension benefits increases.

This paper shows how social security benefits should be spent. Generally, the government levies on the labor income of younger people to provide pension benefits for older people in economically developed countries such as Japan. As long as this pension system is adopted, an increase in labor supply can keep the pension benefit level shown by the replacement rate of pension. Although an increase in fertility with child care policies is regarded as maintaining the pension benefit, the policy by which younger people increase to buy elderly care services in the market and increase their labor supply can be adopted to maintain the pension benefits even if fertility, which indicates the working population size in future, remains at a low level.

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Appendix

Proof of $\frac{d\tilde{a}}{d\epsilon} > 0$

The demand for elderly care service in the market E_t is shown as

$$E_t = \int_{\hat{a}}^{\bar{a}} \frac{(1-\alpha-\beta)\left((1-\tau)aw - T + \frac{S_{t+1}}{1+r}\right)}{(1-\epsilon)p_t} \frac{1}{\bar{a}-\underline{a}} da. \tag{32}$$

Given a constant price of elderly care service p_t, we examine that an increase in the subsidy for elderly care ϵ at $\epsilon = 0$ raises the demand for elderly care services.

$$\begin{aligned} \frac{dE_t}{d\epsilon} &= \frac{1-\alpha-\beta}{(\bar{a}-\underline{a})p_t} \left(\frac{(1-\tau)w(\bar{a}^2-\hat{a}^2)}{2} + \frac{(\bar{a}-\hat{a})S_{t+1}}{1+r} \right) \left(1 - \frac{(\bar{a}-\hat{a})(1-\alpha-\beta)}{\bar{a}-\underline{a}} \right) \\ &- \frac{(1-\alpha-\beta)\left((1-\tau)\hat{a}w + \frac{S_{t+1}}{1+r}\right)}{p_t(\bar{a}-\underline{a})} \frac{d\hat{a}}{d\epsilon} + \frac{\bar{a}-\hat{a}}{p_t(\bar{a}-\underline{a})(1+r)} \frac{dS}{d\epsilon}. \end{aligned} \tag{33}$$

The first term and second term have a positive sign. However, the sign of $\frac{dS}{d\epsilon}$ is not determined here. Therefore, if $\frac{dS}{d\epsilon} < 0$ and this effect is large, $\frac{dE_t}{d\epsilon}$ can be negative. However, we consider the case in which the subsidy for elderly care service raises aggregate demand for care services, i.e., $\frac{dE_t}{d\epsilon} > 0$. Then, as shown in Fig. the quantity of purchasing elderly care service increases. Therefore, labor input for elderly care service increases, i.e., we obtain $\frac{d\tilde{a}}{d\epsilon} > 0$.

[Insert Fig around here.]

Condition of Local Stability

This appendix derives the condition of local stability at $T = 0$ and $\epsilon = 0$. Completely differentiating (22) by S_t, S_{t+1} and $\tilde{a}$, one obtains

$$\frac{n\tau}{\bar{a}-\underline{a}} \left(\left((\alpha+\beta)(3\tilde{a}+\underline{a})w - \frac{1-\alpha-\beta}{1-\tau} \frac{S}{1+r} \frac{1}{\rho\phi(1-\tau)} \right) d\tilde{a} - \frac{1-\alpha-\beta}{1-\tau} \frac{\hat{a}-\underline{a}}{1+r} dS_{t+1} \right) = dS_t. \tag{34}$$

Completely differentiating (21) by S_{t+1} and $\tilde{a}$, one obtains

$$d\tilde{a} = \frac{\frac{(1-\alpha-\beta)(\bar{a}-\hat{a})}{1+r}}{(2\tilde{a}-\underline{a})w + \frac{1-\alpha-\beta}{\rho\phi(1-\tau)}\left((1-\tau)\tilde{a}w + \frac{S}{1+r}\right)} dS_{t+1}. \tag{35}$$

Substituting (35) into (34), $\frac{dS_{t+1}}{ds_t}$ is obtained as

$$\frac{dS_{t+1}}{dS_t} = \frac{1}{\frac{n\tau}{\bar{a}-\underline{a}} \left(\frac{\frac{(1-\alpha-\beta)(\bar{a}-\hat{a})}{1+r}\left((\alpha+\beta)(3\tilde{a}+\underline{a}) - \frac{1-\alpha-\beta}{1-\tau}\frac{S}{1+r}\frac{1}{\rho\phi(1-\tau)}\right)}{(2\tilde{a}-\underline{a})w + \frac{1-\alpha-\beta}{\rho\phi(1-\tau)}\left((1-\tau)\tilde{a}w + \frac{S}{1+r}\right)} - \frac{1-\alpha-\beta}{1-\tau}\frac{\hat{a}-\underline{a}}{1+r} \right)}. \tag{36}$$

With $-1 < \frac{dS_{t+1}}{dS_t} < 1$, the steady state is locally stable.

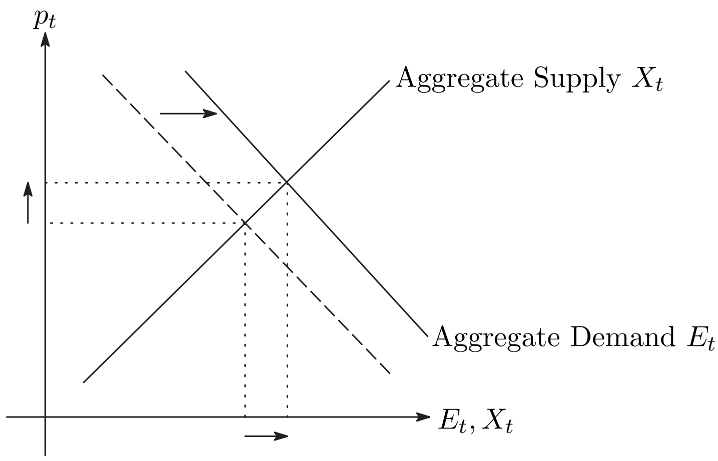

Fig.: Subsidy for Elderly Care Service and Aggregate Demand.