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## Optimal Term Length for an Overconfident Central Banker

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# Optimal Term Length for an Overconfident Central Banker\*

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## Abstract

This paper discusses the implications of overconfidence when it affects a monetary policy-maker. We consider two forms of overconfidence: the illusion of precision and the illusion of control. Embedding them in a standard New Keynesian framework, we derive the optimal term length of a central banker and examine how it depends on the types and degrees of overconfidence. In particular, we show that the legal mandate should be lengthened when these two types of biases increase concurrently.

*Keywords:* central banker; overconfidence; legal mandate; optimal term length

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# 1 Introduction

Overconfidence among policymakers is recognized as a very prevalent phenomenon, especially among top-level managers (DellaVigna, 2009). Central bankers today would certainly belong to such a category. Although the behavioral science literature has identified many different types of biases, overconfidence is probably one of the most prominent types to affect monetary policy-makers as Kahneman (2011, p. 219) states: “the person who acquires more knowledge develops an enhanced illusion of her skill and becomes unrealistically overconfident.” In our view, monetary policy-makers are all the more susceptible to fall under this behavioral trait of becoming overconfident, if only because “[t]he people who have the greatest influence on the lives of others are likely to be optimistic and overconfident, and to take more risks than they realize (Kahneman, 2011, p. 252).” With monetary policy an important tool of today’s macroeconomic management, it would be unthinkable for central bankers not to have fallen prey to overconfidence.

Such a behavioral trait has been well-documented in a large literature, which covers a wide range of decision-makers from hedge-fund managers to marrying couples. However, when it comes to monetary policy-makers, the literature is quite sparse. A notable exception is by Claussen et al. (2012), who look at the workings of a monetary policy committee whose members suffer from overconfidence and analyze its impact on the resulting monetary policy. They also show how certain organizational structures mitigate the issue of disagreement that are more likely to occur among overconfident policy-makers.

In contrast, our analysis here investigates the macroeconomic consequences of monetary policy decisions made by a single overconfident policy-maker. Given the influences of overconfidence, we focus on the legal term length that a society should give to its central banker. As we discuss below, overconfidence can have conflicting effects on the economic performance, yielding no straightforward answer as regards the optimal time length.

The socially optimal term length for central bankers has in particular been studied by Waller and Walsh (1996) and Lin (1999). In contrast to these previous works, we consider the issue in a forward-looking New Keynesian framework, and add the influences of a central banker’s overconfidence. More precisely, we integrate into the modern macroeconomic model two specific types of overconfidence biases. The first one is of

the “illusion of control”-type (DellaVigna, 2009), where a decision-maker believes that his/her current action has a stronger influence on the future economy than it actually has. This bias encourages the central banker to attempt to influence the economy in a more significant way. The second type of overconfidence bias we consider is often called the “illusion of precision”. Under this trait, the central banker thinks that he/she is more able to process the information, and can assess more precisely the actual size of a shock that hits an economy.

This study contributes by formally modeling the behavioral biases and deriving their consequences. It not only shows how the impacts of such overconfidence proceed through the whole macro-economy, but it also helps to understand an empirical puzzle, as central bankers, even though their institutions tend to become more similar (Crowe and Meade, 2007), still have different term lengths in their mandates and actual turnover rates (Hayat and Farvaque, 2011, Dreher et al., 2008, 2010).

In effect, our results indicate that the overconfidence biases of a central banker can have important impacts on the performance of an economy, which justifies the reassessment of the optimal term length of a central banker. First, in a forward-looking framework, the “illusion of control”-type of overconfidence has a lasting harmful impact, while the other type of overconfidence has a positive effect. Second, even though the two biases have conflicting consequences, we show that their combined impacts should induce a longer optimal term length in our framework. We also show that, when the output gap in the economy becomes more persistent, the optimal term length should be extended in the presence of overconfidence, whereas it should be unchanged without it.

The paper is organized as follows. In the next section, we set up a standard New Keynesian model on which our analysis is based, and also describe how this framework is modified to account for the overconfidence of a central banker. The following section addresses the issue of determining the optimal term length. Then, in section 4, we obtain a series of comparative statics results with respect to exogenous variables and briefly discuss their policy implications. The final section concludes the article.

## 2 The Model

### 2.1 The Macroeconomic Model

Our argument relies on a workhorse New Keynesian model, as is described in e.g., Froyen and Guender (2007) and Walsh (2010). This facilitates comparison with the earlier literature and also ensures that our conclusions do not rely on non-standard assumptions. Specifically, the economy is summarized by the forward-looking Phillips curve and the New Keynesian IS curve:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + \varepsilon_t, \quad (1)$$

$$y_t = -\alpha (i_t - E_t \pi_{t+1}) + E_t y_{t+1} + v_t, \quad (2)$$

where  $\pi_t$  denotes the inflation rate at time  $t$ ,  $\beta$  the discount factor,  $E_t$  the expectations operator,  $\kappa$  the degree of price rigidity,  $y_t$  the output gap,  $i_t$  the policy rate, and  $\alpha$  is a positive parameter. Moreover,  $\varepsilon_t$  and  $v_t$  are, respectively, cost and demand shocks that are independently and identically distributed (in particular, we assume  $\varepsilon_t \sim N(0; \sigma_\varepsilon^2)$ ).

Assuming customarily that the stabilities in the inflation rate and the output gap are appropriate objectives of monetary policy, the central banker attempts to minimize a quadratic expected loss function of the form:

$$L_t = \frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i (\pi_{t+i}^2 + \lambda y_{t+i}^2), \quad (3)$$

where  $\lambda$  denotes the relative weight given by the central banker to the output gap stability.

In a discretionary regime, the central banker attempts to minimize the loss value of a single period in (3) subject to (1), which gives the first-order condition for loss minimization as:

$$\pi_t = -\frac{\lambda}{\kappa} y_t.$$

By using the fact that the shock persists over time, i.e.,  $E_t y_{t+1} = \rho y_t$ , where  $\rho < 1$  is the degree of persistence of the shock (see Walsh, 2010), one gets the equilibrium inflation rate,  $\pi_t^*$ , as:

$$\pi_t^* = \frac{\lambda}{\lambda(1 - \beta\rho) + \kappa^2} \varepsilon_t. \quad (4)$$

Then, the expected value of the one-period loss function is equal to:

$$L^* = \frac{1}{2} \left( \frac{\lambda}{[\lambda(1 - \beta\rho) + \kappa^2]^2} \right) (\lambda + \kappa^2) \sigma_\varepsilon^2. \quad (5)$$

## 2.2 An Overconfident Central Banker

The literature on overconfidence identifies several distinctive types of behavioral traits (DellaVigna, 2009). Here, we focus on the two specific traits that seem quite likely to affect a central banker: the first relates to a positively-biased estimation of one’s own influences, giving rise to an illusion of control, while the second concerns a supposed higher capacity to extract and/or interpret information in any given situation, which is a bias related to an illusion of precision. In our overconfidence setting below, we consider that these two forms of overconfidence coexist.<sup>1</sup>

More specifically, as for the illusion of control type of bias, we assume that the central banker mistakenly believes that he/she has a better assessment of the “deep” parameters of the economy. In our framework, this bias intervenes through the persistence parameter:

$$0 < \rho < \bar{\rho} < 1, \quad (6)$$

where  $\rho$  is the actual value of the persistence parameter and  $\bar{\rho}$  is the overconfident central banker’s assessment of this deep parameter of the economy. This belief in a stronger influence of his/her current action upon the future economy results in a higher degree of manipulation of monetary policy instruments in the current period.

Concerning the information gathering and processing ability, we assume that the central banker makes a judgment error in perceiving the volatilities induced on the economy through the cost shock. Specifically, the subjective estimate of the variance of the shocks by the central banker is stipulated as:

$$\hat{\varepsilon}_t \sim N \left( 0; \frac{\sigma_\varepsilon^2}{\gamma} \right), \quad (7)$$

where  $\gamma$  ( $> 1$ ) represents the significance of this bias held by the central banker.<sup>2</sup>

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<sup>1</sup>We do not consider a third type of overconfidence, the over-placement (or “better-than-average”) effect, as it does not particularly influence managers. See Benoît and Dubra (2011).

<sup>2</sup>A key difference from Claussen et al. (2012) is that, although they introduce a degree of overconfidence of the latter form, their model does not consider the first type of judgment bias.

As a consequence, the equilibrium inflation rate and the expected value of the one-period loss of the economy under an overconfident central banker are respectively equal to:

$$\pi_t^{OC} = \frac{\lambda}{\lambda(1 - \beta\bar{\rho}) + \kappa^2} \hat{\varepsilon}_t, \quad (8)$$

$$L^{OC} = \frac{1}{2} \left( \frac{\lambda}{[\lambda(1 - \beta\bar{\rho}) + \kappa^2]^2} \right) (\lambda + \kappa^2) \frac{1}{\gamma} \sigma_\varepsilon^2, \quad (9)$$

where the superscript *OC* identifies the presence of overconfidence. As can be seen by comparing (5) and (9), overconfidence of the central banker has two opposing effects on the economy as a whole.

The first effect, stemming from the illusion of control parameter,  $\bar{\rho}$ , is a negative one as it increases the expected value of the period-wise loss function. The mechanism can be explained intuitively as follows: as the central banker overestimates the persistence of the output gap shock, as is stipulated in (6), and, given that he/she has a forward-looking behavior, he/she would seek to influence the future of the economy. In effect, the central banker has a tendency to overindulge in manipulating its policy instrument. For example, confronted with a positive output shock, he/she increases the policy rate substantially in order to tame any inflationary pressure. This increases the instantaneous volatility of the economy, i.e., we have:  $\frac{\partial L^{OC}}{\partial \bar{\rho}} > 0$ . The overconfident central banker accepts this immediate costly consequence of the policy move so as to benefit from a lower future volatility of the economy, conditionally on the occurrence of future unpredictable shocks. All in all, the illusion-of-control bias imposes an immediate cost on the economy, which suffers from a higher volatility in the policy rate, and, consequently, higher actual volatilities of the output gap and the inflation rate.

The second effect comes via the illusion of information precision bias, which is captured by  $\gamma$  in (7), and it is a positive one, meaning that it decreases the value of the loss function, as  $\frac{\partial L^{OC}}{\partial \gamma} < 0$ . As the central banker judges the cost shocks to be smaller than they really are (more precisely, he/she considers them to belong to a smaller interval), he/she underreacts. In other words, this type of bias induces less frequent and, potentially, smaller changes in the policy rates to offset any shock, a behavior that results in lower volatilities of the inflation and output variables, and thus reduces the expected value of the period-wise loss function.

Because of these two conflicting effects, overconfidence by itself has no systematically adverse (nor positive) effect on the economy, and a trade-off emerges in setting the optimal term length of a central banker when both of these behavioral traits are present.

### 3 The Optimal Term Length

In addressing the issue of the optimal term length, we suppose that the expected intertemporal social loss is different from the loss perceived by the central banker in the following two aspects. Firstly, there is some fixed cost associated with replacing an incumbent central banker with a new one,  $F$ . This cost can be associated with the search, hearings, and other processes involved during the hiring procedure. Secondly, we assume that there is some societal cost that grows over time while the same person is in the office,  $C(j) = c^{j-1}$ , where  $c > 1$  and  $j$  denotes the number of the periods for which a banker holds the policy-making position. The latter cost can be associated with increased rent-seeking activities or to informal pressures that come with an increased proximity to politicians, as is exemplified by chairmen at the Fed (Axilrod, 2011), for instance, and sets in from the second period on. In total, the “global” loss function to the society,  $G(T)$ , consists of the discounted sum of the one-period loss value, i.e.,  $E_0(L_t) = E_0(\sum_{t=0}^{\infty} \beta^t L_t)$ , where  $L_t$  is the relevant one-period loss, and also of these two additional costs to the society.

#### 3.1 The Case without Overconfidence

In the absence of overconfidence, the expected loss value of one term that spans  $T$  periods is written as:

$$\Lambda(T) = L^* + F + \beta(L^* + c) + \beta^2(L^* + c^2) + \dots + \beta^{T-1}(L^* + c^{T-1}), \quad (10)$$

where  $L^*$  is the value given in (5). This can be simplified to:

$$\Lambda(T) = \sum_{t=0}^{T-1} \beta^t L^* + F + \sum_{t=1}^{T-1} \beta^t c,$$

or:

$$\Lambda(T) = \left[ \frac{1}{2} \left( \frac{\lambda}{[\lambda(1 - \beta\rho) + \kappa^2]^2} \right) (\lambda + \kappa^2) \sigma_\varepsilon^2 \right] \frac{1 - \beta^T}{1 - \beta} + \frac{(\beta c - \beta^T c^T)}{1 - \beta c} + F. \quad (11)$$



The socially optimal term length ( $T^*$ ) can be determined by considering the timing of appointing a new policy-maker in light of the discounted loss function over the infinite time-horizon,  $G(T)$ . The global loss function writes as:

$$\begin{aligned} G(T) &= \Lambda(T) + \beta^T \Lambda(T) + \beta^{2T} \Lambda(T) + \dots \\ &= \Lambda(T) + \beta^T \{ \Lambda(T) + \beta^T \Lambda(T) + \beta^{2T} \Lambda(T) + \dots \}. \end{aligned} \quad (12)$$

This can be rewritten as:

$$G(T) = \Lambda(T) + \beta^T G(T),$$

and, therefore:

$$G(T) = \frac{1}{1 - \beta^T} \cdot \Lambda(T). \quad (13)$$

Since the value of (13) has to be minimized in order to define the socially optimal term length in the absence of overconfidence,  $T^*$ , we impose the condition,  $\frac{\partial G(T)}{\partial T} = 0$ , which gives:<sup>3</sup>

$$\begin{aligned} &\frac{1}{(1 - \beta^{T^*})^2} \left[ \frac{\lambda(\lambda + \kappa^2) \sigma_\varepsilon^2 (1 - \beta^T)}{2(\lambda(1 - \beta\rho) + \kappa^2)^2 (1 - \beta)} + \frac{(\beta - \beta^{T^*} c^{T^*})}{1 - \beta c} + F \right] \beta^{T^*} \ln \beta \\ &- \frac{1}{1 - \beta^{T^*}} \left[ \frac{\lambda(\lambda + \kappa^2) \sigma_\varepsilon^2 \beta^{T^*} \ln \beta}{2(\lambda(1 - \beta\rho) + \kappa^2)^2 (1 - \beta)} + \frac{(\beta c \ln \beta - \beta^{T^*} c^{T^*} \ln c)}{1 - \beta c} \right] = 0, \end{aligned} \quad (14)$$

which must hold for the optimal term length,  $T^*$ .

### 3.2 The Case with Overconfidence

Overconfidence emerges, starting from the second period in office, as we suppose that in the first period of his appointment a central banker does not exhibit an overconfidence bias.<sup>4</sup> Furthermore, we consider that overconfidence sets in from the second period in office and remains as long as he/she assumes the position. Thus, in the first period, a newly appointed central banker is completely free from the behavioral bias, but he/she will be afflicted with overconfidence after this initial period.

<sup>3</sup>For simplicity, we ignore the integer problem, as in Waller and Walsh (1996) and Lin (1999).

<sup>4</sup>Thus, we basically consider that the overconfidence in this case is something one acquires after he/she spends a certain period of time in office and not a trait one carries over from a previous position.

For each newly appointed central banker, the social inter-temporal loss can be written as a function of the legal mandate ( $T$ ). In particular, we write the loss of one term that covers the length of  $T$  periods as:

$$\Lambda^{OC}(T) = L^{App} + F + \beta(L^{OC} + c) + \beta^2(L^{OC} + c^2) + \dots + \beta^{T-1}(L^{OC} + c), \quad (15)$$

where the superscripts respectively denote the appointment period ( $App$ ) with  $L^{App} \equiv L^*$  in (5) as its period-wise loss value, and the subsequent periods ( $OC$ ), where overconfidence exists. This can be re-expressed as:

$$\Lambda^{OC}(T) = L^{App} + F + \sum_{t=1}^{T-1} \beta^t (L^{OC} + \eta c), \quad (16)$$

or:

$$\begin{aligned} \Lambda^{OC}(T) &= \frac{1}{2} \left( \frac{\lambda}{[\lambda(1 - \beta\rho) + \kappa^2]^2} \right) (\lambda + \kappa^2) \sigma_\varepsilon^2 + F \\ &+ \left[ \frac{1}{2} \left( \frac{\lambda}{[\lambda(1 - \beta\bar{\rho}) + \kappa^2]^2} \right) (\lambda + \kappa^2) \frac{\sigma_\varepsilon^2}{\gamma} \right] \frac{\beta - \beta^T}{1 - \beta} + \frac{(\beta c - \beta^T c^T)}{1 - \beta c}. \end{aligned} \quad (17)$$

Just as in the previous subsection, the global loss function is written as (13). Combining the global loss function with (17) and differentiating with respect to  $T$ , we obtain the value of the optimal term length for this overconfidence case,  $T^{**}$ , which must satisfy:

$$\begin{aligned} \frac{1}{(1 - \beta^{T^{**}})^2} \left\{ \left[ \frac{\lambda(\lambda + \kappa^2) \sigma_\varepsilon^2}{2} \left[ \frac{1}{(\lambda(1 - \beta\rho) + \kappa^2)^2} + \frac{\beta - \beta^{T^{**}}}{\gamma(\lambda(1 - \beta\bar{\rho}) + \kappa^2)^2(1 - \beta)} \right] + F \right. \right. \\ \left. \left. + \frac{(\beta c - \beta^{T^{**}} c^{T^{**}})}{1 - \beta c} \right] \beta^{T^{**}} \ln \beta \right\} - \frac{\lambda(\lambda + \kappa^2) \sigma_\varepsilon^2 \beta^{T^{**}} \ln \beta}{2\gamma(\lambda(1 - \beta\bar{\rho}) + \kappa^2)^2(1 - \beta)(1 - \beta^{T^{**}})} \\ + \frac{(-\beta^{T^{**}} c^{T^{**}} \ln \beta - \beta^{T^{**}} c^{T^{**}} \ln c)}{(1 - \beta c)(1 - \beta^{T^{**}})} = 0. \end{aligned} \quad (18)$$

Thus, the equation (18) implicitly determines the value of  $T^{**}$ .

Having obtained (14) and (18), we are now ready to obtain comparative statics results with respect to some exogenous variables.

## 4 Comparative Statics Analysis

In order to obtain comparative statics results, we rely on the implicit function theorem, applied to (14) or (18), depending on the presence of overconfidence. Let us denote (14)

and (18) as  $Z = 0$  and  $Z^{OC} = 0$ , respectively. As we focus on the range of the term length which is optimally defined by these first-order conditions, we make use of the second-order conditions for loss minimization, and basically examine the signs of the partial derivatives of the relevant first-order conditions to see how each exogenous variable affects the optimal term length in these two respective cases. First, we look at the impact of each type of bias individually.

**Lemma 1.** *An increase in the the degree of the “illusion of the precision”-type overconfidence of a central banker increases the duration of the socially optimal term length.*

*Proof.* The sign of the partial derivative of  $Z^{OC}$  with respect to  $\gamma$  is:

$$\frac{\partial Z^{OC}}{\partial \gamma} = \frac{\lambda^2 (\lambda + \kappa^2) \sigma_\varepsilon^2 \ln \beta \cdot \beta^{T^{**}}}{2\gamma^2 (-\lambda + \lambda\beta\bar{\rho} - \kappa^2)^2 (\beta^{T^{**}} - 1)^2} < 0. \quad (19)$$

At  $T^{**}$ , the second-order condition for minimization of the loss, i.e.,  $\frac{\partial Z^{OC}}{\partial T^{**}} > 0$ , must hold. Hence, we can obtain the following:

$$\frac{dT^{**}}{d\gamma} = -\frac{\frac{\partial Z^{OC}}{\partial \gamma}}{\frac{\partial Z^{OC}}{\partial T^{**}}} > 0. \quad (20)$$

**Q.E.D.**

As the greater “illusion of precision”-type overconfidence lowers the one-period loss value as in (9), an increase in  $\gamma$  actually raises the benefit of keeping an incumbent central banker.

The next result is related to the “illusion of control”-type of overconfidence.

**Lemma 2.** *An increase in the the degree of the “illusion of the control”-type overconfidence of a central banker decreases the duration of the socially optimal term length.*

*Proof.* The sign of the partial derivative of  $Z^{OC}$  with respect to  $\bar{\rho}$  is:

$$\frac{\partial Z^{OC}}{\partial \bar{\rho}} = -\frac{\lambda^2 (\lambda + \kappa^2) \sigma_\varepsilon^2 \ln \beta \cdot (-\beta^{T^{**}+1})}{\gamma (-\lambda + \lambda \beta \bar{\rho} - \kappa^2)^3 (\beta^{T^{**}} - 1)^2} > 0. \quad (21)$$

Then, similarly to the above lemma, we can obtain the following:

$$\frac{dT^{**}}{d\bar{\rho}} = -\frac{\frac{\partial Z^{OC}}{\partial \bar{\rho}}}{\frac{\partial Z^{OC}}{\partial T^{**}}} < 0. \quad (22)$$

**Q.E.D.**

As can be expected from the one-period loss value of (9), an increase in the value of  $\bar{\rho}$  leads to a shorter optimal term length. More frequent replacement of central bankers becomes more attractive as the loss value from the second period onward increases due to this type of overconfidence. This is a quite contrasting result to an increase in the degree of the “illusion of precision”-type overconfidence. Thus, when the two types of typical overconfidence to a central banker both increase or decrease at the same time, we must be aware of the relative magnitudes of these changes in determining if an incumbent policy-maker should be kept in the office a little longer or not.

In order to answer this question more precisely, we focus on a case where an increase in overconfidence of a central banker implies the increases of the values of  $\gamma$  and  $\bar{\rho}$  by the same percentages. Under this supposition, we can derive the following clear-cut proposition:

**Proposition 1.** *When both types of overconfidence increase by the same magnitudes, the legal mandate should be lengthened.*

*Proof.* In order to make a comparison between the effects of the changes due to two different types of overconfidence, we first convert the derivatives in (19) and (21) into the respective elasticities,  $\eta_\gamma$  and  $\eta_{\bar{\rho}}$ :

$$\eta_\gamma = \frac{\partial T^{**}}{\partial \gamma} \cdot \frac{\gamma}{T^{**}} = \left( -\frac{\frac{\partial Z^{OC}}{\partial \gamma}}{\frac{\partial Z^{OC}}{\partial T^{**}}} \right) \cdot \frac{\gamma}{T^{**}}, \quad (23)$$

$$\eta_{\bar{\rho}} = \frac{\partial T^{**}}{\partial \bar{\rho}} \cdot \frac{\bar{\rho}}{T^{**}} = \left( -\frac{\frac{\partial Z^{OC}}{\partial \bar{\rho}}}{\frac{\partial Z^{OC}}{\partial T^{**}}} \right) \cdot \frac{\bar{\rho}}{T^{**}}, \quad (24)$$

Then, the combined effect,  $\Omega$ , can be measured by the sum of these two elasticities:

$$\Omega = \eta_{\gamma} + \eta_{\bar{\rho}} = \left( -\frac{1}{\frac{\partial Z^{OC}}{\partial T^{**}} \cdot T^{**}} \right) \cdot \left( \frac{\partial Z^{OC}}{\partial \gamma} \cdot \gamma + \frac{\partial Z^{OC}}{\partial \bar{\rho}} \cdot \bar{\rho} \right). \quad (25)$$

Using the expressions in (19) and (21), we can get:

$$\Omega = \left( -\frac{1}{\frac{\partial Z^{OC}}{\partial T^{**}} \cdot T^{**}} \right) \cdot \left( \frac{\lambda^2 (\lambda + \kappa^2) \sigma_{\varepsilon}^2 \ln \beta \cdot \beta^{T^{**}}}{2\gamma (-\lambda + \lambda\beta\bar{\rho} - \kappa^2)^2 (\beta^{T^{**}} - 1)^2} \right) \cdot \left( 1 - \frac{\beta\bar{\rho}}{2\{\lambda(\beta\bar{\rho} - 1) - \kappa^2\}} \right), \quad (26)$$

which is always positive for the loss-minimizing  $T$ , i.e.,  $T^{**}$ . **Q.E.D.**

This proposition implies that, when the magnitudes of  $\gamma$  and  $\bar{\rho}$  increase by the same percentages, it is necessarily the case in our framework that such a increase in overconfidence calls for a longer term length. Therefore, if the relative significance of the two types of overconfidence is unchanged, the optimal term length should always be increased when an incumbent central banker becomes more overconfident, irrespective of the exact values of the other variables.

Additionally, we have the following comparative statics result concerning the actual persistence of the output gap in the economy.

**Proposition 2.** *As the output becomes more persistent, the optimal term length should be extended only under overconfidence.*

*Proof.* The signs of the partial derivatives of  $Z$  and  $Z^{OC}$  with respect to  $\rho$  are respectively:

$$\frac{\partial Z}{\partial \rho} = 0, \quad (27)$$

$$\frac{\partial Z^{OC}}{\partial \rho} = -\frac{\lambda^2 (\lambda + \kappa^2) \sigma_{\varepsilon}^2 \beta^{1+T^{**}} \ln \beta}{(-1 + \beta^{T^{**}})^2 (-\lambda + \lambda\beta\rho - \kappa^2)^3} < 0. \quad (28)$$

Thus, we can obtain the followings:

$$\frac{dT^*}{d\rho} = -\frac{\frac{\partial Z}{\partial \rho}}{\frac{\partial Z}{\partial T^*}} = 0, \quad (29)$$

$$\frac{dT^{**}}{d\rho} = -\frac{\frac{\partial Z^{OC}}{\partial \rho}}{\frac{\partial Z^{OC}}{\partial T^{**}}} > 0. \quad (30)$$

**Q.E.D.**

While the increase in  $\rho$  does not affect  $T^*$  without the “illusion of control”-type overconfidence, this increases the optimal term length in the presence of the overconfidence because this change in fact lowers the cost of having an overconfident central banker in office by reducing the gap between  $\bar{\rho}$  and  $\rho$ . This effectively reduces the degree of overconfidence.<sup>5</sup> Indeed, as  $\bar{\rho}$  is strictly less than one, even the value of  $\bar{\rho}$  for the most overconfident central banker is bounded from above. This also raises the issue of considering how the optimal term length had been set initially. If it is determined by taking the effect of this overconfidence into account, the legal mandate should be lengthened as the output shock persists in a greater scale.

## 5 Concluding Remarks

Our results indicate that the overconfidence biases of a central banker can have important qualitative impacts on the performance of an economy, which justifies the reassessment of the optimal term length of a central banker. In a forward-looking framework, the “illusion of control”-type of overconfidence has a lasting harmful impact. On the contrary, the “illusion of precision”-type overconfidence has a beneficial effect by inducing lower volatilities in output and inflation levels. Our results show that concurrent increases in these two types of behavioral biases by the same percentages should always induce a longer optimal term length in our framework.

In this paper, we exclusively dealt with a single policy-maker. In reality, though, almost all the major central banks have adopted committee systems in making important

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<sup>5</sup>This argument does not work if  $\bar{\rho}$  is pushed up proportionally as  $\rho$  increases.

monetary decisions. It would be interesting to examine the effect of overconfidence of the sorts we analyzed here in a richer organizational context.

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