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Task Assignment under Agent Loss Aversion*

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Abstract

We analyze a simple task-assignment model in which a principal assigns a task to one of two agents depending on the state. If the agents have standard concave utility, the principal assigns the task to an agent with the highest productivity in each state. In contrast, if the agents are loss averse, in order to alleviate their expected losses the principal may assign the task to a single agent in all states. Furthermore, the optimal contract may specify the same effort level across states. Our results imply that such simple contracts can be optimal even when employers can write contingent contracts at no cost.

JEL Classification Numbers: D03, D86, M12, M52

Keywords: task assignment, loss aversion, reference-dependent preferences

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1 Introduction

Assigning a task to an employee who is most appropriate for implementing the task is a major determinant of firm performance. This issue can be even more important when a task requires a different skill depending on uncertain future states. According to contract theory, when a principal does not face any asymmetric-information problem, she should offer a contingent contract where she assigns a task to an agent whose productivity is the highest in each state.[1] In working environments, however, a task is often assigned to a particular agent regardless of situations even if such a contingent contract is possible. We investigate this issue by incorporating a prominent behavioral aspect, *loss aversion*: people are more sensitive to losses than to same-sized gains. This paper analyzes a simple task-assignment problem in which future states are uncertain and agents have expectation-based loss aversion à la Kőszegi and Rabin (2006, 2007). The utility of each agent depends not only on material payoffs, such as wages and effort costs, but also on psychological gain-loss payoffs from comparing his realized outcome with his expected outcomes.

In our model, a risk- and loss-neutral principal assigns a task to two agents, A and B. We suppose that agent A is more productive than agent B in state 1 whereas agent A is less productive in state 2. Both the principal and agents are uncertain regarding the state at the contracting stage. However, the principal can write a complete contract that specifies wages, which agent works on the task, and his effort level depending on the state. Then the state is realized, and the contract is implemented. We call a contract *state-independent* if the principal assigns the task to a single agent in all states; otherwise, we call it *state-dependent*. When agents are not loss averse, the optimal contract is always state-dependent. In contrast, we show that when agents are loss averse, state-independent contracts can become optimal. The intuition is as follows. Since a less productive agent works in some state, a state-independent contract is less efficient in terms of productivity than a state-dependent contract. On the other hand, a state-independent contract reduces the principal's wage payment by alleviating the expected losses of an agent who exerts effort in one state but not in another state. Thus, a trade-off between improving productivity and alleviating expected losses arises, and the state-independent contract is optimal if the latter effect outweighs. We also show that when the degree of loss aversion is large, the optimal state-independent contract specifies the same effort levels in both states to eliminate the expected losses. This result is in sharp contrast with the standard concave utility case where the principal specifies different effort levels as long as the productivities of the agents are different. This may help explain, for example, why fixed working-hour contracts are so popular even when employers can adjust the working hours of their employees depending on the market environment.

[1] We use the male pronoun to refer to agents and the female pronoun to refer to the principal.

Our result is also qualitatively different from the result of models with cost complementarity for assignments. Although these models could explain why the principal assigns a task to a single agent, we predict that the principal is more likely to specify the same effort levels across states when a single agent works on the task in all states than when different agents do depending on the state.

This paper belongs to the literature on contract theory and mechanism design where agents have expectation-based reference-dependent preferences. The study by Herweg and Mierendorff (forthcoming) is most closely related to ours. They build a model of firm pricing where consumers are expectation-based loss averse and uncertain about their future demands, and show the optimality of flat-rate pricing. Heidhues and Kőszegi (2005, 2008) also derive sticky and focal pricing under expectation-based loss aversion. Similar to our results, these papers show the optimality of state-independent policies. However, each of these studies focuses only on a particular state-independent policy whereas we focus on two types of state-independent contracts and find both of them can be optimal depending on situations.[2]

Herweg and Schmidt (2012) investigate a model of renegotiation under loss aversion by assuming that—as developed by Hart and Moore (2008)—two parties write a deterministic initial contract, and the initial contract serves as a reference point at the renegotiation stage. They show that the renegotiated outcome becomes sticky and materially inefficient, and derive implications to hold-up problems.[3] While their paper and ours are independently conducted and the settings are different, our work can be interpreted as a complement of their paper: sticky and materially inefficient contracts can be optimal even when a principal can write a detailed contract in the initial stage. Moreover, we show that the state-independent contracts can be optimal under expectation-based reference-dependent preferences.

Some papers on incomplete contracts also show the optimality of state-independent contacts.[4] With regard to the difference, we show that state-independent contracts can be optimal even when our setting has no factor of incomplete contracts often considered in the literature.[5] In our model, the optimality of state-independent contracts is solely coming from an agent's risk preferences. Our result provides a new insight for state-independent contracts due to risk preferences rather than due to incomplete contracting.

The remainder of the paper is organized as follows. Section 2 sets up the model, explains the expectation-

[2]A recent literature analyzes moral-hazard problems with agent loss aversion (Daido and Itoh 2007; Herweg, Müller and Weinschenk 2010; Macera 2011; Daido and Murooka 2011). Lange and Ratan (2010), Hahn, Kim, Kim, and Lee (2010), Carbajal and Ely (2012), and Eisenhuth (2012) examine screening and auction problems. In contrast to them, we highlight that even when a principal does not face any asymmetric-information problems, loss aversion can have a crucial role in contracting.

[3]Relatedly, Mukerji (1998) shows that an incomplete contract may be optimal when an agent has ambiguity beliefs in the setting of a hold-up problem.

[4]See, for example, Tirole (2009).

[5]Tirole (1999) indicates out three possible factors that make a contract incomplete: (i) unforeseen contingencies, (ii) cost of writing contracts, and (iii) cost of enforcing contracts.

based reference-dependent preferences, and introduces the solution concept of the *choice-acclimating personal equilibrium* (CPE). Section 3 characterizes the optimal contract and provides the results of comparative statics. Section 4 concludes.

2 The Model

2.1 Setup

We consider a model in which one risk- and loss-neutral principal assigns a task to one of two agents.[6] All of them are uncertain about the future state at the contracting stage. There are two states, $s = 1, 2$, and one of the states is realized after contracting. State 1 (resp. state 2) is realized with probability $q \in (0, 1)$ (resp. $1 - q$). The value of the task depends on the state, and the principal can write a contract that specifies the task assignment *contingent on the state*. Agent $i = A, B$ works on the task if and only if the principal assigns the task to him, and only one agent can work on the task in each state. The agent who is in charge of the task exerts effort $e \in \mathbb{R}_+$ with an effort cost $c(e) = e^2/2$.[7] If agent A (resp. agent B) is assigned to the task in state $s \in \{1, 2\}$ and exerts effort e_s^A (resp. e_s^B), the principal earns $\alpha_s e_s^A$ (resp. $\beta_s e_s^B$) from the task. Assume that $\alpha_1 > \beta_1 > 0$ and $\beta_2 > \alpha_2 > 0$: the productivity of agent A is higher (resp. lower) than that of agent B in state 1 (resp. state 2).

Since our focus is not on moral hazard issues, we consider a case in which *the effort level in each state is contractible*. The principal offers a contract that specifies a wage scheme to each agent depending on the state $w = (w_1^A, w_2^A, w_1^B, w_2^B)$, the effort level in each state $e = (e_1, e_2)$, and which agent works on the task contingent on the state.[8] The states that agent A works on the task is denoted by $D \in \mathbb{D} \equiv \{\varnothing, \{1\}, \{2\}, \{1, 2\}\}$; for example, $D = \{1\}$ means agent A works on the task in state 1 but agent B works in state 2. The contract is denoted by $C(w, e; D) \in \mathbb{R}^4 \times \mathbb{R}_+^2 \times \mathbb{D}$. Each agent accepts the contract if his expected utility is larger than or equal to his reservation utility, which is assumed to be zero. A task assignment is *state-independent* if the principal assigns the task to one agent in both states; otherwise it is *state-dependent*. We call a contract *full assignment to agent* i if the principal adopts a state-independent contract and assigns the task to agent i in both states.

The timing is given below:

[6] Although we choose this multi-agent setup for practical interpretations, none of our results relies on the multi-agent structure. For example, our main messages hold in an alternative single-agent model where a principal either delegates the task to an agent or not depending on the state.

[7] None of our results will qualitatively change even when we adopt effort cost functions that varies across agents. If the effort cost depends on the state, then the principal may not specify the same effort level across states. However, even in that case the principal will still assign the task to a single agent in both states under agent loss aversion.

[8] Note that for each state, an agent who is not in charge of the task exerts zero effort.

1. The principal offers a contract to the agents.

2. Each agent chooses whether to accept the contract.

3. The state is realized.

4. The task assignment, the effort provision, and the payment are carried out according to the contract.

2.2 Reference-Dependent Preferences

A key assumption of our model is that each agent's overall utility comprises intrinsic consumption utility and psychological gain-loss utility. We assume that each agent has expectation-based reference-dependent preferences à la Kőszegi and Rabin (2006, 2007). In our model, the agents have two consumption dimensions: wage and effort. For each consumption dimension, the agents feel a psychological gain or loss by comparing a realized outcome with reference outcomes. For deterministic reference points, denote each agent's reference point for his wage and his effort cost by $\hat{w}$ and $\hat{e}$, respectively. If his actual wage and effort are w and e, then his overall utility is given by:

$$w - e + \mu(w - \hat{w}) + \mu(-e + \hat{e}),$$

where $\mu(\cdot)$ is a gain-loss function that corresponds to Kahneman and Tversky's (1979) value function. In what follows, we assume that $\mu(\cdot)$ is piecewise linear to focus on the effect of loss aversion. Then, we can simply define the gain-loss function when consumption is x and the reference point is r as

$$\mu(x - r) = \begin{cases} \eta(x - r) & \text{if } x - r \geq 0, \\ \eta\lambda(x - r) & \text{if } x - r < 0. \end{cases}$$

where $\eta \geq 0$ represents the weight given to the gain-loss payoff and $\lambda \geq 1$ is the degree of loss aversion. The agent is loss-neutral when $\lambda = 1$. For ease of notation, we set $\eta = 1$ hereafter.[9]

Following Kőszegi and Rabin (2006, 2007), we assume that the reference point is determined by rational beliefs regarding outcomes and that the reference point itself is stochastic if the outcome is stochastic. Each agent feels a gain-loss by comparing every possible outcome with every reference point. For example, suppose that the principal assigns the task to agent i in $s = 1$ but not in $s = 2$ and pays a fixed wage w^i. Then, the agent's expected gain-loss utility is a combination of the following four cases. First, agent i has expected to incur effort cost $c(e_1)$ with probability q, and $s = 1$ is realized and he actually incurs the cost with

[9] We can set $\eta = 1$ without loss of generality if we adopt the equilibrium condition introduced below (CPE) because η and λ are not jointly separable under CPE. However, this is not true if we adopt the Preferred Personal Equilibrium (PPE), which is another equilibrium concept introduced by Kőszegi and Rabin (2006, 2007). Although additional technical complications arise in the analysis, our main results hold under PPE.

probability q. There is no expected gain-loss in this case. Second, agent i has expected to incur effort cost $c(e_1)$ with probability q, but $s = 2$ is realized and he does not incur the cost with probability $1 - q$. This leads to an expected gain of $q(1 - q)c(e_1)$. Third, agent i has not expected to incur effort cost $c(e_1)$ with probability $1 - q$, but $s = 1$ is realized and he incurs the cost with probability q. This causes the expected loss of $\lambda q(1 - q)c(e_1)$. Fourth, agent i has not expected to incur effort cost $c(e_1)$ with probability $1 - q$, and $s = 2$ is realized and he does not incur the cost with probability $1 - q$. There is no expected gain-loss in this case. Ex-ante the agent correctly anticipates all the above four cases, and his expected gain-loss utility in the effort dimension is $-q(1 - q)(\lambda - 1)c(e_1)$. Because the loss of $c(e_1)$ looms larger than the gain of $c(e_1)$, the expected gain-loss utility is negative. The expected gain-loss utility in the wage dimension is zero because the agent anticipates fixed wage w and actually gets it.

Formally, given $C(w, e; D)$ let $\mathbf{1}_s^i$ be the indicator function that takes one if agent i incurs effort cost in state s and takes zero otherwise. We denote agent i's belief of D, e, and $\mathbf{1}_s^i$ by $\hat{D}$, $\hat{e}$, and $\hat{\mathbf{1}}_s^i$, respectively. Agent i's reference point is his expectation regarding which state will be realized and whether to implement the task in the realized state. The expected utility of agent i in our model is represented by

$$
\begin{aligned}
U^i(w, e; D|\hat{w}, \hat{e}; \hat{D}) = & qw_1^i + (1-q)w_2^i - \mathbf{1}_1^i qc(e_1) - \mathbf{1}_2^i(1-q)c(e_2) \\
& + q^2\mu(w_1^i - \hat{w}_1^i) + q(1-q)\mu(w_1^i - \hat{w}_2^i) + (1-q)q\mu(w_2^i - \hat{w}_1^i) + (1-q)^2\mu(w_2^i - \hat{w}_2^i) \\
& + q^2\mu\left(-\mathbf{1}_1^i c(e_1) + \hat{\mathbf{1}}_1^i c(\hat{e}_1)\right) + q(1-q)\mu\left(-\mathbf{1}_1^i c(e_1) + \hat{\mathbf{1}}_2^i c(\hat{e}_2)\right) \\
& + (1-q)q\mu\left(-\mathbf{1}_2^i c(e_2) + \hat{\mathbf{1}}_1^i c(\hat{e}_1)\right) + (1-q)^2\mu\left(-\mathbf{1}_2^i c(e_2) + \hat{\mathbf{1}}_2^i c(\hat{e}_2)\right).
\end{aligned} \tag{1}
$$

We derive the optimal contract based on the equilibrium concept defined by Kőszegi and Rabin (2007): the *choice-acclimating personal equilibrium* (CPE). This equilibrium concept is plausible when agents accept a contract long before they actually exert effort. Because each agent knows that his beliefs will be acclimated to his accepted contract before he actually chooses his action, he takes this change into account when he decides whether to accept a contract. Thus, agent i's accepted contract itself determines his reference points under CPE, and the condition for accepting a contract $C(w, e; D)$ under CPE is represented by

$$U^i(w, e; D|w, e; D) \geq 0. \tag{CPE-IR}$$

Condition (CPE-IR) implies that the agent's utility when his reference point is D and he actually does the task according to D is no less than when he expected to decline the contract and actually does. According to Equation (1), Condition (CPE-IR) is represented as follows:

$$\underbrace{qw_1^i + (1-q)w_2^i - \mathbf{1}_1^i qc(e_1) - \mathbf{1}_2^i(1-q)c(e_2)}_{\text{intrinsic utility}}$$
$$-\underbrace{q(1-q)(\lambda-1)\left(|w_1^i - w_2^i| + |\mathbf{1}_1^i c(e_1) - \mathbf{1}_2^i c(e_2)|\right)}_{\text{gain-loss utility}} \geq 0.$$

3 Analysis

3.1 The Optimal Contract without Loss Aversion

First, we study the standard case where each agent is loss neutral ($\lambda = 1$). We denote the effort level by which agent A (resp. B) maximizes social surplus in state s by $\bar{e}_s^A \equiv \text{argmax}_e\{\alpha_s e - c(e)\}$ (resp. $\bar{e}_s^B \equiv \text{argmax}_e\{\beta_s e - c(e)\}$). Note that $\bar{e}_1^A = \alpha_1, \bar{e}_2^A = \alpha_2$, $\bar{e}_1^B = \beta_1$, and $\bar{e}_2^B = \beta_2$.

Since the principal can maximize social surplus without considering the agents' feeling of losses, she assigns the task to agent A in state 1 and to agent B in state 2. The optimal wages, denoted by $\bar{w}$, are simply determined by $q\bar{w}_1^A + (1-q)\bar{w}_2^A = qe_1^2/2$ and $q\bar{w}_1^B + (1-q)\bar{w}_2^B = (1-q)e_2^2/2$ from Condition (CPE-IR) with equality. As a result, the principal's expected payoff in the optimal contract can be written as:

$$\pi(\bar{w}, e; \{1\}) = q\left(\alpha_1 e_1 - \frac{e_1^2}{2}\right) + (1-q)\left(\beta_2 e_2 - \frac{e_2^2}{2}\right).$$

Let $\bar{e}$ denote the optimal effort level. It is straightforward that $\bar{e}_1 = \bar{e}_1^A = \alpha_1$ and $\bar{e}_2 = \bar{e}_2^B = \beta_2$. The following proposition summarizes the key result in this benchmark case:

Proposition 1. Suppose the agents are loss neutral. The state-dependent contract $C(\bar{w}, \bar{e}; \{1\})$ is optimal.

When each agent is not loss averse, the task assignment is determined by which the agent maximizes the social surplus in each state. As a result, state-independent contracts are never optimal in our setting. Further, the effort levels specified in the optimal contract vary across states. This is because agents' productivity depends on the state; even if full assignment to agent A was optimal, the principal would still specify different effort levels across states.

It is worth emphasizing that state-independent contracts are never optimal even if the agent's consumption utility is concave. Let $u(\cdot)$ be a function that is strictly increasing, concave, and $u(0) = 0$. First, if the agent has concave consumption utility on income that is separable from the effort cost, $u(w) - c(e)$, then the principal assigns the task to agent A if and only if state 1 is realized in the optimal contract. However, the optimal wages in this case are modified by the inverse of the utility function, and a constant wage across states becomes optimal: $\bar{w}_1^A = \bar{w}_2^A = u^{-1}(qc(\bar{e}_1))$ and $\bar{w}_1^B = \bar{w}_2^B = u^{-1}((1-q)c(\bar{e}_2))$. Second, if the agent has concave consumption utility with a unitary consumption dimension, $u(w - c(e))$, then in the optimal contract the principal assigns the task to agent A if and only if state 1 is realized. Each agent obtains a positive wage if and only if he actually works on the task: $\bar{w}_1^A = c(\bar{e}_1), \bar{w}_2^A = \bar{w}_1^B = 0$, and $\bar{w}_2^B = c(\bar{e}_2)$.

3.2 The Optimal Contract with Loss Aversion

Next, we examine the case where each agent is loss averse ($\lambda > 1$). Lemma 1 shows that each agent obtains a constant wage across states in the optimal contract:

Lemma 1. Suppose the agents are loss averse. The optimal contract specifies that $w_1^{A*} = w_2^{A*}$ and $w_1^{B*} = w_2^{B*}$.

Lemma 1 is coming from the fact that the agents are averse to wage uncertainty due to loss aversion and the principal can completely eliminate wage uncertainty by offering a constant wage. Hence, without loss of generality we focus only on the contract with constant wage schemes across states: $w_s^i = w^i$ for all i and all s. For each task-assignment scheme D, the expected utility of agent A if he accepts a contract $C(w, e; D)$ becomes

$$\begin{aligned}
U^A(w, e; \varnothing | w, e; \varnothing) &= w^A, \\
U^A(w, e; \{1\} | w, e; \{1\}) &= w^A - q\frac{e_1^2}{2} - q(1-q)(\lambda - 1)\frac{e_1^2}{2}, \\
U^A(w, e; \{2\} | w, e; \{2\}) &= w^A - (1-q)\frac{e_2^2}{2} - q(1-q)(\lambda - 1)\frac{e_2^2}{2}, \\
U^A(w, e; \{1,2\} | w, e; \{1,2\}) &= w^A - q\frac{e_1^2}{2} - (1-q)\frac{e_2^2}{2} - q(1-q)(\lambda - 1)\frac{|e_1^2 - e_2^2|}{2}.
\end{aligned}$$

The expected utility of agent B can be described in the same manner. Note that in the optimal contract, Condition (CPE-IR) holds with equality; otherwise the principal can decrease the wage with holding Condition (CPE-IR). For each D, we denote the optimal fixed wage by $w^*(D)$ satisfying Condition (CPE-IR) with equality; we abbreviate the notation slightly in the following manner: $w^*(\varnothing) = w^*_\varnothing, w^*(\{1\}) = w_1^*, w^*(\{2\}) = w_2^*$, and $w^*(\{1,2\}) = w_{12}^*$. By substituting the optimal wage into the principal's payoff function in each case, we can represent the principal's payoff in each task-assignment scheme in the following way:

$$\pi(w^*_{\varnothing}, e; \varnothing) = q\beta_1 e_1 + (1-q)\beta_2 e_2 - q\frac{e_1^2}{2} - (1-q)\frac{e_2^2}{2} - q(1-q)(\lambda-1)\frac{|e_1^2 - e_2^2|}{2},$$
$$\pi(w^*_1, e; \{1\}) = q\alpha_1 e_1 + (1-q)\beta_2 e_2 - q\frac{e_1^2}{2} - (1-q)\frac{e_2^2}{2} - q(1-q)(\lambda-1)\frac{e_1^2 + e_2^2}{2},$$
$$\pi(w^*_2, e; \{2\}) = q\beta_1 e_1 + (1-q)\alpha_2 e_2 - q\frac{e_1^2}{2} - (1-q)\frac{e_2^2}{2} - q(1-q)(\lambda-1)\frac{e_1^2 + e_2^2}{2},$$
$$\pi(w^*_{12}, e; \{1,2\}) = q\alpha_1 e_1 + (1-q)\alpha_2 e_2 - q\frac{e_1^2}{2} - (1-q)\frac{e_2^2}{2} - q(1-q)(\lambda-1)\frac{|e_1^2 - e_2^2|}{2}.$$

It is straightforward that $D = \{2\}$ never becomes an optimal contract because it is always dominated by $D = \{1\}$ specifying the same effort levels as $D = \{2\}$; hence, we ignore this case in the following account. By solving the principal's problem in each case, we derive the optimal effort levels for each task-assignment scheme, $(e_1^*(w^*(D)), e_2^*(w^*(D)))$:

Proposition 2. Suppose the agents are loss averse. Let $\bar{\lambda}_A = \frac{\alpha_1 + q(\alpha_1 - \alpha_2)}{\alpha_2 + q(\alpha_1 - \alpha_2)}$ and $\bar{\lambda}_B = \frac{\beta_2 + (1-q)(\beta_2 - \beta_1)}{\beta_1 + (1-q)(\beta_2 - \beta_1)}$.

(i) Under a state-independent task-assignment scheme $D = \{1,2\}$, $\bar{e}_1^A > e_1^*(w^*_{12}) > e_2^*(w^*_{12}) > \bar{e}_2^A$ where $e_1^*(w^*_{12}) = \frac{\alpha_1}{1+(1-q)(\lambda-1)}$ and $e_2^*(w^*_{12}) = \frac{\alpha_2}{1-q(\lambda-1)}$ if $\lambda < \bar{\lambda}_A$; $e_1^*(w^*_{12}) = e_2^*(w^*_{12}) = q\alpha_1 + (1-q)\alpha_2$ if $\lambda \geq \bar{\lambda}_A$.

(ii) Under a state-independent task-assignment scheme $D = \varnothing$, $\bar{e}_2^B > e_2^*(w^*_{\varnothing}) > e_1^*(w^*_{\varnothing}) > \bar{e}_1^B$ where $e_1(w^*_{\varnothing}) = \frac{\beta_1}{1-(1-q)(\lambda-1)}$ and $e_2(w^*_{\varnothing}) = \frac{\beta_2}{1+q(\lambda-1)}$ if $\lambda < \bar{\lambda}_B$; $e_1^*(w^*_{\varnothing}) = e_2^*(w^*_{\varnothing}) = q\beta_1 + (1-q)\beta_2$ if $\lambda \geq \bar{\lambda}_B$.

(iii) Under a state-dependent task-assignment scheme $D = \{1\}$, $e_1^*(w_1^*) = \frac{\alpha_1}{1+(1-q)(\lambda-1)} < \bar{e}_1^A$ and $e_2^*(w_1^*) = \frac{\beta_2}{1+q(\lambda-1)} < \bar{e}_2^B$.

Proposition 2 (i) and (ii) indicate that if the principal assigns the task to one agent in both states, then the difference in the effort levels across states is smaller than that of the case without loss aversion. Moreover, the effort levels are same across states when the degree of loss aversion is large. This is in sharp contrast with the loss-neutral case where the principal specifies different effort levels as the productivities are different. Intuitively, because the agent dislikes the effort-cost uncertainty at the first order due to loss aversion, the principal needs to compensate for the expected losses to make the agent accept the contract. As λ is larger, the benefit of alleviating expected losses by specifying the same effort levels becomes larger than that of improving productivity by specifying different effort levels. To explain this aspect clearly, consider the case of full assignment to agent A (i.e., $D = \{1,2\}$). In this case, $e_1^*(w^*_{12})$ moves downward from $\bar{e}_1^A$ and $e_2^*(w^*_{12})$ moves upward from $\bar{e}_2^A$ as λ increases. Furthermore, if λ is larger than or equal to $\bar{\lambda}_A$ then $e_1^*(w^*_{12})$ coincides with $e_2^*(w^*_{12})$ at $q\alpha_1 + (1-q)\alpha_2$ and the effort-cost uncertainty disappears. This result may help explain, for example, why fixed working-hour contracts are so popular even when employers can adjust the working hours of their employees contingent on situations.

Proposition 2 (iii) states that if the principal chooses a state-dependent task-assignment scheme, then the effort levels are lower than those in the loss-neutral case. In this task-assignment scheme, each agent works in one state but does not work in the other state. This uncertainty of the task assignment generates expected losses in the effort-cost dimension, and the principal needs to compensate for these losses. Hence, the principal has an incentive to reduce the amount of effort in each state in order to decrease expected losses. Indeed, the principal sets lower effort levels than the loss-neutral case for any $\lambda > 1$ because the benefit of reducing expected losses by decreasing effort levels from $\bar{e}_s^i$ is the first order whereas the social-surplus cost by changing the effort levels from $\bar{e}_s^i$ is the second order.

Next, we analyze the optimal contract for loss-averse agents. We focus on the case $\bar{\lambda}_A \geq \bar{\lambda}_B$; the other case can be derived by the same procedure. By substituting the optimal effort levels into the principal's profit function and comparing each of the profits, we obtain the following proposition:

Proposition 3. Suppose the agents are loss averse.

(i) Suppose $\lambda \geq \bar{\lambda}_A$. Then, the state-independent contract $C(w_{12}^*, e^*; \{1, 2\})$ is optimal if and only if

$$(q\alpha_1 + (1-q)\alpha_2)^2 \geq \frac{q\alpha_1^2}{1 + (1-q)(\lambda - 1)} + \frac{(1-q)\beta_2^2}{1 + q(\lambda - 1)} \tag{2}$$

and

$$q\alpha_1 + (1-q)\alpha_2 \geq q\beta_1 + (1-q)\beta_2 \tag{3}$$

hold. The state-independent contract $C(w_\varnothing^*, e^*; \varnothing)$ is optimal if and only if

$$(q\beta_1 + (1-q)\beta_2)^2 \geq \frac{q\alpha_1^2}{1 + (1-q)(\lambda - 1)} + \frac{(1-q)\beta_2^2}{1 + q(\lambda - 1)} \tag{4}$$

holds but (3) does not hold. Otherwise, the state-dependent contract $C(w_1^*, e^*; \{1\})$ is optimal.

(ii) Suppose $\bar{\lambda}_A > \lambda \geq \bar{\lambda}_B$. Then, the state-independent contract $C(w_{12}^*, e^*; \{1, 2\})$ is optimal if and only if

$$\frac{\alpha_2^2}{1 - q(\lambda - 1)} \geq \frac{\beta_2^2}{1 + q(\lambda - 1)} \tag{5}$$

and

$$\frac{q\alpha_1^2}{1 + (1-q)(\lambda - 1)} + \frac{(1-q)\alpha_2^2}{1 - q(\lambda - 1)} \geq (q\beta_1 + (1-q)\beta_2)^2 \tag{6}$$

hold. The state-independent contract $C(w_\varnothing^*, e^*; \varnothing)$ is optimal if and only if Inequity (4) holds but (6) does not hold. Otherwise, the state-dependent contract $C(w_1^*, e^*; \{1\})$ is optimal.

(iii) Suppose $\bar{\lambda}_B > \lambda$. Then, the state-independent contract $C(w^*_{12}, e^*; \{1,2\})$ is optimal if and only if Inequity (5) and

$$\frac{q\alpha_1^2}{1+(1-q)(\lambda-1)} + \frac{(1-q)\alpha_2^2}{1-q(\lambda-1)} \geq \frac{q\beta_1^2}{1-(1-q)(\lambda-1)} + \frac{(1-q)\beta_2^2}{1+q(\lambda-1)} \tag{7}$$

hold. The state-independent contract $C(w^*_{\varnothing}, e^*; \varnothing)$ is optimal if and only if

$$\frac{\beta_1^2}{1-(1-q)(\lambda-1)} \geq \frac{\alpha_1^2}{1+(1-q)(\lambda-1)} \tag{8}$$

holds but (7) does not hold. Otherwise, the state-dependent contract $C(w^*_1, e^*; \{1\})$ is optimal.

In contrast to Proposition 1, state-independent contracts can be optimal: the principal may assign the task to a single in all states. To see the intuition, note that on the one hand a state-independent contract is less efficient than a state-dependent contract in terms of productivity because a less productive agent works in some state. On the other hand, the state-independent contract reduces the principal's wage payment because it alleviates the expected losses of an agent who exerts effort in one state but not in another state. Hence, the trade-off between improving productivity and alleviating expected losses arises if agents are loss averse. The state-independent contract is optimal if the latter effect supersedes. In addition, as described in Proposition 2, when the degree of loss aversion is large the optimal state-independent contract specifies the same effort levels across states, because it eliminates expected losses for the agent who dislikes effort uncertainty at the first order due to loss aversion.

We summarize the comparative statics results below.

Proposition 4. Suppose agents are loss averse.

(i) State-independent contracts are more likely to be optimal as λ increases.

(ii) Under state-independent contracts, full assignment to A (resp.B) is more likely to be optimal as α_1, α_2 or q increases (resp. decreases), and as β_1 or β_2 decreases (resp. increases).

Proposition 4 (i) shows that state-independent contracts are more likely to be optimal as λ increases because such contracts alleviate the agents' expected losses. Related to this, state-independent contracts become optimal even when the degree of loss aversion is small if the difference in productivities across states is small. This implies that the existence of loss aversion can lead to qualitatively different economic outcomes even when λ is not large. Proposition 4 (ii) illustrates that when the principal chooses a state-independent contract, which agent works on the task depends on productivities and the distribution of future states: agent A is more likely to be in charge of the task when relative benefits are larger, that is, when α_1, α_2, or q are large and β_1 or β_2 are small.

4 Conclusion

This paper analyzed a task assignment model where the agents have expectation-based reference-dependent preferences à la Kőszegi and Rabin (2006, 2007) and future states are uncertain. We showed that state-independent contracts—the principal assigns a task to a single agent regardless of future states—can be optimal when agents are loss averse, although these contracts are not optimal when agents have a standard concave utility. State-independent contracts become optimal when the positive effect from alleviating expected losses for agents outweighs the negative effect from assigning a task to a less-productive agent in some state. We also found that the optimal state-independent contract specifies the same effort levels across states when the degree of loss aversion is large. Our model and results could be applicable to relevant issues related to labor contracts, such as task specialization vs. multitasking, uneven workload, work sharing, and over-time premium.

Appendix

Proof of Proposition 1

In the text. □

Proof of Lemma 1

Suppose: $w_1^i \neq w_2^i$, $i = A, B$, in the optimal contract. Then, by setting $\tilde{w}_1^i = \tilde{w}_2^i = qw_1^i + (1-q)w_2^i$, the principal can make agent i sign the contract by keeping the expected payment the same and relaxing Condition (CPE-IR)—a contradiction. □

Proof of Proposition 2

For each task-assignment scheme D, agent A's expected utility if he accepts contract $C(w, e; D)$ is described in the text; agent B's expected utility can be expressed in the same manner. Because Condition (CPE-IR) holds with equality in the optimal contract, given effort level and task assignment the optimal wages become:

$$\begin{aligned}
&w_{\varnothing}^{A*} = w_{12}^{B*} = 0,\\
&w_1^{A*} = w_2^{B*} = q\frac{e_1^2}{2} + q(1-q)(\lambda-1)\frac{e_1^2}{2},\\
&w_2^{A*} = w_1^{B*} = (1-q)\frac{e_2^2}{2} + q(1-q)(\lambda-1)\frac{e_2^2}{2},\\
&w_{12}^{A*} = w_{\varnothing}^{B*} = q\frac{e_1^2}{2} + (1-q)\frac{e_2^2}{2} + q(1-q)(\lambda-1)\frac{|e_1^2 - e_2^2|}{2}.
\end{aligned}$$

The optimal effort levels for maximizing the principal's payoff in each task assignment scheme (except for $D = \{2\}$) are given below.

For $D = \{1\}$,

$$e_1^*(w_1^*) = \frac{\alpha_1}{1 + (1-q)(\lambda - 1)} < \bar{e}_1^A,$$
$$e_2^*(w_1^*) = \frac{\beta_2}{1 + q(\lambda - 1)} < \bar{e}^B.$$

For $D = \{1, 2\}$, suppose first the principal specifies $e_1(w_{12}^*) \geq e_2(w_{12}^*)$. If $\lambda \geq 1 + \frac{1}{q}$, then the principal's payoff is increasing in e_2; hence $e_1^*(w_{12}^*) = e_2^*(w_{12}^*)$ should hold and

$$e_1^*(w_{12}^*) = e_2^*(w_{12}^*) = q\alpha_1 + (1-q)\alpha_2.$$

If $\lambda < 1 + \frac{1}{q}$, the first-order condition yields

$$e_1(w_{12}^*) = \frac{\alpha_1}{1 + (1-q)(\lambda - 1)},$$
$$e_2(w_{12}^*) = \frac{\alpha_2}{1 - q(\lambda - 1)}.$$

Note that $\frac{\alpha_1}{1+(1-q)(\lambda-1)} \geq \frac{\alpha_2}{1-q(\lambda-1)}$ if and only if $\lambda \leq \bar{\lambda}_A \equiv \frac{\alpha_1 + q(\alpha_1 - \alpha_2)}{\alpha_2 + q(\alpha_1 - \alpha_2)}$. Because

$$1 + \frac{1}{q} - \frac{\alpha_1 + q(\alpha_1 - \alpha_2)}{\alpha_2 + q(\alpha_1 - \alpha_2)} = \frac{\alpha_2}{q\{\alpha_2 + q(\alpha_1 - \alpha_2)\}} > 0,$$

the principal specifies the same effort level if and only if $\lambda \geq \frac{\alpha_1 + q(\alpha_1 - \alpha_2)}{\alpha_2 + q(\alpha_1 - \alpha_2)}$.

Next, we show that $D = \{1, 2\}$ with $e_1(w_{12}^*) < e_2(w_{12}^*)$ is never optimal. Suppose otherwise. It is straightforward that $e_1(w_{12}^*) = 0$ is never optimal; then, the principal's payoff is given by

$$\pi(w_{12}^*, e; \{1, 2\}) = q\alpha_1 e_1 + (1-q)\alpha_2 e_2 - q\frac{e_1^2}{2} - (1-q)\frac{e_2^2}{2} - q(1-q)(\lambda - 1)\frac{e_2^2 - e_1^2}{2}.$$

Note that

$$\frac{\partial \pi(w_{12}^*, e; \{1, 2\})}{\partial e_1} = q[\alpha_1 - \{1 - (1-q)(\lambda - 1)\}e_1],$$
$$\frac{\partial \pi(w_{12}^*, e; \{1, 2\})}{\partial e_2} = (1-q)[\alpha_2 - \{1 + q(\lambda - 1)\}e_2].$$

If $1 - (1-q)(\lambda - 1) < 0$, then the principal's payoff can increase its expected profits by simply increasing e_1. Suppose $1 - (1-q)(\lambda - 1) \geq 0$. By the first-order conditions, the optimal effort levels are $e_1(w_{12}^*) = \frac{\alpha_1}{1-(1-q)(\lambda-1)}$ and $e_2(w_{12}^*) = \frac{\alpha_2}{1+q(\lambda-1)}$. These lead that $e_1(w_{12}^*) \geq e_2(w_{12}^*)$ must hold—a contradiction.

For $D = \varnothing$, we can derive the result by the same procedure as the case $D = \{1, 2\}$, and the optimal effort levels are given by: if $\lambda \geq \bar{\lambda}_B$,

$$e_1(w_\varnothing^*) = e_2(w_\varnothing^*) = q\beta_1 + (1-q)\beta_2,$$

and if $\lambda < \bar{\lambda}_B$,

$$e_1(w^*_\varnothing) = \frac{\beta_1}{1-(1-q)(\lambda-1)}$$
$$e_2(w^*_\varnothing) = \frac{\beta_2}{1+q(\lambda-1)},$$

where $\bar{\lambda}_B = \frac{\beta_2+(1-q)(\beta_2-\beta_1)}{\beta_1+(1-q)(\beta_2-\beta_1)}$. □

Proof of Proposition 3

Substituting the optimal effort levels into principal's payoff in each task assignment scheme, we have:

$$\pi(w^*_\varnothing, e^*; \varnothing) = \begin{cases} \frac{(q\beta_1+(1-q)\beta_2)^2}{2} & \text{if } e_1^*(w^*_\varnothing) = e_2^*(w^*_\varnothing), \\ q\frac{\beta_1^2}{2(1-(1-q)(\lambda-1))} + (1-q)\frac{\beta_2^2}{2(1+q(\lambda-1))} & \text{if } e_1^*(w^*_\varnothing) < e_2^*(w^*_\varnothing), \end{cases}$$
$$\pi(w^*_1, e^*; \{1\}) = q\frac{\alpha_1^2}{2(1+(1-q)(\lambda-1))} + (1-q)\frac{\beta_2^2}{2(1+q(\lambda-1))},$$
$$\pi(w^*_{12}, e^*; \{1,2\}) = \begin{cases} \frac{(q\alpha_1+(1-q)\alpha_2)^2}{2} & \text{if } e_1^*(w^*_{12}) = e_2^*(w^*_{12}), \\ q\frac{\alpha_1^2}{2(1+(1-q)(\lambda-1))} + (1-q)\frac{\alpha_2^2}{2(1-q(\lambda-1))} & \text{if } e_1^*(w^*_{12}) > e_2^*(w^*_{12}). \end{cases}$$

The proposition directly follows from comparisons among above payoffs. □

Proof of Proposition 4

Comparative statics on $\lambda, \alpha_1, \alpha_2, \beta_1$, and β_2 are straightforward.

For the comparative statics on q, note that for the case $\lambda \geq \bar{\lambda}_A$, the left hand side of (3) is increasing in q and the right hand side is decreasing in q.

For the case $\bar{\lambda}_A > \lambda \geq \bar{\lambda}_B$, the right hand side of (6) is decreasing in q. The left hand side of (6) is increasing in q because its first derivative with respect to q is given by

$$\lambda\frac{\alpha_1^2}{(1+(1-q)(\lambda-1))^2} + (\lambda-2)\frac{\alpha_2^2}{(1-q(\lambda-1))^2} > 2(\lambda-1)\frac{\alpha_2^2}{(1-q(\lambda-1))^2} > 0,$$

where the first inequality follows from that $\frac{\alpha_1}{1+(1-q)(\lambda-1)} > \frac{\alpha_2}{1-q(\lambda-1)}$ if $\bar{\lambda}_A > \lambda$.

For the case $\bar{\lambda}_B > \lambda$, the left hand side of (7) is increasing in q as shown above. The right hand side of (7) is decreasing in q because its first derivative with respect to q is given by

$$-(\lambda-2)\frac{\beta_1^2}{(1-(1-q)(\lambda-1))^2} - \lambda\frac{\beta_2^2}{(1+q(\lambda-1))^2} < -2(\lambda-1)\frac{\beta_1^2}{(1-(1-q)(\lambda-1))^2} < 0,$$

where the first inequality follows from that $\frac{\beta_2}{1+q(\lambda-1)} > \frac{\beta_1}{1-(1-q)(\lambda-1)}$ if $\bar{\lambda}_B > \lambda$. □

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